IE 230   Seat # ____________   (1 point) Name (clearly) __ < KEY > __

Closed book and notes. No calculators.

Designed for 60 minutes, but time is essentially unlimited.

Cover page, four pages of exam.

This test covers through Section 2.7 of Montgomery and Runger, fourth edition.

Throughout, you will receive full credit if someone with no understanding of probability or set theory could simplify your answer to obtain the correct numerical solution.

A true/false question is true only if it is always true. If any conditions exist for which the statement is not true, then mark "false".

Score ______________________________

IE 230 Seat # ____________ (1 point) Name (clearly) __ < KEY > __

Closed book and notes. 60 minutes.

1. True or false. (2 points each) (Here all upper-case letters are events.)

(a) T ← F An experiment results in exactly one outcome.

(b) T F ← Every event contains at least one outcome.

(c) T ← F For every event A, the sample space is $A \cup A'$.

(d) T ← F If $A \subset B$, then $\mathrm{P}(A) \le \mathrm{P}(B)$.

(e) T ← F $\mathrm{P}(B') = \mathrm{P}(B' \mid A)\,\mathrm{P}(A) + \mathrm{P}(B' \mid A')\,\mathrm{P}(A')$.

(f) T ← F If $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$, then A and B are mutually exclusive.

(g) T F ← If $\mathrm{P}(A \cup B) = \mathrm{P}(A)\,\mathrm{P}(B)$, then A and B are independent.

(h) T F ← The number of ways to choose 6 things from a set of 9 things (without regard to order) is $C_6^9 = 9!\,/\,6!$.

(i) T F ← $0! = 0$.

2. (Montgomery and Runger, Edition 4, Problem 2–63). Samples of emissions from three suppliers are classified for conformance to air-quality specifications. The results from 100 samples are summarized in the following table.

		Conforms	
		yes	no
Supplier	1	22	8
	2	25	5
	3	30	10

Let C denote the event that the sample conforms. Let W_i denote the event that the supplier is i, for $i = 1, 2, 3$.

(a) (6 points) State the experiment?

Randomly choose one of the one-hundred samples.

(b) (6 points) Determine the value of $\mathrm{P}(C' \cap W_2)$.

$\mathrm{P}(C' \cap W_2) = 5\,/\,100$ ←

3. (Montgomery and Runger, Edition 4, Problem 2–79). The following table summarizes the analysis of samples for galvanized steel for coating weight and surface roughness.

		Coating weight high	Coating weight low
surface roughness	high	12	16
	low	88	34

(a) (6 points) If the coating weight of a sample is high, what is the probability that the surface roughness is low?

$P(R' \mid W)$	$= P(R' \cap W) \,/\, P(W)$	def of cond. probability
	$= [88/150]/[(12+88)/150]$	equally likely
	$= 0.88 \leftarrow$	simplify

(b) (6 points) Define notation and use it to state the conditional probability in Part (a).

Let R denote high roughness.
Let W denote high weight.
Then Part (a) finds the value of $P(R' \mid W)$.

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4. (6 points) Sketch a Venn diagram to illustrate $B = (B \cap E) \cup (B \cap E')$.

Sketch a rectangle; label it S.
Place an area B in the rectangle; typically B is a circle.
Place an area E in the rectangle overlapping B; typically B is a circle.
Shade $B \cap E$. Use different shading for $B \cap E'$.

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5. (Montgomery and Runger, Edition 4, Problem 2–98). A lot of 100 semiconductor chips contains twenty that are defective. Two are selected at random, without replacement.

(a) (6 points) Define notation sufficient for Parts (b)–(d).

Let D_i denote the event that the ith chip selected is defective, $i = 1, 2$.

(b) (6 points) Using your notation, write the event that the second chip is defective and the first chip is not defective.

$D_2 \cap D'_1 \leftarrow$

(No probability here.)

(c) (6 points) Determine the probability that the second chip is defective, conditional that the first is defective.

$\mathrm{P}(D_2 \mid D_1) = 19 / 99 \leftarrow$

(d) (6 points) Determine the probability that the second chip is defective.

$\mathrm{P}(D_2)$	$= \mathrm{P}(D_2 \mid D_1)\,\mathrm{P}(D_1) + \mathrm{P}(D_2 \mid D'_1)\,\mathrm{P}(D'_1)$	total probability
	$= (19/99)(20/100) + (20/99)(80/100)$	without replacement
	$= 0.2$	simplify

6. The Multiplication Rule

(a) (6 points) State the multiplication rule for P($A \cap B \cap C$).

$$P(A \cap B \cap C) = P(A)\,P(B \mid A)\,P(C \mid A \cap B)$$

(Comment: Order of A, B, and C is unimportant.)

(b) (5 points) Explain, as intuitively as you can, why the multiplication rule is true.

For all of A, B, and C to occur, at least A must occur.
If A occurs, then B still needs to occur.
If A and B both occur, then C still needs to occur.
(Comment: Other explanations could be correct.)

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7. Consider 100 independent coin tosses, each with probability 0.6 of the head side landing up. Let H_i denote the event that toss i results in a head facing up.

(a) (6 points) Write the event of exactly one head in the 100 tosses.
(You may use ellipses.)

$$\begin{aligned} & (H_1 \cap H'_2 \cap H'_3 \cap \cdots \cap H'_{99} \cap H'_{100}) \\ \cup\ & (H'_1 \cap H_2 \cap H'_3 \cap \cdots \cap H'_{99} \cap H'_{100}) \\ & \cdots \\ \cup\ & (H'_1 \cap H'_2 \cap H'_3 \cap \cdots \cap H'_{99} \cap H_{100}) \end{aligned}$$

(b) (6 points) Determine the probability of at least one head in the 100 tosses.

P(at least one head in the 100 tosses)

$= 1 - P(H'_1 \cap H'_2 \cap \cdots \cap 1_{100)}$	complement
$= 1 - P(H'_1)\,P(H'_2) \cdots P(H'_{100})$	independent events
$= 1 - (1 - P(H_1))\,(1 - P(H_2)) \cdots (1 - P(H_{100}))$	complements
$= 1 - (1 - 0.6)\,(1 - 0.6) \cdots (1 - 0.6))$	given
$= 1 - (0.4)^{100} \leftarrow$	simplify
