Exam 1 MA353 - 12.30 Section
October 5, 2015

Part 1.

1. Let T: $P_3(R) \to P_2(R)$, T(f)=f' (T is the derivation).
   a. Find Im(T) and dim(Im(T)).
   b. Apply the Dimension Theorem to find dim(Ker(T)).
   c. Ker(T)=? (Describe Ker(T)).

2. Let T: $P_2(R) \to R$, T(f)=f(2). Let α and β be the canonical bases of $P_2(R)$ and R respectively.
   a. Compute $[T]_\alpha^\beta$.
   b. Using the definition of T find $[T(2-3x+5x^2)]_\beta$.
   c. Using Th. 2.14 ($[T(v)]_\beta=[T]_\alpha^\beta[v]_\alpha$) find $[T(2-3x+5x^2)]_\beta$ and compare to what you obtained at b.

3. Suppose the linear application T: V→W is injective (one-to-one). Let S be a subset of V. Prove that S is linearly independent in V iff T(S) is linearly independent in W.

4. Let T: $R^3 \to R$ be a linear application.
   a. Prove that there exist three real numbers a,b,c such that T(x,y,z)=ax+by+cz.
   b. What are the possible dimensions of Im(T)? Why?
   c. For each of these possible dimensions of Im(T), how much is the corresponding dimension of Ker(T)?
   d. Can T be injective? Why?

5. Give an example of linear transformation T: $R^2 \to R^2$ for which Ker(T)=Im(T).

Part 2: True-False Questions.

6. $[T(v)]_\beta=\{T]_\alpha^\beta[v]_\beta$ for all vectors v in V, where T: V→V is a linear transformation and α and β are bases in V.

a. True b. False

7. $L(V,W)$ is isomorphic to $M_{mxn}(R)$, where $L(V,W)$ is the space of all linear transformations from V to W and m represents the dim(V), n represents the dim(W).

a. True b. False

8. Let T: $P_3(R) \to P_2(R)$, $T(f)=f'$ (the derivation). If S is a basis in $P_3(R)$, then T(S) is a basis in $P_2(R)$.

a. True b. False

9. If V is the direct sum of the subspaces W_1 and W_2, then every vector v in V can be uniquely written as u+w, with u in W_1 and w in W_2.

a. True b. False

10. The spaces L(V, W), where dim(V)=dim(W)=2, and R^4 are isomorphic.

11. Let T, U be two different elements of L(V,W) and β a basis in V. Then there exists a vector v in β such that T(v) is different from U(v).

a. True b. False

12. Let V and W be vector spaces, $\{v_1,v_2,...,v_n\}$ a basis in V and $\{w_1,w_2,...,w_m\}$ vectors in W. Assume m>n. Then there exists a linear transformation T: V→W such that $T(\{v_1,v_2,...,v_n\})$ is a subset of $\{w_1,w_2,...,w_m\}$.

a. True b. False

13. If $[T]_\alpha^\beta$, the matrix of T in the bases α and β, is invertible, then $Ker(T)=\{0_V\}$.

a. True b. False

14. There exists a linear transformation T: $P(R) \to P_2(R)$ which is different from the zero transformation (P(R) is the space of all polynomials with real coefficients).

a. True b. False

15. Let T, U : V→V be two linear transformation and α, β two bases in V. Then $[TU]_\alpha=[T]_\alpha^\beta[U]_\alpha^\beta$.

a. True b. False

Part 3.

16. Prove that for any linear transformation T: V→W, Im(T)=SpanT(β), where β is a given basis in V.