IE 230   Seat # ________   Name ___ < KEY > ___

Closed book and notes. 60 minutes.

Cover page and four pages of exam.
No calculator.

This exam covers event probabilities and the definition of random variables.
Chapter 2 of Montgomery and Runger, fourth edition.

A true/false question is true only if it is always true;
that is, if it ever is false, then it is false.

No need to simplify answers.

(1 point) Write your name neatly, on all five pages.

(1 point) Circle your family name.

Score ____________________________

Closed book and notes. 60 minutes.

1. True or false. (3 points each) Consider an experiment. Assume that events A and B and random variable X are defined. Recall that $\varnothing$ denotes the empty set and S denotes the sample space.

   (a) T ← F ← Every replication of the experiment results in one or more outcomes.

   (b) T ← F The random variable X assigns a real number to every outcome.

   (c) T F ← The random variable is said to "occur" whenever $X = x$.

   (d) T F ← $\mathrm{P}(B) = \mathrm{P}(A \mid B)\,\mathrm{P}(A) + \mathrm{P}(A' \mid B)\,\mathrm{P}(A')$

   (e) T F ← $\mathrm{P}(A \mid B') = 1 - \mathrm{P}(A \mid B)$.

   (f) T ← F $\mathrm{P}(B \mid A)\,\mathrm{P}(A) = \mathrm{P}(A \mid B)\,\mathrm{P}(B)$.

   (g) T F ← If A and B are mutually exclusive, then $\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$.

   (h) T ← F The four events $(A \cap B)$, $(A' \cap B')$, $(A \cap B')$, and $(A' \cap B)$ partition the sample space.

   (i) T F ← For every replication of the experiment, both A and B occurring is impossible.

2. Consider the HIV/AIDS example. Recall that T_{ij} denotes the event of (not necessarily the first) virus transmission on the jth contact with partner i. Let n_i denote the number of contacts with partner i. Let n denote the number of partners.

   (a) (6 points) Write the event that virus is transmitted from partner 2.

   $T_{2,1} \cup T_{2,2} \cup \cdots \cup T_{2,n_2}$ ←

   (b) (3 points) T F ← $T_{1,1}$ and $T_{1,2}$ are mutually exclusive.

   (c) (6 points) In words, state the meaning of $\cup_{i=1}^{n} \cup_{j=1}^{n_i} T_{ij}$.

   The event that there was virus transmission on
   at least one contact with at least one partner. ←
   (That is, the person being analyzed has contracted the virus.)

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3. Three cable providers service a certain metropolitan area. Provider A has 30% of the households; Provider B has 35% of the households; Provider C has 10% of the households; the other households have no cable service.

All households with cable service were surveyed. Each household with cable service reported its provider and a satisfaction rating of "satisfied" or "unsatisfied". For Providers A, B, and C, results were that 80%, 50%, and 45% were satisfied.

We are interested in the using probability to discuss the fraction of households with "satisfactory" cable service.

(a) (5 points) Define the experiment.

Randomly choose a household from the metropolitan area. ←

(b) (5 points) Define notation and write the given information in terms of your notation.

Let A, B, C, and N denote the events that the household has service from Provider A, B, C or no service.
Let R denote the event that the household has rated "satisfactory".
Then
$\mathrm{P}(R \mid A) = 0.8$, $\mathrm{P}(R \mid B) = 0.5$, $\mathrm{P}(R \mid C) = 0.45$, $\mathrm{P}(R \mid N) = 0$
$\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.35$, $\mathrm{P}(A) = 0.1$.
$\mathrm{P}(N) = 1 - \mathrm{P(A)} - \mathrm{P(B)} - \mathrm{P(C)} = 0.15$.

(c) (7 points) Determine the fraction of all households with "satisfactory" cable service.

$\mathrm{P}(R) = \mathrm{P}(\mathrm{R} \mid A)\,\mathrm{P}(A) + \mathrm{P}(\mathrm{R} \mid B)\,\mathrm{P}(B) + \mathrm{P}(\mathrm{R} \mid C)\,\mathrm{P}(C) + \mathrm{P}(\mathrm{R} \mid N)\,\mathrm{P}(N)$ ←

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4. Result: If event A is a subset of event B, then $\mathrm{P}(A) \le \mathrm{P}(B)$.

(4 points each) For each of the four lines, state why the corresponding step is true.

Each blank requires one reason; reasons may be reused. Choose the reasons from this list:

(i) $A \subset B$.

(ii) Events are complementary.

(iii) Events are mutually exclusive.

(iv) Events are independent.

(v) Events are the same.

(vi) Definition of conditional probability.

(vii) Events partition the sample space.

(viii) Total Probability.

(ix) All probabilities are between zero and one.

(x) Bayes's Rule.

$\mathrm{P}(B)$	$= \mathrm{P}[(A \cap B) \cup (A' \cap B)]$	(a) ___ < v > ___
	$= \mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B)$	(b) ___ < iii > ___
	$= \mathrm{P}(A) + \mathrm{P}(A' \cap B)$	(c) ___ < i> or < v > ___
	$\ge \mathrm{P}(A)$	(d) ___ < ix > ___

5. (from `//www.businessinsider.com/a-simple-rationality-test-that-almost-everyone-gets-wrong-2011-2#ixzz1DEe4Kkcm`)

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student she was deeply concerned with issues of discrimination and social justice and also participated in anti-nuclear demonstrations.

Let T denote the event that Linda is a bank teller. Let F denote the event that Linda is active in the feminist movement.

Consider the following two alternatives.

A) Linda is a bank teller.
B) Linda is a bank teller who is active in the feminist movement.

(a) (5 points) Write alternative B) in terms of the notation.

$T \cap F \leftarrow$

(b) (3 points) Circle the less-probable alternative. A B ←

(c) (3 points) T F ← Alternative A) is less probable, because (obviously) Linda is a feminist.

6. (Montgomery and Runger, 4th edition, Problem 2–115) Consider a circuit that operates if and only if there is a path of functional devices from left to right.

The devices are arranged in three pairs, A and B, C and D, and E and F. At least one device in every pair must function for the circuit to operate; that is, A or B must function, and C or D must function, and E or F must function.

Assume that the probability that a device functions does not depend on whether or not other devices function. Assume that the values of P(A), P(B), P(C), P(D), P(E), P(F) are given.

(a) (5 point) Determine the value of $\text{P}(A \mid B \cup C)$.

The devices function independently, so

$$\text{P}(\text{A} \mid B \cup C) = \text{P}(A) \leftarrow$$

(b) (7 points) Determine the value of the probability that the circuit operates.

$$\begin{aligned} &\text{P}[(A \cup B) \cap (C \cup D) \cap (E \cup F)] \\ &= \text{P}(A \cup B) \times \text{P}(C \cup D) \times \text{P}(E \cup F) \\ &= [\text{P(A)} + \text{P}(B) - \text{P}(A \cap B)] \times [\text{P(C)} + \text{P}(D) - \text{P}(C \cap D)] \times [\text{P(E)} + \text{P}(F) - \text{P}(E \cap F)] \\ &= [\text{P(A)} + \text{P}(B) - \text{P}(A)\,\text{P}(B)] \times [\text{P(C)} + \text{P}(D) - \text{P}(C)\,\text{P}(D)] \times [\text{P(E)} + \text{P}(F) - \text{P}(E)\,\text{P}(F)] \end{aligned}$$
