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Everywhere in what follows V will denote an arbitrary vector space. For each of the following questions specify if it is true or false (mark a) for true or b) or false).

1. If c is a real number and x, y, z are vectors in V such that either $x + z = y + z$ or $c \cdot x = c \cdot y$, then $x = y$.

   a) b)

2. $W = \left\{ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \in \mathbb{R}^3; a_2 = a_1 + 5 \right\}$ is a subspace of $\mathbb{R}^3$.

   a) b)

3. Let W be a subset of V. Then W is a subspace of V iff it is nonempty and whenever $a \in \mathbb{R}$ and $x, y \in W$ we have $a \cdot x + y \in W$.

   a) b)

4. A function $f : \mathbb{R} \to \mathbb{R}$ is called odd if $f(-x) = -f(x)$ for all x. A function $f : \mathbb{R} \to \mathbb{R}$ is called even if $f(-x) = f(x)$ for all x. Let $\mathcal{F}(\mathbb{R})$ be the vector space $\{f; f : \mathbb{R} \to \mathbb{R}\}$. Then $W = \{f; f : \mathbb{R} \to \mathbb{R}, f \text{ odd or even}\}$ is a subspace of $\mathcal{F}(\mathbb{R})$.

   a) b)

5. Recall that if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $A^t = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$. A matrix A is called skew-symmetric if $A^t = -A$. Then $W = \{A \in \mathcal{M}_2(\mathbb{R}) | A \text{ skew-symmetric}\}$ is a subspace of dimension 1 of $\mathcal{M}_2(\mathbb{R})$.

   a) b)

6. Let S be a finite subset of V such that whenever $\vec{v}_1, \ldots, \vec{v}_n \in S$ and $a_1\vec{v}_1 + \ldots + a_n\vec{v}_n = \vec{0}$, we have $a_1 = \ldots = a_n = 0$. Then every vector in span(S) can be uniquely written as a linear combination of vectors in S.

   a) b)

7. Let $V = \mathcal{P}(\mathbb{R})$. Then the polynomial $-x^2 + 2x^2 + 3x + 3$ is in span$\{x^3 + x^2 + x + 1, x^2 + x + 1, x + 1\}$

   a) b)

8. $\left\{\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}2\\3\\-1\end{pmatrix},\begin{pmatrix}0\\0\\2\end{pmatrix}\right\}$ spans $\mathbb{R}^3$, but $\left\{\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}2\\3\\-1\end{pmatrix},\begin{pmatrix}0\\0\\2\end{pmatrix},\begin{pmatrix}10\\0\\1\end{pmatrix}\right\}$ does not.

   a) b)

9. $\left\{\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}2\\3\\-1\end{pmatrix},\begin{pmatrix}0\\0\\2\end{pmatrix}\right\}$ are linearly independent, but $\left\{\begin{pmatrix}1\\0\\1\end{pmatrix},\begin{pmatrix}2\\3\\-1\end{pmatrix},\begin{pmatrix}0\\0\\2\end{pmatrix},\begin{pmatrix}10\\0\\1\end{pmatrix}\right\}$ are not.

   a) b)

10. $\left\{\begin{pmatrix}1\\1\\-1\end{pmatrix},\begin{pmatrix}0\\1\\2\end{pmatrix},\begin{pmatrix}3\\4\\-1\end{pmatrix},\begin{pmatrix}9\\13\\-1\end{pmatrix}\right\}$ can be reduced to a basis for $\mathbb{R}^3$.

   a) b)

11. $\left\{\begin{pmatrix}1\\1\\-1\end{pmatrix}\right\}$ can be extended to a basis for $\mathbb{R}^3$.

   a) b)

12. Let $\vec{v}_1, \vec{v}_2, \vec{v}_3 \in \mathbb{R}^3$ such that whenever $a_1\vec{v}_1 + a_2\vec{v}_2 + a_3\vec{v}_3 = \vec{0}$ we have $a_1 = a_2 = a_3 = 0$. Then the $\text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\} = \mathbb{R}^3$.

   a) b)

13. $\{1 + x, 1 + x - x^2, -1 + x + x^3, x^2 + x, 1\}$ is linearly independent in $\mathcal{P}_3(\mathbb{R}^3)$.

   a) b)

14. If $\{\vec{u}, \vec{v}\}$ is a basis for V, then $\{\vec{u} + \vec{v}, 3\vec{u}\}$ is a also a basis for V.

   a) b)

15. A basis for $W = \left\{\begin{pmatrix}a\\a-b\\b\end{pmatrix}; a, b \in \mathbb{R}\right\}$ is $\left\{\begin{pmatrix}2\\2\\0\end{pmatrix},\begin{pmatrix}0\\-4\\4\end{pmatrix}\right\}$.

   a) b)

16. Let $\mathcal{P}_2(\mathbb{R})$ be the vector space of all polynomials of degree less or equal to 2. Then $W = \{f \in \mathcal{P}_2(\mathbb{R}) | f(3) = 0\}$ is a subspace of dimension 2 of $\mathcal{P}_2(\mathbb{R})$.

   a) b)

17. Let W_1, W_2 be subspaces of V such that $V = W_1 \oplus W_2$. Then $\dim(V) = \dim(W_1) + \dim(W_2)$.

a) b)

18. If dim(V)=1, any generating set of V containing strictly more than n vectors can be reduced to a basis of V.

a) b)

19. If dim(V)=1, any n linearly independent vectors will span V.

a) b)

20. Let $W = \left\{ \begin{pmatrix} a \\ a-b \\ b \end{pmatrix}, a, b \in \mathbb{R} \right\}$. Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation. Then $\{v \in \mathbb{R}^4, T(v) \in W\}$ is a subspace of $\mathbb{R}^4$.

a) b)

21. Let T be the following linear transformation from $\mathcal{P}_3(\mathbb{R})$ to $\mathcal{P}_2(\mathbb{R})$: $T(f) = f'$. Then T is both injective (one-to-one) and surjective (onto).

a) b)

22. If $T : V \to W$ is a linear application, where V and W have finite and equal dimensions, then $\text{Ker}(T) = \{0\}$ iff Rank (T) =dim(W)

a) b)

23. If $T : V \to W$ is a linear application, where V and W are finite-dimensional and dim(V) <dim(W), then T can not be surjective.

a) b)

24. If $T : V \to W$ is a linear application, where V and W are finite-dimensional. Then Rank(T) $\leq$dim(W), and Rank(T) $\leq$dim(V).

a) b)

25. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation. Then T is injective iff T is surjective iff Rank(T) $= 3$.

a) b)

26. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection wrt the x-axis: $T(a,b) = (a,-b)$, then $\mathrm{Ker}(T) = \{(0,0)\}$.

    a) b)

27. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the following linear transformation $T(a,b) = (a,0,b)$, then Im (T) is the xy-plane in $\mathbb{R}^3$.

    a) b)

28. The above linear application T is surjective.

    a) b)

29. The matrix of the above inear application T in the canonical basis of $\mathbb{R}^2$ and respectively $\mathbb{R}^3$ is

$$[T]_\alpha^\beta = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

    a) b)

30. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the following linear transformation $T(a,b,c) = (a,0,0)$. Then $\mathrm{Ker}(T) = \{(0,b,c), b,c \in \mathbb{R}\}$.

    a) b)