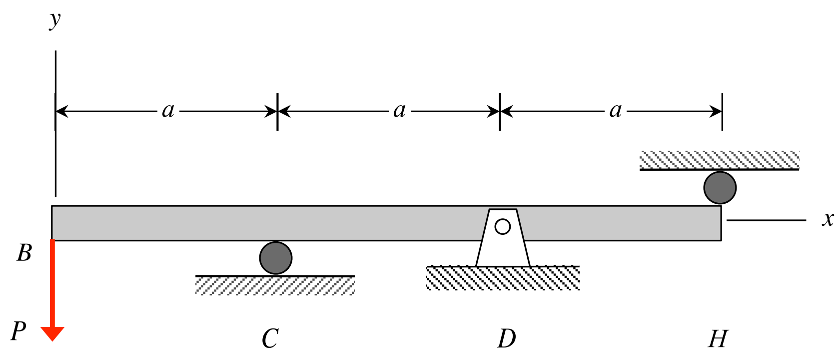
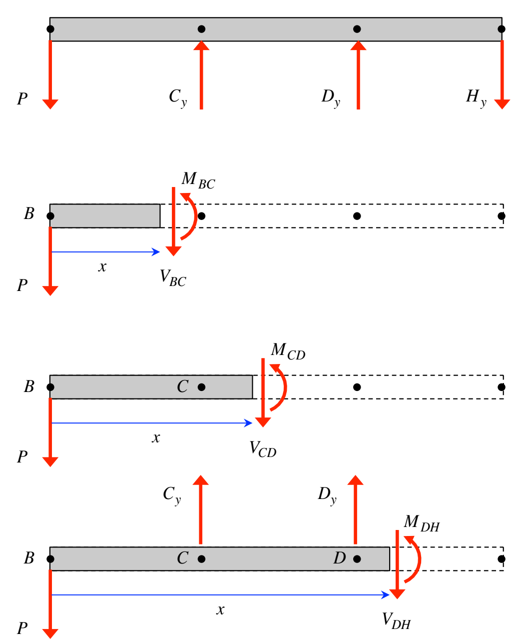
ME 323 Final Examination **SOLUTION**

August 1, 2019



PROBLEM NO. 1 – 20 points max.

A beam made up of a material with a Young's modulus of E and having a cross section with a second area moment of I is supported by a pin joint at D, and by roller supports at C and H. A point load P acts at end B of the beam. Using Castigliano's method, determine the reaction on the beam at support C. Express your answer in terms of P. You may ignore shear effects in your analysis.



<u>*Entire beam*</u>:

$$\sum M_H = P(3a) - C_y(2a) - D_y(a) = 0 \quad \Rightarrow \quad D_y = 3P - 2C_y$$

$$\sum F_y = -P + C_y + D_y - H_y \quad \Rightarrow \quad H_y = D_y + C_y - P = 2P - C_y$$

Indeterminate…choose C_y as the redundant reaction.

<u>*Section BC*</u>:

$$\sum M_{cut} = M_{BC} + Px = 0 \quad \Rightarrow \quad M_{BC}(x) = -Px$$

<u>*Section CD*</u>:

$$\sum M_{cut} = M_{CD} + Px - C_y(x-a) = 0 \quad \Rightarrow$$

$$M_{CD}(x) = (-P + C_y)x - C_y a$$

<u>*Section DH*</u>:

$$\sum M_{cut} = M_{DH} + Px - C_y(x-a) - D_y(x-2a) = 0 \quad \Rightarrow$$

$$M_{DH}(x) = (-P + C_y + D_y)x - (C_y + 2D_y)a$$

$$= (2P - C_y)x - (6P - 3C_y)a$$

Strain energy:

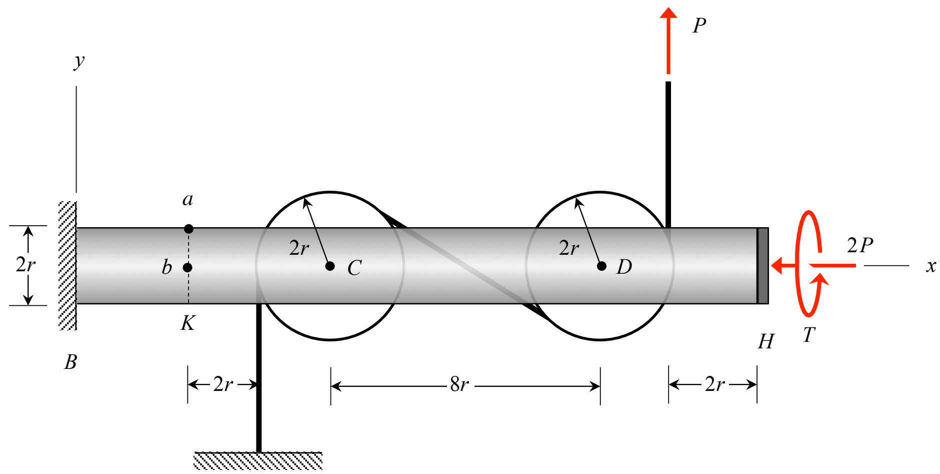
$$U = U_{BC} + U_{CD} + U_{DH} = \frac{1}{2EI}\int_0^a M_{BC}^2\,dx + \frac{1}{2EI}\int_0^a M_{CD}^2\,dx + \frac{1}{2EI}\int_0^a M_{DH}^2\,dx$$

Castigliano's theorem

$$0 = \frac{\partial U}{\partial C_y} = \frac{1}{EI}\int_0^a M_{BC}\frac{\partial M_{BC}}{\partial C_y}\,dx + \frac{1}{EI}\int_a^{2a} M_{CD}\frac{\partial M_{CD}}{\partial C_y}\,dx \frac{1}{EI}\int_{2a}^{3a} M_{DH}\frac{\partial M_{DH}}{\partial C_y}\,dx$$

$$= 0 + \frac{1}{EI}\int_a^{2a}\left[\left(-P + C_y\right)x - C_y a\right]\left(x - a\right)dx + \frac{1}{EI}\int_{2a}^{3a}\left[\left(2P - C_y\right)x - \left(6P - 3C_y\right)a\right]\left(-x + 3a\right)dx$$

Expand terms, integrate and solve the one equation for one unknown, C_y.



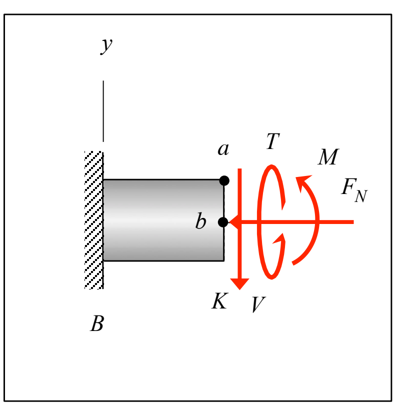
Structural member BH (made up of a ductile material with a yield strength of σ_Y) has a fixed support at end B. Two ideal pulleys (each having an outer radius of 2r) are attached to the member at C and D. A cable (connected to ground at one end) is pulled over these two pulleys, with a load of P acting at the free end of the cable. At end H, a compressive axial force 2P and a torque of $T = 4Pr$ acts on the member at end H, as shown in the figure. The circular cross section of the member at location K is solid with an outer radius of r.

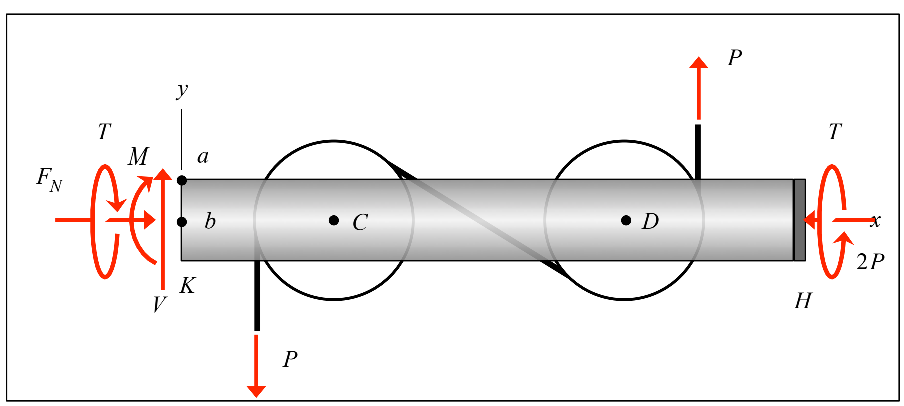
a) Draw a free body diagram of section KH on the figure provided below. From this FBD, determine the internal resultants in the member at K.

b) Show the stress components acting on the cross section at K in the figures provided on the following page. Write down expressions for these stress components at point "a" in the table provided on the following page.

c) Show the stress components on a stress element at K on the stress element for point "a" provided on the following page.

d) Using the maximum shear stress theory, determine the maximum value of P for which the structural component does not yield at point "a".





FBD

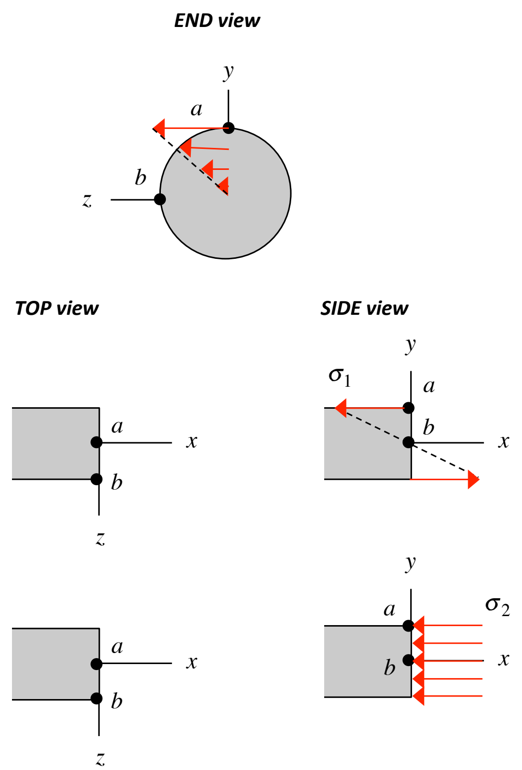
PROBLEM NO. 2 (continued)

$$\sum F_y = V = 0$$

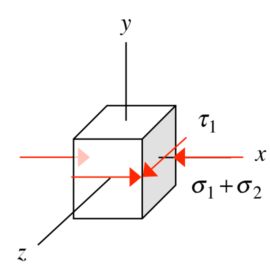
$$\sum M_{Kz} = P(12r) - M = 0 \quad \Rightarrow \quad M = 12Pr$$

$$T = 4Pr \quad (given)$$

$$F_N = 2P$$



load	stress at "a"
M	$\sigma_1 = \frac{Mr}{I} = 12\frac{Pr^2}{\pi r^4/4} = \frac{48}{\pi}\frac{P}{r^2}$
F_N	$\sigma_2 = \frac{F_N}{A} = \frac{2P}{\pi r^2} = \frac{2}{\pi}\frac{P}{r^2}$
T	$\tau_1 = \frac{Tr}{I_P} = \frac{4\Pr^2}{\pi r^4/2} = \frac{8}{\pi}\frac{P}{r^2}$



stress element at "a"

$$\sigma_{ave} = \frac{\sigma_1 + \sigma_2}{2} = \frac{25}{\pi}\frac{P}{r^2}$$

$$R = \sqrt{\left(\frac{\sigma_1 + \sigma_2}{2}\right)^2 + \tau_1^2} = \sqrt{\left(\frac{25}{\pi}\frac{P}{r^2}\right)^2 + \left(\frac{8}{\pi}\frac{P}{r^2}\right)^2} = \frac{\sqrt{689}}{\pi}\frac{P}{r^2}$$

Since $R > \sigma_{ave}$, σ_{P1} and σ_{P2} have opposite signs. Therefore:

$$\left|\tau_{max}\right|_{abs} = \left|\tau_{max}\right|_{in-plane} = R$$

On the failure boundary:

$$\frac{\sigma_Y}{2} = \left|\tau_{max}\right|_{abs} = R = \frac{\sqrt{689}}{\pi}\frac{P}{r^2} \quad \Rightarrow \quad P_{max} = \frac{\pi \sigma_Y r^2}{2\sqrt{689}}$$

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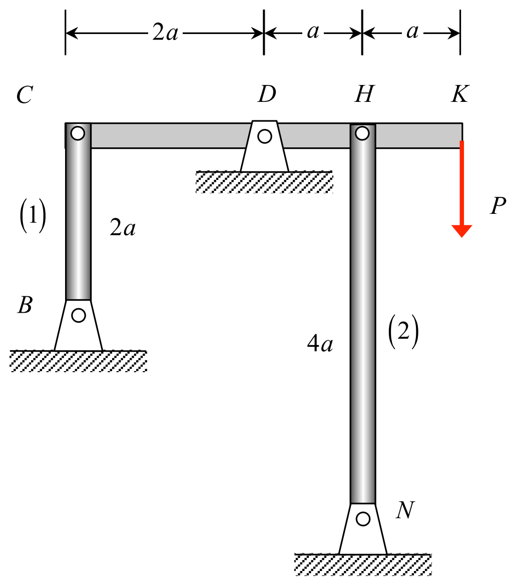
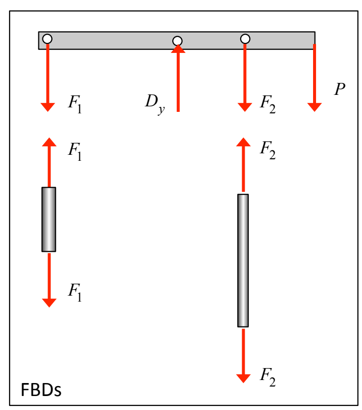


PROBLEM NO. 3 – 20 points max.

Members (1) and (2) above are each made of a material with a Young's modulus of E and a coefficient of thermal expansion of α, and having a cross section with an area of A. Members (1) and (2) are pinned to a rigid structural member CK, with CK being pinned to ground at D. The temperatures of members (1) and (2) are both *increased* by an amount of ΔT. A concentrated load P is applied at end, with $P = 3\alpha\Delta TEA$.

Determine the stress in member (1) as a result of these loads. State whether this stress is tensile or compressive. *Clearly show the four steps in your solution process.*



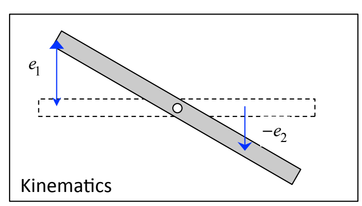
1. Equilibrium:

(1) $\sum M_D = F_1(2a) - F_2(a) - P(2a) = 0 \Rightarrow 2F_1 - F_2 = 2P$

2. Force/elongation:

(2) $e_1 = \dfrac{F_1(2a)}{EA} + \alpha\Delta T(2a)$

(3) $e_2 = \dfrac{F_2(4a)}{EA} + \alpha\Delta T(4a)$



3. Compatibility:

(4) $\dfrac{e_1}{2a} = -\dfrac{e_2}{a} \Rightarrow e_1 = -2e_2$

4. Solve:

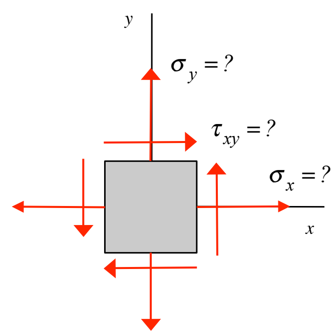
Equations (2)-(4): $\dfrac{2F_1 a}{EA} + 2\alpha\Delta T a = -2\left(\dfrac{4F_2 a}{EA} + 4\alpha\Delta T a\right) \Rightarrow$

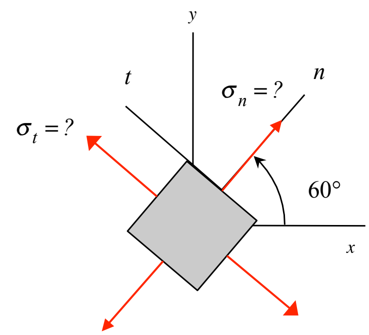
(5) $2F_1 + 8F_2 = -10\alpha\Delta TEA$

Equations (1) and (5):

$$2F_1 + 8\left(2F_1 - 2P\right) = -10\alpha\Delta TEA \Rightarrow F_1 = \frac{1}{18}\left(16P - 10\alpha\Delta TEA\right) = \frac{19}{9}\alpha\Delta TEA$$

Therefore: $\sigma_1 = \dfrac{F_1}{A} = +\dfrac{19}{9}\alpha\Delta TE \ \left(tensile\right)$





PROBLEM NO. 4 – 20 points max.

A structural component is made up of a material having a yield strength of $\sigma_Y = 60\ ksi$. A state of plane stress at a point in this structureal component is represented by the (unknown) stress components σ_x, σ_y and τ_{xy}, as shown in the stress element aligned with the *xy*-axes and as shown in the above-left figure. When the stress element is rotated counterclockwise through an angle of 60°, the shear stress on the surface perpendicular to the *n*-axis is zero, as shown in the above-right figure.

The following is known about this state of stress. The magnitudes of the maximum in-plane and maximum absolute shear stress are $\left|\tau_{max}\right|_{in-plane} = 10\ ksi$ and $\left|\tau_{max}\right|_{abs} = 20\ ksi$, respectively. It is also known that the *largest principal component of stress is positive*.

a) Determine the principal components of stress.

b) Draw the in-plane Mohr's circle for this state of plane stress. Show the location of the *x*-axis on the in-plane Mohr's circle.

c) Determine the σ_x, σ_y and τ_{xy} components of stress.

d) If the *maximum distortional energy theory* is used in the failure analysis, what is the factor of safety against yielding?

SOLUTION

Since $\left|\tau_{max}\right|_{abs} > \left|\tau_{max}\right|_{in-plane}$ and $\sigma_{P1} > 0$, we know:

$$\sigma_{P1} = +2\left|\tau_{max}\right|_{abs} = 40\ ksi$$

$$\sigma_{P2} = \sigma_{P1} - 2R = \sigma_{P1} - 2\left|\tau_{\max}\right|_{in-plane} = 40 - 2(10) = 20\ ksi$$

It is given that $\theta_{P1} = 60°$. With this, the Mohr's circle, including the x-axis, is as shown on the next page. From the Mohr's circle, we see that:

$$\sigma_x = \sigma_{ave} - R\sin 30° = 30 - 10\sin 30° = 25\ ksi$$

$$\sigma_y = \sigma_{ave} + R\sin 30° = 30 + 10\sin 30° = 35\ ksi$$

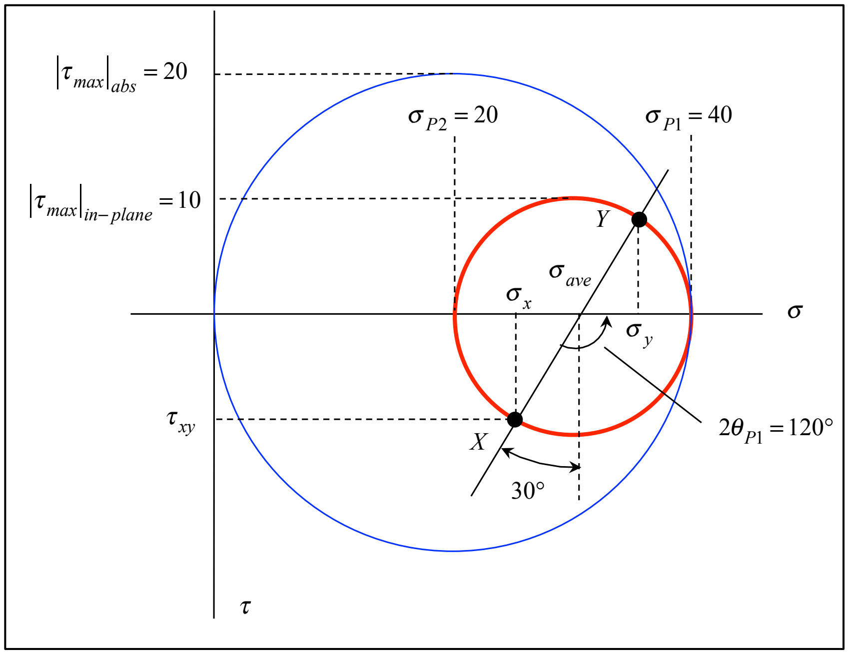
$$\tau_{xy} = +R\cos 30° = 10\cos 30° = 5\sqrt{3}\ ksi$$

From above, we have:

$$\sigma_M = \sqrt{\sigma_{P1}^2 - \sigma_{P1}\sigma_{P2} + \sigma_{P2}^2} = \sqrt{40^2 - (40)(20) + 20^2} = 34.6\ ksi$$

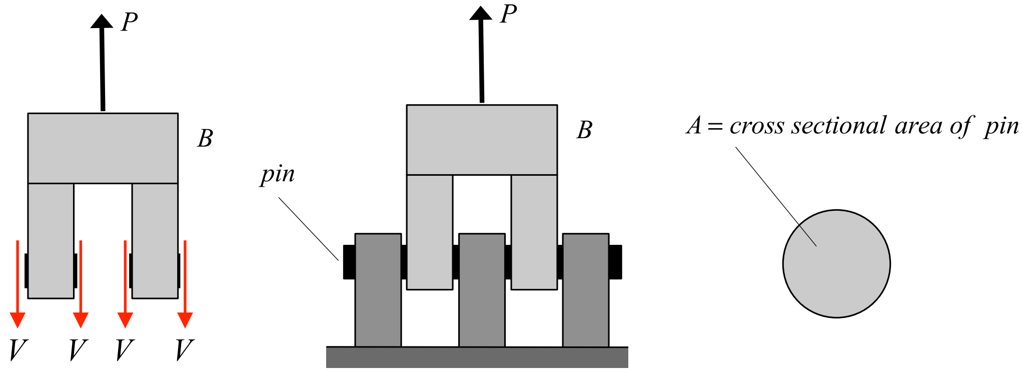
and:

$$FS = \frac{\sigma_Y}{\sigma_M} = \frac{60}{34.6} = 1.73$$



PROBLEM NO. 5 - PART A – 3 points max.

Consider the hinge shown below that is supported by a single pin whose cross-sectional area is A. A load P is applied to end B of the hinge. What is the maximum shear stress in the pin?



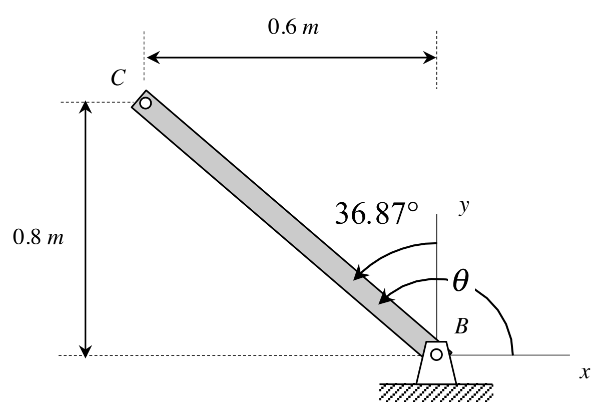
$$\sum F = -4V + P = 0 \;\Rightarrow\; V = P/4 \;\Rightarrow\; \tau = \frac{V}{A} = \frac{P}{4A}$$

PROBLEM NO. 5 - PART B – 4 points max.

Point C moves 12 mm to the LEFT and 16 mm DOWN from the position shown. What is the *displacement* of point C? What is the *change in length* of member BC? Does this change in length represent *elongation*, *compression* or *no change*?

$$d = distance = \sqrt{12^2 + 16^2} = 20\ mm$$

$$\begin{aligned} e &= u_C cos\theta + v_C sin\theta \\ &= -12cos(90° + 36.87°) + (-16)sin(90° + 36.87°) \\ &= -12(-0.6) - 16(0.8) = -5.6\ mm\ (compression) \end{aligned}$$



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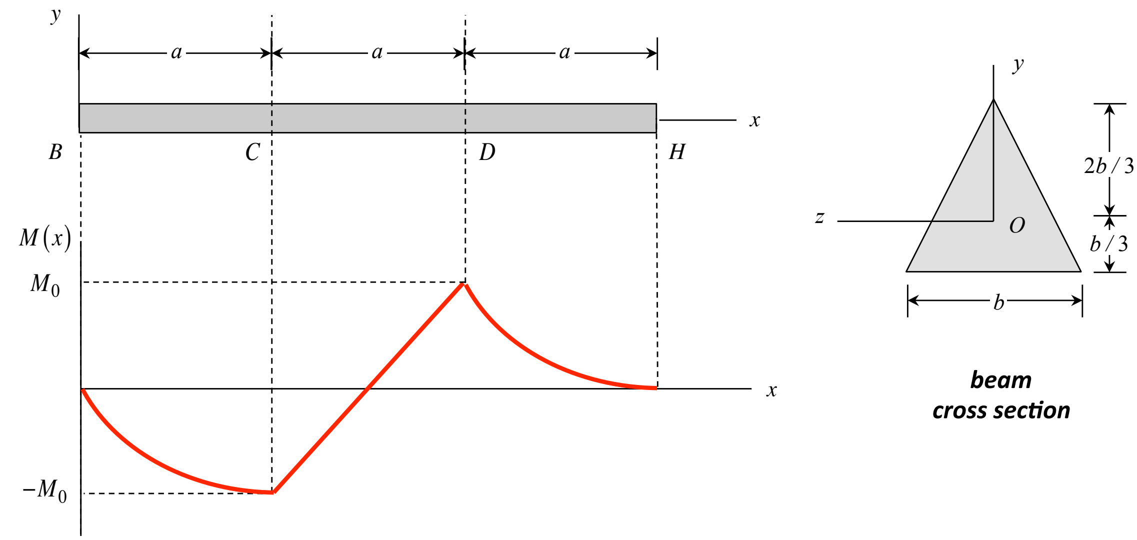
PROBLEM NO. 5 - PART C – 4 points max.



beam cross section

A beam with a triangular cross section is shown above. The loading on the beam produces the distribution of bending moment $M(s)$ shown below. (Neither the loading on the beam nor the beam supports are shown in the figure.) Consider the four locations of B, C, D and H along the length of the beam.

i. Circle the location to the right at which the largest *compressive* stress exists: B C (D) H.

ii. Circle the location to the right at which the largest *tensile* stress exists: B (C) D H.

The stress is compressive on the *top* surface at D, and on the bottom surface at C. The magnitude of the stress on the top surface is always larger than on the bottom surface since the top surface is further from the neutral axis. Therefore for i., the answer is D

The stress is tensile on the *top* surface at C, and on the bottom surface at D. The magnitude of the stress on the top surface is always larger than on the bottom surface since the top surface is further from the neutral axis. Therefore for ii., the answer is C.

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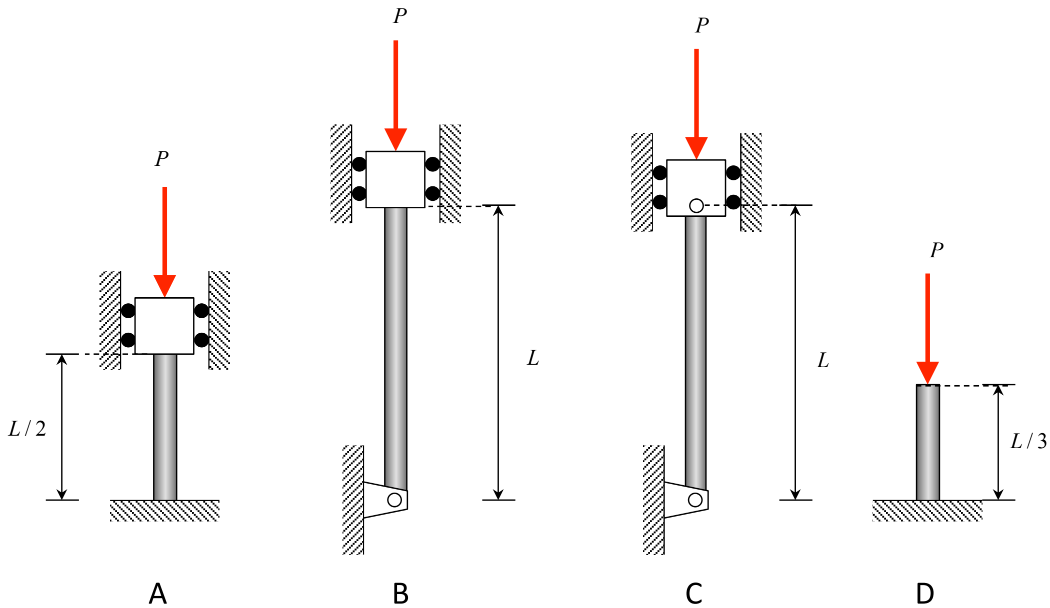
PROBLEM NO. 5 - PART D – 3 points max.



Consider the four columns (A, B, C and D) shown above with differing boundary conditions and lengths. The loading is the same for each column, each column is made up of the same material having a Young's modulus and each column has the same circular cross section.

a) Which column has the ***largest*** critical Euler buckling load? (A) B C D

b) Which column has the ***second largest*** critical Euler buckling load? A B C (D)

c) Which column has the ***smallest*** critical Euler buckling load? A B (C) D

The Euler buckling loads for A-D are:

A: $P_{cr} = \pi^2 \frac{EI}{L_{eff}^2} = \pi^2 \frac{EI}{\left[0.5\left(L/2\right)\right]^2} = 16\pi^2 \frac{EI}{L^2}$

B: $P_{cr} = \pi^2 \frac{EI}{L_{eff}^2} = \pi^2 \frac{EI}{\left[0.7\left(L\right)\right]^2} = \frac{100}{49}\pi^2 \frac{EI}{L^2}$

C: $P_{cr} = \pi^2 \frac{EI}{L_{eff}^2} = \pi^2 \frac{EI}{\left[L\right]^2} = \pi^2 \frac{EI}{L^2}$

D: $P_{cr} = \pi^2 \frac{EI}{L_{eff}^2} = \pi^2 \frac{EI}{\left[2\left(L/3\right)\right]^2} = \frac{9}{4}\pi^2 \frac{EI}{L^2}$

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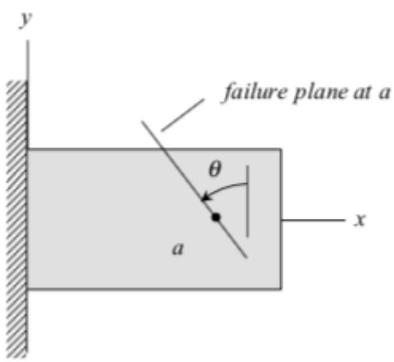
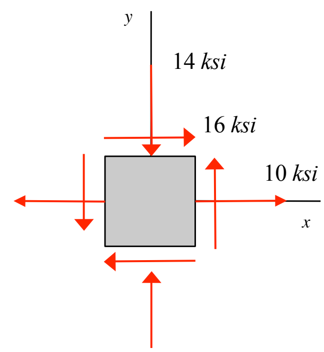
PROBLEM NO. 5 - PART E – 6 points max.

The ductile material making up the structural component shown is known to have failed in yielding at point "a", where the state of stress at "a" is shown below right. What is the angle θ that defines the plane of yielding in the material?





SOLUTION

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{10-14}{2} = -2\ ksi$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{10+14}{2}\right)^2 + (16)^2} = 20\ ksi$$

Since $R > |\sigma_{ave}|$, we know that σ_{P1} and σ_{P2} have opposite signs, and from that:

$$|\tau_{\max}|_{abs} = |\tau_{\max}|_{in-plane} = R = 20\ ksi$$

Failure will occur at the angle θ corresponding to when $\tau = \pm|\tau_{\max}|_{abs} = \pm 20\ ksi$. From the Mohr's circle shown below:

$$2\theta_S = \phi + 90° = tan^{-1}\left(\frac{\tau_{xy}}{\sigma_x}\right) + 90° = tan^{-1}\left(\frac{16}{10+2}\right) + 90° = 143.0° \Rightarrow \theta_S = \frac{143.0°}{2} = 72°$$