MATH 265 FINAL EXAM, Spring 2007

Name and ID:

Instructor: **Section or class time:**

Instructions: Calculators are not allowed. There are 25 multiple choice problems worth 8 points each, for a total of 200 points.

1		14	
2		15	
3		16	
4		17	
5		18	
6		19	
7		20	
8		21	
9		22	
10		23	
11		24	
12		25	
13			

1. For what values of h and k the system $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions?

$$A = \begin{bmatrix} 1 & 1 & 4 \\ -3 & -3 & h \\ 1 & 8 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -2 \\ k \\ 0 \end{bmatrix}.$$

A. $h \neq 12$ and k any number

B. $h = -12$ and k any number

C. $h = -12$ and $k = 6$

D. $h = -11$ and $k = 6$

E. $h =\neq -11$ and $k \neq 6$

2. The inverse of the matrix

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \quad \text{is} \quad A^{-1} = \begin{bmatrix} a & 1/3 & 1/3 \\ -2/3 & b & 1/3 \\ -2/3 & 1/3 & c \end{bmatrix}.$$

What is $a + b + c$?

A. 0
B. $-1/3$
C. $-2/3$
D. $1/3$
E. $2/3$

3. Let A, B and C be invertible $n \times n$ matrices. If $A^{-1}B^{-1} = C^{-1}$, then what is A?

   A. $A = CB^{-1}$

   B. $A = C^{-1}B^{-1}$

   C. $A = BC^{-1}$

   D. $A = B^{-1}C$

   E. $A = BC$

4. If (x_1, x_2, x_3) is the solution of the following system of equations

$$x_1 + 3x_2 + x_3 = 1$$
$$2x_1 + 4x_2 + 7x_3 = 2$$
$$3x_1 + 10x_2 + 5x_3 = 7$$

then $x_2 =$

A. 29/9
B. 8/9
C. 59/9
D. 9/8
E. 20/9

5. Which of the following statements are true?

   (i). A linear system of four equations in three unknowns is always inconsistent

   (ii). A linear system with fewer equations in than unknowns must have infinitely many solutions

   (iii). If the system $A\mathbf{x} = \mathbf{b}$ has a unique solution, then A must be a square matrix.

   A. all of them

   B. (i) and (ii)

   C. (ii) and (iii)

   D. (iii) only

   E. none of them

6. If

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} -11 \\ 1 \end{bmatrix}$$

what is $a + b$?

A. -13

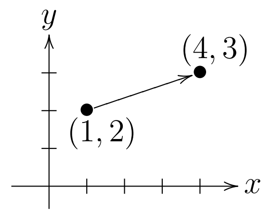
B. -5

C. -1

D. 5

E. 13

7. What vector is represented by the following:



A. $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

B. $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

C. $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} -3 \\ -1 \end{bmatrix}$

E. $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

8. Which of the following are subspaces of $\mathcal{P}_3$ (the vector space of all polynomials of degree ≤ 3)?

(I) $\{1 + t^2\}$

(II) $\{at + bt^2 + (a + b)t^3\}$ with a, b real numbers

(III) $\{a + bt + abt^2\}$ with a, b real numbers

(IV) $\{$polynomials $p(t)$ with $p(2) = 0\}$

A. (II) and (III) only.

B. (I) only.

C. (II) and (IV) only.

D. (I) and (IV) only.

E. (I), (II), and (III) only.

9. Which of the following sets of vectors in $M_{2\times 2}$ (the vector space of 2×2 matrices) are linearly independent?

$$\text{(I)} \quad \left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \right\}$$

$$\text{(II)} \quad \left\{ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix} \right\}$$

$$\text{(III)} \quad \left\{ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \right\}$$

$$\text{(IV)} \quad \left\{ \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right\}$$

A. (III) and (IV) only.

B. (IV) only.

C. (II) and (IV) only.

D. (I) and (II) only.

E. All of them are linearly independent.

10. Which of the following span $\mathbb{R}^2$?

(I) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix} \right\}$

(II) $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

(III) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$

(IV) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$

A. (II) only.

B. (I), (III), and (IV) only.

C. (III) only.

D. (I) and (IV) only.

E. (III) and (IV) only.

11. For four vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4 \in \mathbb{R}^4$, suppose that the 4×4 matrix $A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 \end{bmatrix}$ has its reduced row echelon form

$$rref(A) = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Then, which of the following pairs gives a basis for the vector space $Span\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$?

A. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

C. $\{\mathbf{v}_1, \mathbf{v}_3\}$

D. $\{\mathbf{v}_1, \mathbf{v}_2\}$

E. Cannot be determined from the given information.

12. Suppose that a 4×4 matrix A has its reduced row echelon form

$$rref(A) = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Let r be the rank of the matrix A, and let d be the determinant of the matrix A. Then, what is the value of $r^2 + d^2$?

A. 4

B. 5

C. 6

D. 8

E. 9

13. Consider the following matrices:

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & p \\ 0 & 0 & q \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}.$$

Then, which of the following statement is false?

A. If $q = 0$, then the nullity of the matrix A is 1.

B. If A is invertible, then the equation $A\mathbf{x} = \mathbf{b}$ has $\mathbf{x} = \begin{bmatrix} -3 & 2 & 0 \end{bmatrix}^T$ as its only solution.

C. The eigenvalues of the matrix A are 1 and q.

D. If $A\mathbf{x} = \mathbf{b}$ has more than one solution, then q must be zero.

E. The rank of the augmented matrix $[A|\mathbf{b}]$ is always 3.

14. Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$ be two vectors, satisfying the following properties:

(i) $\mathbf{x} \cdot \mathbf{y} = 0$.

(ii) $||\mathbf{x}|| = 2, ||\mathbf{x}|| = 1$.

Then, for real numbers a, b, what is the expression for $||a\mathbf{x} + b\mathbf{y}||^2$?

A. $a^2 + b^2$

B. $2a^2 + b^2$

C. $4a^2 + b^2$

D. $4a^2 + 4ab + b^2$

E. $a^2 + 4ab + 4b^2$

15. Let W be a subspace of $\mathbb{R}_3$ spanned by $(1,2,3),(2,k,3),(4,5,k+8)$. Determine the values of k so that $W^{\perp}$ has dimension zero.

   A. $k \neq 7$

   B. $k \neq 7,\ k \neq -1$

   C. $k \neq 7,\ k \neq 1$

   D. $k = 7,\ k = 1$

   E. $k = 7,\ k = -1$

16. Let A be the standard matrix representing the linear transformation $L : \mathbb{R}_3 \to \mathbb{R}_3$. Let $\mathbf{v}_1 = (2, 1, 4), \mathbf{v}_2 = (0, 5, 2), \mathbf{v}_3 = (0, 0, 1)$ be eigenvectors of the matrix A associated with eigenvalues $\lambda_1 = 1, \lambda_2 = -3, \lambda_3 = -2$ respectively. Find $L(\mathbf{v}_1 - \mathbf{v}_2 + 3\mathbf{v}_3)$.

A. $(2, -4, 5)$

B. $(2, -14, -8)$

C. $(2, 16, 16)$

D. $(2, 16, 4)$

E. $(2, -14, 4)$

17. Let W be a subspace of $\mathbb{R}_3$ with an basis $\{(1,1,0),(0,1,-1)\}$, and let $\mathbf{v} = (2,0,-4)$. Find the vector $\mathbf{w}$ in W closest to $\mathbf{v}$.

A. $(1,3,-2)$

B. $(0,2,-2)$

C. $(2,-2,-2)$

D. $(1,-3,-2)$

E. $(1,2,1)$

18. If A and B are $n \times n$-matrices, which statement is false?

A. $\det(AB) = \det(A)\det(B)$

B. $\det(A^{\mathrm{T}}) = \det(A)$

C. If k is a nonzero scalar, then $\det(kA) = k\det(A)$.

D. If A is nonsingular, then $\det(A^{-1}) = 1/\det(A)$.

E. If A and B are similar matrices, then $\det(A) = \det(B)$.

19. Compute the $\det(A)$.

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

A. 5

B. 16

C. 0

D. -5

E. 11

20. Find the values of α for which A is singular.

$$A = \begin{bmatrix} 2 & 1 & 3\alpha & 4 \\ 0 & \alpha - 1 & 4 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & \alpha & 4 \end{bmatrix}$$

A. $\alpha = 0$

B. $\alpha = 1$

C. $\alpha = 2$ and $\alpha = 3$

D. $\alpha = 1$ and $\alpha = 8$

E. $\alpha = 0$ and $\alpha = 1$

21. What is the coefficient of the x^3 term in the polynomial

$$q(x) = \begin{vmatrix} 3x & 5 & 7 & 1 \\ 2x^2 & 5x & 6 & 2 \\ 1 & x & 0 & 3 \\ 2 & 1 & 4 & 7 \end{vmatrix}$$

A. 17

B. -17

C. 90

D. -90

E. 0

22. Let A^{-1} be the inverse of the following matrix A.

$$A = \begin{bmatrix} 1+i & -1 \\ 1 & i \end{bmatrix}$$

What is

$$A^{-1} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}?$$

A. $\begin{bmatrix} 1+i & 1 \\ 1 & 1-i \end{bmatrix}$

B. $\begin{bmatrix} 1-i & -1 \\ -1 & 1+i \end{bmatrix}$

C. $\begin{bmatrix} 2 & -i \\ i & 2-i \end{bmatrix}$

D. $\begin{bmatrix} 1-i & 1 \\ 1 & 1+i \end{bmatrix}$

E. $\begin{bmatrix} 4 & 1-i \\ i & 2-i \end{bmatrix}$

23. The matrix A is

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 2 & 1 & 0 \end{bmatrix}.$$

The eigenvalues of A are

A. $0, 1, 2$

B. $0, -1, 2$

C. $0, 1, -2$

D. $0, -1, -2$

E. $-1, 0, 1$

24. Let matrix A be the following 3×3 matrix.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Which matrix P below gives us the following result?

$$P^T A P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$

where P^T is the transpose of matrix P.

A. $P = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} & 0 \end{bmatrix}$

B. $P = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ -\frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \end{bmatrix}$

C. $P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -2 & 0 & 1 \end{bmatrix}$

D. $P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ -2 & 1 & 0 \end{bmatrix}$

E. $P = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \end{bmatrix}$

25. The eigenvectors of $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$ are $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ with eigenvalues 1 and 4 respectively. If $x_1(t)$ and x_2 are the solution of

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},$$

$$x_1(0) = 90, \quad x_2(0) = 150$$

then

$$x_1(1) + x_2(1) \quad \text{is equal to}$$

(a) $240e$

(b) $200e$

(c) $230e$

(d) $60e$

(e) $360e$