Please fill in the following information.

THEN PUT YOUR EMAIL ADDRESS ON EVERY PAGE OF THE EXAM !

NAME**:** **SOLUTION**

Email (please put complete address):

Neatness counts. We will not grade what we cannot read.

Exam is worth 100 points. Please phrase your answer succinctly. Write your answer on the same page, on which the question is given. There are three questions and a total of five pages. Make sure you turn in all these pages.

Do not attempt to look at other students' work. Keep your answers to yourself.
Any sort of cheating will result in a zero grade.

Read and sign the statement below. Wait for instructions to start the examination before continuing to the next page.

"I signify that the work shown in this examination booklet is my own and that I have not received any assistance from other students nor given any assistance to other students."

_______________________________(Signature)

Q1. Binary Search Tree (25 points)

(a) (12 points) Draw the binary search tree that results from inserting the following keys into an initially empty tree, in the order given: 25, 18, 17, 29, 52, 75, 65, 33, 11, 13.

Answer:

25

18 **29**

17 **52**

11 **33** **75**

13 **65**

(b) (13 points) Give two permutations of the above keys (in Part a) that will result in a maximal height binary search tree. Is there a third permutation? If so, give it. If not, explain why not.

Answer:

1, 3, 7, 8, 15, 19, 33, 42, 55, 65
65, 55, 42, 33, 19, 15, 8, 7, 3, 1
There are others; simply delete the parent of an only child which is a leaf and then reinsert the deleted node.
1, 3, 7, 8, 15, 19, 33, 42, 65, 55
65, 55, 42, 33, 19, 15, 8, 7, 1, 3

Q2. Hashing (total points 45)

Suppose n records are stored in a hash table of size m with $m > n$ using chaining, and suppose that a good hash function is used so that the probability that a key is hashed into any of the m slots is $1/m$

(a) **(10 points)** For a particular slot in the chained hash table, what is probability that the slot is empty? **Give a brief reason for your answer.**

Answer: $(1-\frac{1}{m})^n$

(b) **(10 points)** Consider a hash table of size 7. For this table, assume that the quadratic probing based on the following rehash function

$$(k+j^2) \bmod 7$$

is used where k is the original hash value of an input key and j = 0, 1,2,3,4,….

Write down a four-element input sequence x_1, x_2, x_3, x_4
(where $0 <= x_i <= 6$) such that **no collision** occurs and the hash values map into the pattern of occupied (shaded) cells shown below (indices 1, 2, 4, and 6).

Elements							
Index	**0**	**1**	**2**	**3**	**4**	**5**	**6**

Answer: 1, 2, 4, 6

(c) (10 points) List one more sequence of four elements x_1, x_2, x_3, x_4 (where $0 <= x_i <= 6$) that maps into the shaded indices of part (b) and is not a permutation of the sequence that you have found in part (b). The total number of collisions in this new sequence should as minimum as possible. Provide the total number of collisions for this sequence.

Answer: **1 4 6 1**
#collisions = 1

(d) (15 points). Consider a table of size 13 where insertion of a key (k) is carried out using a double hashing, $h(k,i) = (h_1(k) + i\, h_2(k))\ mod\ 13$, where $i = 0, 1, 2, \ldots, 12$, with $h_1(k) = k\ mod\ 13$ and $h_2(k) = 1 + (k\ mod\ 11)$.

Write an ordered sequence of input keys stored in this table that results in the mapping as shown below. **Give your reasoning for full credit.**

Elements		92			82	241		85		157		63	
Index	0	1	2	3	4	5	6	7	8	9	10	11	12

Answer:

92, 82, 85, and 63 can come in any order. 241 and 157 are be mapped at the locations shown in figure. 241 can be re-mapped to location 5 after colliding with 85. Similarly, 157 can be re-mapped to location 9, after first colliding with 92 and then 241.

Q3. Dynamic Programming and Greedy Algorithms (30 points)

Consider the problem of minimizing the number of coins that amounts to a change of *x* cents, where $0.01 \le x \le 0.99$. Assume we have 4 quarters, 10 dimes, 20 nickels, and 100 pennies to make change with. Prove or disprove that this problem has the optimal substructure property. In case, the problem has such property, devise a recurrence in terms of C(*x*), representing the minimum number of coins needed for amount *x*.

Answer:

Suppose I am given an optimal amount of change for *x*, requiring only *m* coins. There is a subproblem of this solution that contains *m*–1 coins for the amount *x*-*y* where *y* represents the removal of the value of some coin from the change given. Suppose that we can come up with a change solution requiring fewer than *m*–1 coins for *x*–*y*, then we would be able to provide a solution smaller than *m* coins for our original problem, a contradiction. Hence, this problem does have the optimal substructure property.

$$C(x) = \begin{cases} 0 & if\ \ x = 0, \\ 1 + min(C(x-25), C(x-10), C(x-5), C(x-1)) & if\ \ x > 0, \end{cases}$$

Note also that if $C(x\text{-}d) < 0$, it should drop out of the minimum test. Assume that there is a test with each term of the form **if** $C(x\text{-}d) \ge 0$ **then** C(x-d) **else** ∞, where *d* is replaced by 25, 10, 5, or 1.