IE 230   Seat # ________   (Neatly, 1 pt) Name ___ < KEY > ___

Closed book and notes. 60 minutes.

Cover page and four pages of exam.
No calculator.

This test covers event probability, Chapter 2 of Montgomery and Runger, fourth edition.

Score ____________________________

Exam #1, September 21, 2010   Schmeiser

Closed book and notes. 60 minutes.

1. True or false. (3 points each)

(a) T ← F If S is the sample space of an experiment, then $P(S') = 0$.

(b) T ← F If A and B are events, then $P(A \cap B) \le P(A)$.

(c) T ← F If E_1, E_2,..., E_n partition the sample space S, then $E_1 \cup E_2 \cup \cdots \cup E_n = S$.

(d) T F ← If $P(A \mid B) = 0.7$, $P(B) = 0.4$, and $P(A) = 0.7$, then B' and A' are mutually exclusive.

(e) T F ← If A is an event, then A and A' are independent.

(f) T F ← If events A and B are independent, then A and B partition the sample space.

(g) T F ← If $P(A \mid B) = 0.5$, $P(B) = 0.5$, and $P(A') = 0.3$, then B' and A' are independent.

(h) T ← F If A and B are independent events, then $P(B' \mid A) = P(B')$.

(i) T ← F If the event A is a subset of B, then $P(A \mid B) \ge P(A)$.

2. Recall: A 52-card deck has four suits (spades, hearts, diamonds, and clubs) and thirteen values (Ace, 2, 3,...,10, Jack, Queen, and King). Spades and clubs are black; hearts and diamonds are red.

Consider the experiment of choosing ten cards without replacement. For draw $i = 1, 2, ..., 10$, let Q_i denote that a queen is chosen, let K_i denote that a king is chosen, and let A_i denote that an ace is chosen.

(a) (3 points) T F ← Q_1 and K_2 are independent.

(b) (3 points) T F ← Q_1, Q_2, ..., Q_{10} partition the sample space.

(c) (3 points) In the given notation, write the event that no queen is drawn.

$Q_1' \cap Q_2' \cap \cdots \cap Q_{10}'$ or $(Q_1 \cup Q_2 \cup \cdots \cup Q_{10})'$

(d) (7 points) Determine the value of $P(Q_1 \cap K_2 \cap K_3)$.

$$P(Q_1 \cap K_2 \cap K_3) = P(Q_1)\,P(K_2 \mid Q_1)\,P(K_3 \mid Q_1 \cap K_2) = (4/52)\,(4/51)\,(3/50) \leftarrow$$

3. (from Montgomery and Runger, 2–97) A batch of twenty-five injection-molded parts contains five that are suffered excessive shrinkage. Suppose that two parts are selected, without replacement.

(a) (7 points) What is the probability that the second part selected is one with excessive shrinkage?

$$P(E_{2)} = 5/25 \leftarrow$$

or

$$P(E_{2)} = P(E_2 \mid E_1)\,P(E_1) + P(E_2 \mid E_1')\,P(E_1')$$
$$= (4/24)\,(5/25) + (5/24)\,(20/25) \leftarrow$$

(b) (7 points) If the first part selected has excessive shrinkage, what is the proability that the second part selected has excessive shrinkage?

$$P(E_2 \mid E_1) = 4/24 \leftarrow$$

(c) (7 points) Determine the probability that both selected parts have excessive shrinkage.

$$P(E_1 \cap E_2) = P(E_1)\,P(E_2 \mid E_1) = (5/25)\,(4/24) \leftarrow$$

4. Recall the problem where the instructor shoots a free throw, either hitting or missing; he is a 60% free-throw shooter. A child reports to you whether the shot was a hit or miss, but is correct only 80% of the time, both when the shot is a hit and when the shot is a miss. Suppose that the child reports a hit. We are interested in the (conditional) probability that the shot was a hit?

(3 pts each) For each of the six lines, state why the corresponding equality is true.

Each blank requires one reason; reasons may be reused. Choose the reasons from this list:

(i) Events partition the sample space.

(ii) Events are complementary.

(iii) Events are mutually exclusive.

(iv) Events are independent.

(v) Events are the same.

(vi) Definition of conditional probability.

(vii) Multiplication Rule.

(viii) Total Probability.

(ix) Substitute known values.

$$\mathrm{P}(H \mid R) = \mathrm{P}(R \cap H) \,/\, \mathrm{P}(R)$$ (a) ___ (vi) ___

$$= \frac{\mathrm{P}(R \mid H)\,\mathrm{P}(H)}{\mathrm{P}(R)}$$ (b) ___ (vii) ___

$$= \frac{\mathrm{P}(R \mid H)\,\mathrm{P}(H)}{\mathrm{P}(R \mid H)\,\mathrm{P}(H) + \mathrm{P}(R \mid H')\,\mathrm{P}(H')}$$ (c) ___ (viii) ___

$$= \frac{\mathrm{P}(C)\,\mathrm{P}(H)}{\mathrm{P}(C)\,\mathrm{P}(H) + \mathrm{P}(C')\,\mathrm{P}(H')}$$ (d) ___ (v) ___

$$= \frac{\mathrm{P}(C)\,\mathrm{P}(H)}{\mathrm{P}(C)\,\mathrm{P}(H) + (1 - \mathrm{P}(C))\,(1 - \mathrm{P}(H))}$$ (e) ___ (ii) ___

$$= \frac{(0.8)\,(0.6)}{(0.8)\,(0.6) + (1 - 0.8)\,(1 - 0.6)}$$ (f) ___ (ix) ___

$$= 6 \,/\, 7$$

5. A system is composed of three components, each of which randomly works or fails. The system works only if no component fails. Let A denote the event that the first component works, B the second component works, and C the third component works.

The first component works one-fourth of the time. The second component works one fifth of the time, independently of the first component. If neither of the first two components work, then the third component works one time in eight; otherwise, the third component works one-sixth of the time.

(a) (4 points) Write the four given probabilities in the defined notation.

$$\mathrm{P}(A) = 1/4$$
$$\mathrm{P}(B) = \mathrm{P}(B \mid A) = 1/5$$
$$\mathrm{P}(C \mid A' \cap B') = \mathrm{P}(C \mid (A \cup B)') = 1/8$$
$$\mathrm{P}(C \mid (A' \cap B')') = \mathrm{P}(C \mid A \cup B) = 1/6$$

(b) (3 points) T F ← The events A and C are mutually exclusive.

(c) (3 points) In the notation given, write the probability that the system works.

$\mathrm{P}(A \cap B \cap C)$ ←

(d) (7 points) Determine the probability that the system works.

$$\mathrm{P}(A \cap B \cap C) \leftarrow = \mathrm{P}(A)\,\mathrm{P}(B \mid A)\,\mathrm{P}(C \mid A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)\,\mathrm{P}(C \mid A \cup B) \leftarrow$$