Solutions

ECE321/ECE595 Exam 3
Spring 2013

Notes: **You must show work for credit.**

This exam has 5 problems and 13 pages.

The last page has handy trig facts.

Note that problem 2 is different specifications depending on if you are in ECE321 or ECE595

Good luck!

1) 20 pts. Consider a machine with stator and rotor windings. The conductor distribution for b-phase stator winding and a-phase rotor winding are

$$\boxed{\frac{P}{2} = 6 \quad P = 12}$$

$$n_{bs} = -100\cos(6\phi_{sm})$$

$$n_{ar} = -10\sin(6\phi_{rm})$$

where ϕ_{sm} is position measured from the stator axis and ϕ_{rm} is position measured from the rotor axis. In this machine the axis definitions are arranged such that

$$\phi_{sm} = \phi_{rm} + \theta_{rm}$$

The machine is 10 cm long, the stator radius is 5 cm, and the airgap is constant and is 1 mm. Express the mutual inductance between the b-phase of the stator and the a-phase of the rotor in terms of θ_{rm}. The permeability of free space is $4\pi 10^{-7}$ H/m.

$$W_{bs} = \frac{1}{2}\int_0^{2\pi/12} (-100\cos(6\phi_{sm}))\, d\phi_{sm} - \int_0^{\phi_{sm}} (-100\cos(6\phi_{sm}))\, d\phi_{sm}$$

5

$c' = s$

$s' = c$

$\int c = s$

$$W_{bs} = -\frac{50}{6}\left[\sin(6\phi_{sm})\right]\Big|_0^{\pi/6} + \frac{100}{6}\sin(6\phi_{sm})\Big|_0^{\phi_{sm}}$$

$$= -\frac{50}{6}\left[\cancel{\sin\pi}^{\,0} - \cancel{\sin 0}^{\,0}\right] + \frac{100}{6}\sin(6\phi_{sm}) - 0$$

$$= \frac{100}{6}\sin(6\phi_{sm})$$

$$W_{ar} = \frac{1}{2}\int_0^{2\pi/12} (-10\sin 6\phi_{rm})\, d\phi_{rm}$$

5

$$- \int_0^{\phi_{rm}} (-10\sin 6\phi_{rm})\, d\phi_{rm}$$

$$W_{ar} = \frac{5}{6}\cos(6\phi_{rm})\Big|_0^{\pi/6} - \frac{10}{6}\cos(6\phi_{rm})\Big|_0^{\pi/6}$$

$$= \cancel{\frac{5}{6}[-1-1]} - \frac{10}{6}\cos(6\phi_{rm}) + \cancel{\frac{10}{6}}$$

$$= -\frac{10}{6}\cos 6\phi_{rm}$$

$$L_{bsar} = \frac{\mu_0 L r}{g}\int_0^{\phi_{sm}} W_{bs} W_{ar}\, d\phi_{sm}$$

$$= \frac{\mu_0 L r}{g}\left(\frac{100}{6}\right)\left(-\frac{10}{6}\right)\int_0^{\phi_{sm}} \sin(6\phi_{sm})\cos(6\phi_{rm})\, d\phi_{sm}$$

10 $$= -\frac{1000\mu_0 L r}{36\, g}\int_0^{\phi_{sm}} \sin(6\phi_{sm})\cos(6\phi_{sm} - 6\theta_{rm})\, d\phi_{sm}$$

$$= \frac{-1000 L r \mu_0}{72\, g}\int_0^{\phi_{sm}} \sin(12\phi_{sm} - 6\theta_{rm}) + \sin(6\theta_{rm})\, d\phi_{sm}$$

$$= \frac{-2000\pi L r \mu_0}{72 g}\sin(6\theta_{rm})$$

$$= -5.48\times 10^{-4}\sin(6\theta_{rm}) \quad H$$

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2.) 20 pts. Consider a machine with a round rotor (and therefore a constant airgap). Let ϕ_{sm} denote position measured from the stator in a counter-clockwise direction. For this machine the winding functions and currents are given by

$$w_{as} = 50\sin\phi_{sm}$$
$$w_{bs} = 50\cos\phi_{sm}$$
$$i_{as} = 5\cos(100t)$$
$$i_{bs} = -5\sin(100t)$$

Part 1. Determine the air-gap such that the peak flux density will be 1 T. Use this airgap for all remaining parts of this question.

Part 2. Give a simple expression for the air gap flux density as a wave equation in terms of time and space (ϕ_{sm}).

Part 3. What is the speed and direction of the flux density wave?

Part 4. [ECE595 Only] If the stator resistance is 1 Ohm, the stator radius is 10 cm, and the machine length is 5 cm, what is the a-phase voltage as a function of time.

① $F = W_{as} I_{as} + W_{bs} I_{bs}$

10 $= 250 \left[\sin\phi_{sm} \cos 100t - \cos\phi_{sm} \sin 100t \right]$

$= 250 \sin(\phi_{sm} - 100t)$

(5) $B = \dfrac{\mu_0 F}{g}$

$g = \dfrac{(250)(4\pi 10^{-7}) \text{ H/m}}{1\text{ T}} = 3.141 \times 10^{-4} \text{ m}$

②

5 $B = \sin(\phi_{sm} - 100t)$

③ $\left.\dfrac{d\phi_{sm}}{dt}\right|_{\text{at peak}} = 100 \text{ rad/s (ccw)}$

5

(4) $\lambda_{as} = r\ell \int_0^{2\pi} W_{as} B \, d\phi_{sm}$

$= r\ell \int_0^{2\pi} 50 \sin \phi_{sm} \sin(\phi_{sm} - 100t) \, d\phi_{sm}$

(5) $= \frac{50 r\ell}{2} \int_0^{2\pi} \cos(-100t) - \cos(2\phi_{sm} - 100t) \, \partial\phi_{sm}$

$= \frac{50 r\ell}{\cancel{2}} \cancel{2}\pi \cos(100t)$

$= 0.785 \cos(100t)$

$\frac{d\lambda_{as}}{dt} = -78.5 \sin(100t) \qquad 78.5 \angle 90$

$r_{as} i_{as} = 5 \cos 100t \qquad 5 \angle 0$

$V_{as} = r_{as} i_{as} + \frac{d\lambda_{as}}{dt}$

$= 5\cos 100t - 25\pi \sin(100t)$

$= 78.66 \cos(100t + 86.35°)$

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3) 20 pts. A three phase brushless DC machine has the following parameters: $r_s = 3\Omega$, $L_{ss} = 10$ mH, $\lambda_m = 0.17$ Vs, $P = 4$. The load torque is given by $T_l = 0.01\omega_{rm}$. It is desired to operate the machine at 4000 rpm. What is the peak machine efficiency possible to satisfy this operating point?

$$\omega_{rm} = 4000 \text{ rpm} \left(\frac{2\pi}{\text{rev}}\right)\left(\frac{\text{min}}{60\text{ s}}\right) = 418.88 \text{ rad/s}$$

$$\downarrow$$

$$\omega_r = 837.8 \text{ rad/s}$$

$$T_e = 0.01\,\omega_{rm} = 4.19 \text{ Nm}$$

$$P_{out} = 1.755 \text{ kW}$$

$$I_q = \frac{T_e}{\frac{3}{2}\frac{P}{2}\lambda_m} = 8.21 \text{ A}$$

$$I_d = 0$$

$$V_q = r_s I_q + \omega_r L_{ss} I_d + \omega_r \lambda_m = 167 \text{ V}$$

$$V_d = r_s I_d - \omega_r L_{ss} I_q = -68.8 \text{ V}$$

$$P_{in} = \frac{3}{2}\left(V_q I_q + \cancel{V_d I_d}^{\,0}\right) = 2.06 \text{ kW}$$

$$\eta = \frac{P_{out}}{P_{in}} = 85.2\,\%$$

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4) Suppose we have a devices in which

$$\lambda_{abcs} = \mathbf{L}_{ss} i_{abcs}$$

$$\mathbf{L}_{ss} = \begin{bmatrix} 6 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 6 \end{bmatrix}$$

Express the q-axis, d-axis, and 0-sequence flux linkage equations in the rotor reference frame. Solve this problem by using the appropriate transformations and performing the needed matrix manipulations. Recall

$$\mathbf{K}_s^r = \frac{2}{3}\begin{bmatrix} \cos(\theta_r) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \sin(\theta_r) & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\left[\mathbf{K}_s^r\right]^{-1} = \begin{bmatrix} \cos(\theta_r) & \sin(\theta_r) & 1 \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

$$\lambda_{qdos}^r = K_s^r L_{ss} K_s^{r^{-1}} I_{qdos}^r$$

$$= \left[\underbrace{K_s^r L_{ss1} K_s^{r^{-1}}}_{\text{Term1}} + \underbrace{K_s^r L_{ss2} K_s^{r^{-1}}}_{\text{Term2}} \right] I_{qdos}^r$$

note $L_{ss} = L_{ss1} + L_{ss2}$

Let $L_{ss1} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$$\text{Term1} = K_s^r \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} K_s^{r-1} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$L_{ss2} = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\text{Term2} = 2\,\frac{2}{3} \begin{bmatrix} c & c^- & c^+ \\ s & s^- & s^+ \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} K_s^{r-1}$$

$$= 2\,\frac{2}{3} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3/2 & 3/2 & 3/2 \end{bmatrix} K_s^{r-1}$$

$$= 2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} c & s & 1 \\ c^- & s^- & 1 \\ c^+ & s^+ & 1 \end{bmatrix}$$

$$= 2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\lambda_{qdos} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 10 \end{bmatrix} i^r_{qdos}$$

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The following are worth 2 points each. No equations allowed – explain verbally.

5a) Using the definitions we had in class, what is the difference between n_{as} and w_{as}?

n_{as} → conductor density w_{as} → winding function (turns)

5b) Why might we want to convert the continuous conductor description to a discrete description?

to determine # of conductors in slots so we can build

5c) What are the chief uses of the winding function?

→ calculate air gap MMF   → compute air gap fields

→ compute inductances

) 2 of 3

5d) What is an advantage of a 3-phase machine over a 1-phase machine?

→ MMF wave only travels in one direction

→ More efficient

5e) In a two phase machine, if the conductor distributions were different (apart from a spatial shift), is it still theoretically possible to achieve a single rotating MMF wave?

→ yes

5f) In class we studied a radial surface-mounted permanent ac machine. Name another type of permanent magnet ac machine.

→ radial buried magnet machine

→ axial machine

5g) Name a disadvantage of a permanent magnet ac machine.

fault current, physical robustness

5h) Why is it important to use a differential voltage probe when measuring the voltage of a open-circuited permanent magnet machine?

the voltage could damage the oscilloscope

5i) Name the benefits of transforming the machine equations to qd0 variables?

→ ac-dc   → magnetic decoupling

→ no rotor position in terms

) name 2

5j) Name advantages of current control of permanent magnet ac machines (brushless dc machines) relative to voltage control.

→ typically better efficiency

→ better transient performance

→ easy to control

→ robust with respect to some faults

) name 2

Handy Facts

$$\mu_0 = 4\pi 10^{-7} \text{ H/m}$$

Table A-1 Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\cos A \cos B = \tfrac{1}{2}[\cos(A+B) + \cos(A-B)]$$
$$\sin A \sin B = \tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$$
$$\sin A \cos B = \tfrac{1}{2}[\sin(A+B) + \sin(A-B)]$$
$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B)$$
$$\sin A - \sin B = 2 \sin \tfrac{1}{2}(A-B) \cos \tfrac{1}{2}(A+B)$$
$$\cos A + \cos B = 2 \cos \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B)$$
$$\cos A - \cos B = -2 \sin \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B)$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$
$$\sin \tfrac{1}{2}A = \sqrt{\tfrac{1}{2}(1-\cos A)} \qquad \cos \tfrac{1}{2}A = \sqrt{\tfrac{1}{2}(1+\cos A)}$$
$$\sin^2 A = \tfrac{1}{2}(1 - \cos 2A) \qquad \cos^2 A = \tfrac{1}{2}(1 + \cos 2A)$$
$$\sin x = \frac{e^{jx} - e^{-jx}}{2j} \qquad \cos x = \frac{e^{jx} + e^{-jx}}{2} \qquad e^{jx} = \cos x + j \sin x$$
$$A \cos(\omega t + \phi_1) + B \cos(\omega t + \phi_2) = C \cos(\omega t + \phi_3)$$

where

$$C = \sqrt{A^2 + B^2 - 2AB \cos(\phi_2 - \phi_1)}$$
$$\phi_3 = \tan^{-1}\left[\frac{A \sin\phi_1 + B \sin\phi_2}{A \cos\phi_1 + B \cos\phi_2}\right]$$
$$\sin(\omega t + \phi) = \cos\left(\omega t + \phi - \frac{\pi}{2}\right)$$

Taken from, *Continuous and Discrete Signal and Systems Analysis, 2nd Edition,* by McGillem & Cooper, 1984, CBS College Publishing, and one heck of a good book.

$$\cos(x) + \cos(x - 2\pi/3) + \cos(x + 2\pi/3) = 0$$
$$\sin(x) + \sin(x - 2\pi/3) + \sin(x + 2\pi/3) = 0$$
$$\cos(x)\cos(y) + \cos(x - 2\pi/3)\cos(y - 2\pi/3) + \cos(x + 2\pi/3)\cos(y + 2\pi/3) = \frac{3}{2}\cos(x - y)$$
$$\sin(x)\sin(y) + \sin(x - 2\pi/3)\sin(y - 2\pi/3) + \sin(x + 2\pi/3)\sin(y + 2\pi/3) = \frac{3}{2}\cos(x - y)$$
$$\sin(x)\cos(y) + \sin(x - 2\pi/3)\cos(y - 2\pi/3) + \sin(x + 2\pi/3)\cos(y + 2\pi/3) = \frac{3}{2}\sin(x - y)$$