MA 265. MIDTERM I. Spring, 2005

Professor: Jose E. Figueroa-Lopez

PRINT your name: ______________________________ **SECTION:** 9:00 or 1:30

Intructions:

1. **The exam is 1-hour long.**
2. The exam is closed book and notes.
3. You are NOT allowed to use graphing calculator.
4. Justify all your answers and work.
5. Be sure to write so that other than yourself can understand your solution.
6. **The bonus problem will be grade only if you try all other problems.**

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
BONUS	10	
Total	100	

Problem 1: Consider the system

$$\begin{array}{rcrcrcl} x & + & y & + & z & = & 2 \\ x & + & 2y & + & z & = & 4 \\ x & + & y & + & (a^2-3)z & = & a \end{array}$$

Using Gaussian Elimination, find the value(s) of a for the system to have

(a) unique solution?

(b) infinitely many solutions?

Show your work.

Problem 2: (a) Find the matrix A given that

$$A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}.$$

Note: All entries in A should be integer numbers!!

(b) Using **only** the matrix A^{-1} in part (a), find the matrix B such that $AB^T = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 0 \end{bmatrix}$.

Problem 3: Let W be the set of vectors of the form $\begin{bmatrix} a+b-c \\ a+2b-4c \\ -a-2b+4c \end{bmatrix}$, where a, b, c can be any real numbers.

(a) W is the *span* of three vectors: $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$. Find those vectors.

(b) Find a basis for $W = \text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. What is the dimension of W?

Note: Even if you didn't find $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ *in part (a), indicate* ***concisely*** *the procedure to find the basis if you had them.*

Problem 4: Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -2 & -2 & 4 \\ -1 & -1 & 2 \end{bmatrix}$$

(a) Find the set of all solutions to $A\mathbf{x} = \mathbf{0}$.

(b) Find a basis for the null-space of A. What is the dimension of the null-space?

Problem 5: Some "quick" questions (explain BRIEFLY your answer):

(a) Are the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}$$

linearly dependent or independent?

(b) Can four polynomials be a basis for $\mathcal{P}_2 = \{at^2 + bt + c, \quad a, b, c \text{ reals}\}$?

(c) Is a homogeneous system always consistent?

(d) If a linear system has more unknowns than equations, can it have a unique solution?

(e) Suppose that the dimension of span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is two. Can you conclude whether or not the vectors are linearly dependent?

Bonus Problem: (10 points)

Prove that if $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent and $\mathbf{v}_3$ is NOT in $\text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$, then $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent.