# ECE 608: Computational Models and Methods, Fall 2003 <br> Test \#2 <br> Wednesday, November 52003 

O Your exam should have 10 (ten) pages.
O Pages 8 and 10 are intentionally left blank.
O Write your name on this page and at least one other page.
O Closed book, closed notes.
O Switch off and put away cell-phones/pagers
O Apportion your time carefully.
O Numbers in brackets represent points for that question. Points add up to 100.

O Good luck.

| Prob. | Max. | Score |
| :---: | :---: | :---: |
| I | 10 |  |
| II | 20 |  |
| III | 10 |  |
| IV | 20 |  |
| V | 10 |  |
| VI | 30 |  |
| Total | 100 |  |

I. (10 points) Assuming both options in the following option pairs can be correctly used for a particular application, explain how choosing one option over the other affects time and space-complexity? Be brief.
a. Top-down Memoization vs. Top-down divide-and-conquer
b. Randomly built binary search trees vs. red-black trees
II. (20 points)
A. (10 points) List the three cases that may arise when TREE_DELETE $(T, z)$ is called on binary search tree $T$ and node $z$. List the actions taken for each case.
B. (10 points) Insert the following keys (in the given order) into an empty red-black tree and show the state of the tree just after insertion of each key and after rebalancing/recoloring that may be needed after each such insertion. Keys $=(8,13,15,3,9,17)$
III. (10 points)
A. (5 points) Explain the difference between primary and secondary clustering in open-address hashing.
B. (5 points) Outline, using pseudocode, an algorithm to locate the predecessor of a given key, "HASH_PREDECESSOR(k)" in an openaddress hash table and comment on its complexity.
IV. (20 points)
A. (15 points) Illustrate the operation of the routine to compute the length of the Longest Common Subsequence (LCS_LENGTH) algorithm with $S 1=(Y \times Z W X Z)$ and $\mathrm{S} 2=(W \times W Z)$ as inputs by filling in the missing entries in the dynamic programming matrix. What is the value returned by LCS_LENGTH(S1, S2) i.e., the length of the longest common subsequence?

|  | - | $Y$ | $X$ | $Z$ | $W$ | $X$ | $Z$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| - | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $W$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| $X$ | 0 | 0 | 1 | 1 | 1 | 2 | 2 |
| $W$ | 0 | 0 |  |  |  |  |  |
| $Z$ | 0 | 0 |  |  |  |  |  |

B. (5 points) Using the filled in matrix in (A), state the value(s) of the longest common subsequence(s) of S1 and S2.
$\qquad$
V. (10 points) For a set of keys $A=\left\{k_{1}, k_{2}, k_{3}, \ldots k_{10}\right\}$ and for a given set of probabilities, the optimal binary search tree T is as shown below. (For simplicity, the dummy leaf nodes that represent unsuccessful searches $D=\left\{d_{0}, d_{1}, d_{2}, \ldots, d_{10}\right\}$ are not shown in the figure.) With out any knowledge of the probabilities of the successful and unsuccessful searches, fill in as many values in the "root" matrix as you can infer from tree T. If any entry cannot be determined from the available information, leave that entry empty.


VI. (30 points)
A. (20 points) Johnny Wanderlust wants to visit a number of cities in a fixed order. He has two choices to go from city to city: He could fly or take a train. Within a city he can take a cab from the railway station to the airport and vice versa. If the cost of air-tickets, train tickets, and cab fares from/to the airport to/from the railway-station are all known a priori, outline a dynamic programming algorithm that charts out the least-cost itinerary for Johnny Wanderlust.

1. Johnny starts and ends his journey at his home.
2. Johnny lives in $\mathrm{C}_{0}$. The cab fares from his home to airport and the railway station is $X A_{0}$ and $X R_{0}$ respectively. The cab fares from the airport and train station to his house are $X A_{n+1}$ and $X R_{n+1}$ respectively.
3. The cities to be visited are $\left\{\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3}, \ldots, \mathrm{C}_{\mathrm{n}}\right\}$ and finally back to $\mathrm{C}_{\mathrm{n}+1}=\mathrm{C}_{0}$.
4. The airfare from $C_{i}$ to $C_{i+1}$ is $A_{i}$ for $i=1,2,3, \ldots n$
5. The train fare from $C_{i}$ to $C_{i+1}$ is $R_{i}$
6. The cab fare from the airport in $\mathrm{C}_{\mathrm{i}}$ to the train station in $\mathrm{C}_{\mathrm{i}}$ is $X R_{i}$.
7. The cab fare from the train station in $\mathrm{C}_{\mathrm{i}}$ to the airport in $\mathrm{C}_{\mathrm{i}}$ is $X A_{i}$.

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B. (10 points) For the following values, show the computation of the leastcost itinerary.

$$
\begin{gathered}
\mathrm{n}=3 \\
\mathrm{XA}[0 . .4]=(1,3,4,6,2) \\
\mathrm{XR}[0 . .4]=(7,2,9,2,1) \\
\mathrm{R}[1 . .3]=(8,5,17) \\
\mathrm{A}[1 . .3]=(12,5,9)
\end{gathered}
$$

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