Closed book and notes. 60 minutes.

1. True or false. (for each, 2 points if correct, 1 point if left blank.)

   (a) T F For every random variable X, "$X = 2$" and "$3 \le X < 4$" are mutually exclusive.

   (b) T F For every random variable X, "X" is also an event.

   (c) T F $C_2^{10} = 90$.

   (d) T F By definition, Bernoulli trials are independent of each other.

   (e) T F For every random variable X, this expression is true: $V(X) = E(X^2) - \mu_X$.

   (f) T F For every random variable X, $F_X(x) = P(X \le x)$ for every real number x.

   (g) T F For every random variable X, $f_X(x) = P(X = x)$.

   (h) T F The "continuous uniform" distribution is *continuous* in the sense that the density function has no discontinuities. (That is, there are no "steps" in the density function.)

   (i) T F A *binomial experiment* is an experiment composed of n Bernoulli trials.

2. Fill in the blanks.

3 pts each

   (a) The binomial distribution answers questions about the number of successes in n Bernoulli trials.

   (b) The geometric distribution answers questions about the number of Bernoulli trials until the first success.

   (c) The negative binomial distribution answers questions about the number of Bernoulli trials until the rth success.

   (d) The Poisson distribution answers questions about the number of counts in an interval.

   (e) The hypergeometric distribution answers questions about the number of successes in a sample of size n from a population of size N containing K successes.

 1 pt Name Key

3. Result: If X is continuous, then $P(a \le X \le b) = F_X(b) - F_X(a)$.
Provide a reason for each line of the proof.

3 pts each

$P(a \le X \le b) = P(X = a \text{ or } a < X \le b)$	same event
$= P(X = a) + P(a < X \le b)$	mutually exclusive
$= P(a < X \le b)$	$P(X=a)$ because X is continuous
$= P(X \le b) - P(X \le a)$	"$X \le a$" $\cup$ "$a < X \le b$" = "$X \le b$"
$= F_X(b) - F_X(a)$	definition of cdf F_X

4. The proportion, X, of people who respond to a certain mail-order solicitation has density function

$$f_X(x) = c\,(x+2) \qquad \text{if } 0 \le x \le 1$$

and zero elsewhere.

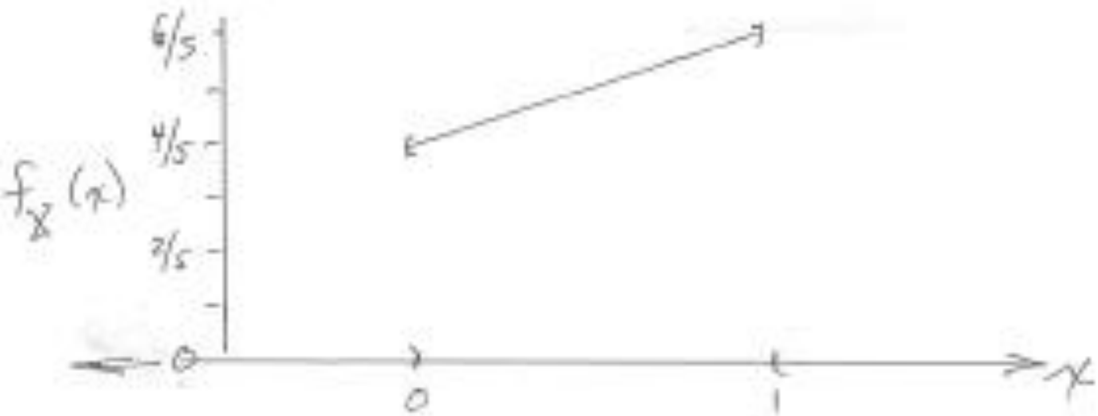
4 pts (a) Find the value of c.

$$\int_0^1 c\,(x+2)\,dx = c\left[\frac{x^2}{2} + 2x\right]_0^1$$

$$= \left(\frac{1}{2} + 2\right) - \left(\frac{0}{2} + 0\right) = 5/2$$

$$\Rightarrow c = \frac{1}{5/2} = \frac{2}{5} \quad \text{because } \int_{-\infty}^{\infty} f_X(x) = 1.$$

4 pts (b) Sketch the density function. Label and scale all axes.



5. A study conducted at George Washington University and the National Institutes of Health examined national attitudes about tranquilizers. The study revealed that approximately 70% believe "tranquilizers don't really cure anything, they cover up the real trouble." We are interested in the probability that less than two of the next (randomly selected) five people will be of this opinion.

2 pts (a) Define, in words, a binomial random variable X useful for solving this problem.

X = "# of people who believe (that tranquilizers don't really cure anything) out of the next 5 people.

2 pts (b) What is the number of Bernoulli trials?

$n = 5$

2 pts (c) Define, in words, the meaning of "success."

success = "person believes that tranquilizers don't really cure anything"

2 pts (d) Write the quantity to be found. (This is a probability statement involving X.)

$P(X < 2)$

5 pts (e) Solve the problem, showing all of your work.

$$P(X < 2) = \sum_{x=0}^{1} \binom{5}{x} (.7)^x (1-.7)^{5-x}$$
$$= \binom{5}{0}(.7)^0(.3)^5 + \binom{5}{1}(.7)^1(.3)^4$$
$$= .3^5 + 5(.7)(.3)^4$$
$$= (.3)^4[.3 + 5(.7)] = 0.308$$

2 pts (f) One reason the distribution is binomial is that there are two random variables. One is listed in Part (a). What is the other?

$Y = n - X$ = "# of people who don't believe..."

6. Suppose that X is a continuous random variable with $F(x) = 2x^2$ over the interval of interest.

4 pts (a) What is the interval of interest?

$$\left.\begin{array}{l} F(0) = 2 \cdot 0^2 = 0 \\ F(1/\sqrt{2}) = 2 \cdot (\frac{1}{\sqrt{2}})^2 = 1 \end{array}\right\} \Rightarrow \text{interval of interest is } [0, \tfrac{1}{\sqrt{2}}] \leftarrow$$

4 pts (b) What is the density function? (Be sure to state its value for every real number.)

$$f(x) = \left.\frac{dF(y)}{dy}\right|_{y=x} = \begin{cases} 4x & \text{if } 0 \le x \le \frac{1}{\sqrt{2}} \\ 0 & \text{elsewhere} \end{cases}$$

3 pts (c) Find $P(X \le 0)$.

$$P(X \le 0) = \int_{-\infty}^{0} f(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{0} 4x\,dx$$
$$= 0 + 0 = 0 \leftarrow$$

7. (Montgomery and Runger, 4-91.) Because of contamination particles, a particular testing instrument occasionally fails. Assume that the failures form a Poisson process with rate 0.02 failure per hour. (Recall: The Poisson pmf is $f(x) = e^{-\lambda}\lambda^x / x!$ for $x = 0, 1, \ldots$).

3 pts (a) What is the mean number of failures in one hour?

$$\lambda = .02 \text{ failures} \leftarrow$$

3 pts (b) What is the mean number of failures in an 8-hour shift?

$$8\lambda = 8(.02) = .16 \text{ failures} \leftarrow$$

4 pts (c) What is the probability that the instrument does not fail during an 8-hour shift?

Let X = "failures in an 8-hour shift"

Then $X \sim \text{Poisson}(\text{mean} = .16)$

$$\Rightarrow P(X = 0) = \frac{e^{-.16}(.16)^0}{0!} = e^{-.16} = .852 \leftarrow$$

3 pts. (d) Let p denote your answer to Part (c). In terms of p, what is the probability of at least one failure during an 8-hour shift?

$$P(X \ge 1) = 1 - P(X < 1)$$
$$= 1 - P(X = 0)$$
$$= 1 - p \leftarrow$$