Name (Print)____

(Last) (First)

ME 323 - Mechanics of Materials
Final Exam
Date: May 3, 2017 Time: 8:00 – 10:00 AM - Location: PHYS 114

Instructions:

Circle your lecturer's name and your class meeting time.

Koslowski	Zhao	Bi
8:30-9:20AM	11:30-12:20AM	1:30-2:20PM

Begin each problem in the space provided on the examination sheets.

Work on one side of each sheet only, with only one problem on a sheet.

Please remember that for you to obtain maximum credit for a problem, you must present your solution clearly.

Accordingly,

- coordinate systems must be clearly identified,
- free body diagrams must be shown,
- units must be stated,
- write down clarifying remarks,
- state your assumptions, etc.

If your solution cannot be followed, it will be assumed that it is in error.

When handing in the test, make sure that ALL SHEETS are in the correct sequential order.

Remove the staple and restaple, if necessary.

Prob. 1 ______________________

Prob. 2 ______________________

Prob. 3 ______________________

Prob. 4 ______________________

Total ________________________

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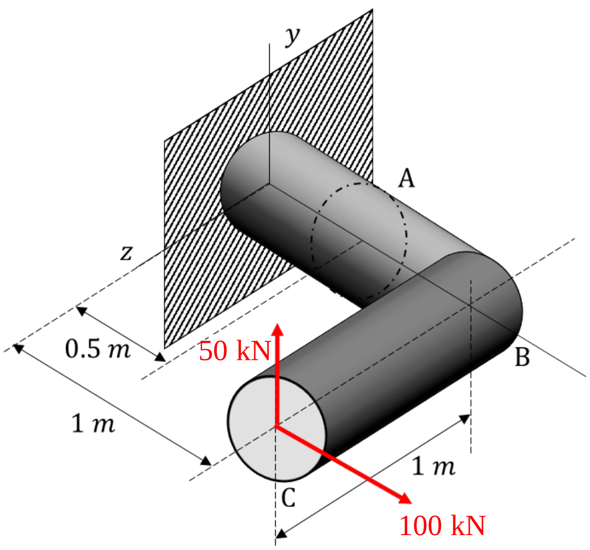
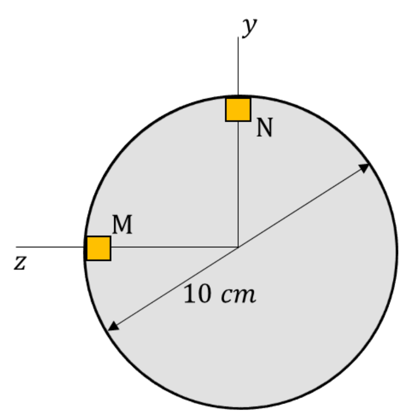
PROBLEM #1 (26 points)

An elbow with a circular cross section of diameter 10 cm is fixed to a wall at the origin of the coordinate system. At the other end C, load 100 kN and 50 kN are applied to the centroid of the cross section in the x and y direction, respectively.

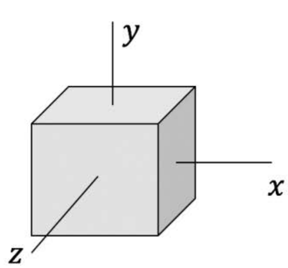
a) Determine the stresses induced at the locations M and N on the cross section A (x = 0.5 m). Draw separate stress elements for M and N, and indicate both direction and magnitude of the stresses.

b) Suppose the elbow is made of a ductile material with yield strength $\sigma_y = 800\ MPa$, according to the maximum shear stress theory, will the material points M and N fail?

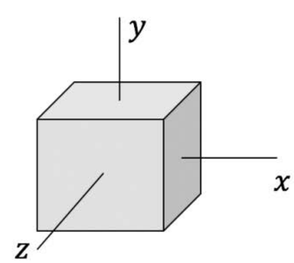




cross section at A



M



N

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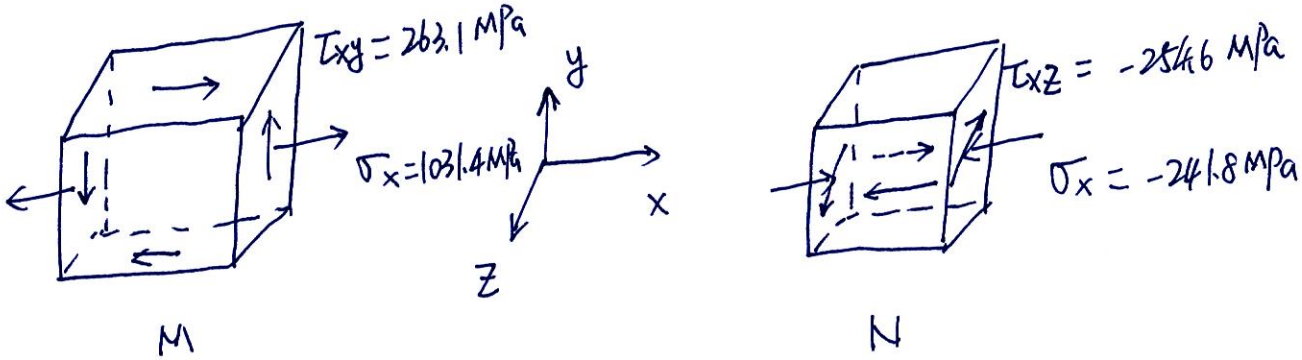
(a)

Load	stress at M	stress at N
$F_x = 100$ kN (Normal)	$\sigma_x = \frac{F_x}{A} = \frac{100\times10^3}{\pi R^2} = 12.8$ MPa (+)	$\sigma_x = \frac{F_x}{A} = 12.8$ MPa (+)
$F_y = 50$ kN (shear)	$\tau_{xy} = \frac{4F_y}{3A} = \frac{4\times 50\text{kN}}{3A} = 8.5$ MPa (+)	$\tau_{xy} = 0$
$F_z = 0$ (shear)	$\tau_{xz} = 0$	$\tau_{xz} = 0$
$M_x = -50$ kN.m (Torque)	$\tau_{xy} = \left\lvert \frac{M_x \cdot R}{I_p} \right\rvert = \frac{50\times10^3 \text{ N.m}}{\pi R^3/2} = 254.6$ MPa (+)	$\tau_{xz} = -\left\lvert \frac{M_x \cdot R}{I_p} \right\rvert = -254.6$ MPa
$M_y = 100$ kN.m (bending)	$\sigma_x = \left\lvert \frac{M_y \cdot R}{I} \right\rvert = \frac{100\times10^3 \text{ N.m}}{\pi R^3/4} = 1018.6$ MPa (+)	$\sigma_x = 0$
$M_z = 25$ kN.m (bending)	$\sigma_x = 0$ (N.A.)	$\sigma_x = -\left\lvert \frac{M_z \cdot R}{I} \right\rvert = -254.6$ MPa

M: $\sigma_x = \overset{1018.6}{\cancel{254.6}} + 12.8 = 1031.4$ MPa, $\tau_{xy} = 254.6 + 8.5 = 263.1$ MPa

N: $\sigma_x = -254.6 + 12.8 = -241.8$ MPa, $\tau_{xz} = -254.6$ MPa

stress element



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(b): For M: $\sigma_{p1} = \sigma_{ave} + R = \frac{1031.4}{2} + \sqrt{(\frac{1031.4}{2})^2 + 263.1^2} = 1094.6$ MPa

$\sigma_{p2} = \sigma_{ave} - R = -63.2$ MPa

Therefore: $\sigma_1 = \sigma_{p1} = 1094.6$ MPa, $\sigma_2 = 0$ MPa, $\sigma_3 = -63.2$ MPa

$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2} = 578.9$ MPa

$\frac{\sigma_Y}{2} = 400$ MPa

$\tau_{max,abs} > \frac{\sigma_Y}{2}$

M will fail

For N: $\sigma_{p1} = \sigma_{ave} + R = 160.9$ MPa

$\sigma_{p2} = \sigma_{ave} - R = -402.9$

Therefore: $\sigma_1 = \sigma_{p1} = 160.9$ MPa, $\sigma_2 = 0$ MPa, $\sigma_3 = -402.9$ MPa

$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2} = 281.8$ MPa $< \frac{\sigma_Y}{2}$

N will Not fail.

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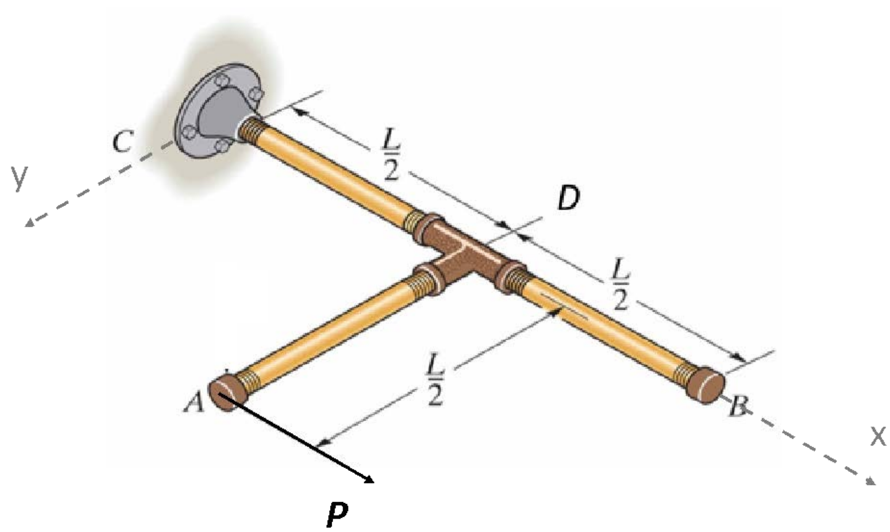
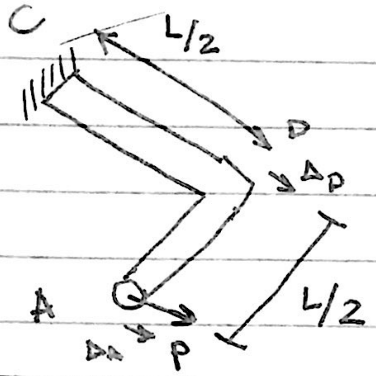
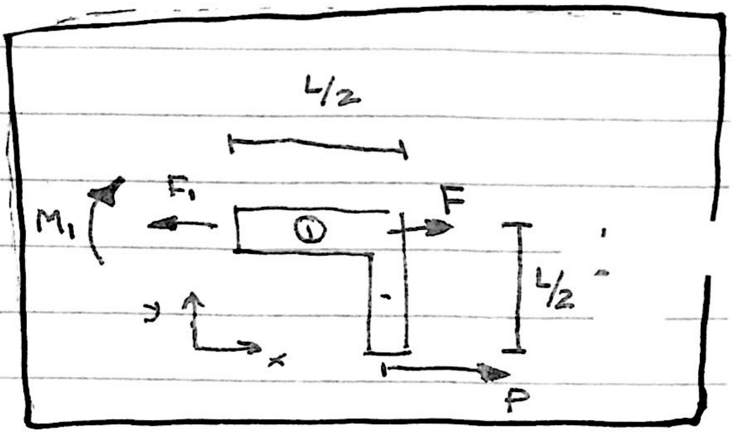
PROBLEM #2 (26 points)

The pipe assembly consists of three equal-sized pipes with flexural stiffness $\boldsymbol{EI}$ and torsional stiffness $\boldsymbol{GI_p}$. Use Castigliano's second theorem to calculate the displacement in the $\boldsymbol{x}$ direction of the points A and D.

Do not include the shear energy due to bending.





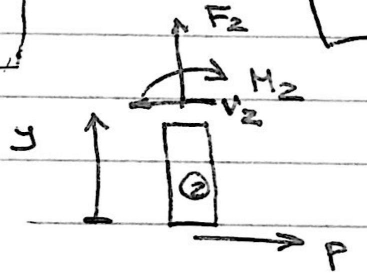


For member 1

$F_1 = F + P$

F is a dummy force.

$M_1 = P\dfrac{L}{2}$



For member 2

$V_2 = P$

$F_2 = 0$

$M_2 = Py$

$$\boxed{\begin{aligned} U &= U^{(1)} + U^{(2)} \\ &= \frac{1}{2}(F+P)^2\frac{L}{2EA} + \frac{1}{2}\left(\frac{PL}{2}\right)^2\frac{1}{EI}\frac{L}{2} \\ &\quad + \frac{1}{2}\int_0^{L/2}\frac{(Py)^2}{EI}\,dy \end{aligned}}$$

$$\left.\frac{y^3}{3}\right|_0^{L/2}$$

$$= \frac{L}{4EA}(F+P)^2 + \frac{L}{4EI}\left(\frac{PL}{2}\right)^2 + \frac{1}{6}\frac{P^2}{EI}(L/2)^3$$

$$\boxed{\Delta_D = \left.\frac{\partial U}{\partial F}\right|_{F=0} = \left.\frac{2L(F+P)}{4EA}\right|_{F=0} = \frac{LP}{2EA}}$$

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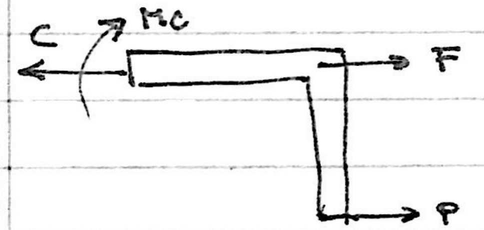
$$\boxed{\Delta_A = \left.\frac{\partial U}{\partial P}\right|_{F=0} =}$$

$$= \frac{\cancel{2}PL}{2\cancel{4}EA} + \frac{\cancel{2}P\,L^2}{2\,\cancel{4}}\,\frac{L}{4EI} + \frac{2P}{6EI}\cdot\frac{L^3}{\cancel{8}4}$$

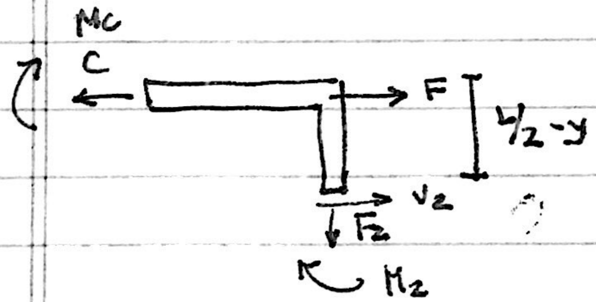
$$\boxed{\Delta_A = P\left(\frac{L}{2EA} + \frac{L^3}{8EI} + \frac{L^3}{24EI}\right)}$$

$$P\left(\frac{L}{2EA} + \frac{L^3}{6EI}\right)$$

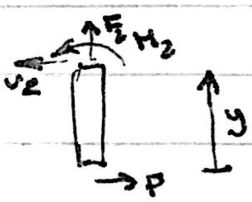


$C = F + P$

$M_c = \frac{PL}{2}$



$$0 = M_2 - C\left(\frac{L}{2} - y\right) + M_c + F\left(\frac{L}{2} -\right.$$



$$M_2 = (F+P)\left(\frac{L}{2} - y\right) - \frac{PL}{2} - F\left(\frac{L}{2} - y\right.$$

$$\boxed{M_2 = -Py}$$

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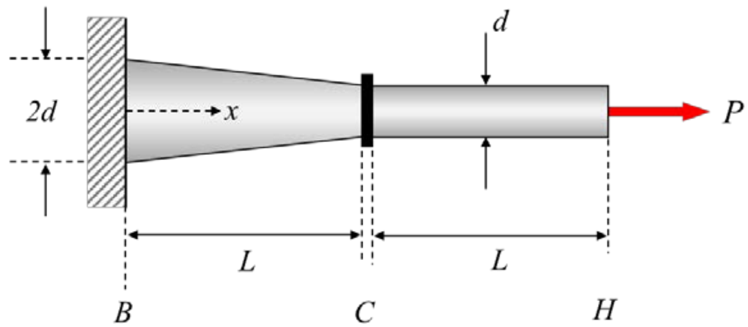
<u>PROBLEM #3 (24 points)</u>

A rod is made up of two circular cross-section segments BC and CH, with each segment having a length of L. Segment BC is tapered with its outer diameter going linearly from $2d$ at B to d at C. Segment CH has constant outer diameter of d. The segments are joined by a rigid connector C, and the rod is fixed on the wall at end B. An axial load P acts at the end H of this rod. The material of the rod has a Young's modulus of E.



a) Using a **two-element** finite element model (one element per segment), construct the global stiffness matrix. Note that the average cross section area of a "cone frustum", whose ends have circular cross sectional areas A_1, and A_2, is given by:

$$A_{avg} = \frac{1}{3}\left(A_1 + \sqrt{A_1 A_2} + A_2\right)$$

b) Construct the force vector, enforce the displacement boundary condition(s), and solve the axial displacements of points C and H in terms of E, P, L, d.

c) Use the work-energy principle to determine the axial displacement of point H. (*Hint*: $\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)} + c$)

d) Compare the result in Part c) with the finite element solution found in Part b). If the two results are different, explain why they are different.

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a). $k_1 = \frac{EA_1}{L}$, $A_1 = \frac{1}{3}\left(\pi\left(\frac{2d}{2}\right)^2 + \pi\left(\frac{d}{2}\right)^2 + \sqrt{\pi\left(\frac{2d}{2}\right)^2 \cdot \pi\left(\frac{d}{2}\right)^2}\right) = \frac{7}{12}\pi d^2$

$= \frac{7}{12} \cdot \frac{E\pi d^2}{L}$

$k_2 = \frac{EA_2}{L} = \frac{1}{4} \cdot \frac{E\pi d^2}{L}$, global stiffness matrix $[K] = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix}$

b). $[F] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot P$ $\quad [K]\cdot[U] = [F] \Rightarrow$

$$\begin{bmatrix} \cancel{k_1} & \cancel{-k_1} & \cancel{0} \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \cdot \begin{bmatrix} \cancel{u_B} \to 0 \\ u_C \\ u_H \end{bmatrix} = \begin{bmatrix} \cancel{0} \\ 0 \\ 1 \end{bmatrix} \cdot P$$

$$\Rightarrow \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} u_C \\ u_H \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} P \Rightarrow \begin{bmatrix} \frac{5}{6} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix}\begin{bmatrix} u_C \\ u_H \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{PL}{E\pi d^2}$$

$$\Rightarrow \begin{cases} u_C = \frac{12}{7}\frac{PL}{E\pi d^2} \\ u_H = \frac{40}{7}\frac{PL}{E\pi d^2} \approx 5.7\frac{PL}{E\pi d^2} \end{cases}$$

c). Both BC and CH have internal axial force P, the total strain energy

$$U = U_{BC} + U_{CH} = \frac{1}{2}\frac{P^2}{E}\int_0^L \frac{1}{A_1(x)}dx + \frac{1}{2}\frac{P^2}{E}\int_0^L \frac{1}{\pi\left(\frac{d}{2}\right)^2}dx$$

$A_1(x) = \frac{\pi}{4}\left[2d - \frac{d}{L}x\right]^2 \quad (0<x<L)$

$$\Rightarrow U = \frac{1}{2}\cdot\frac{P^2}{E}\int_0^L \frac{1}{\frac{\pi d^2}{4L^2}(2L-x)^2}dx + \frac{1}{2}\cdot\frac{4P^2L}{E\pi d^2}$$

$$= \frac{1}{2}\cdot\left[\frac{4P^2L^2}{E\pi d^2}\cdot\frac{1}{2L-x}\Big|_0^L + \frac{4P^2L}{E\pi d^2}\right]$$

$$= \frac{1}{2}\cdot\left[\frac{2P^2L}{E\pi d^2} + \frac{4P^2L}{E\pi d^2}\right] = \frac{1}{2}\cdot P\cdot u_H \Rightarrow u_H = \frac{6PL}{E\pi d^2}$$

d). The result in part c) is different from the FEM solution in part b). FEM is an numerical method which gives approximate results, especially when the cross-section area of BC is changing quadratically. If more # of elements are used, the result will be closer to the result in c).

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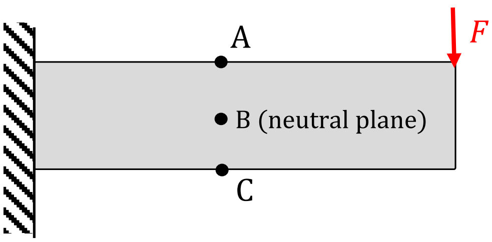
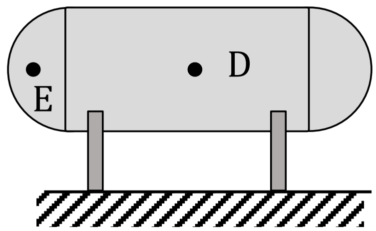
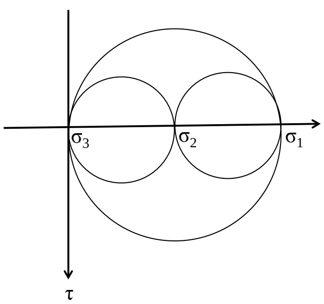
<u>PROBLEM #4 (24 Points)</u>:
<u>4.1. (15 Points)</u>



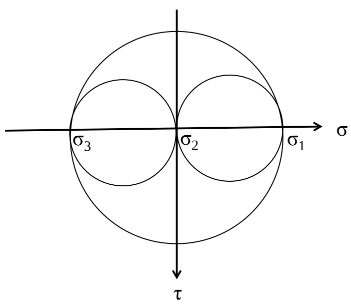


The points A, B, and C are located in a beam subjected to the vertical load F, and the points D and E are located in the thin-wall pressure vessel subjected to the internal pressure p. The state of stress of each point is represented by a Mohr's circle with the principal stresses $\sigma_1 \geq \sigma_2 \geq \sigma_3$.

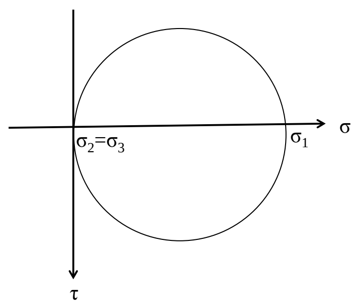
Circle the letter that corresponds to the state of stress represented in the Mohr's circle.



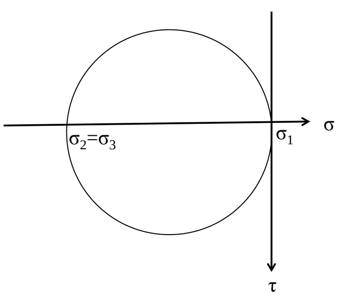
A B C D E



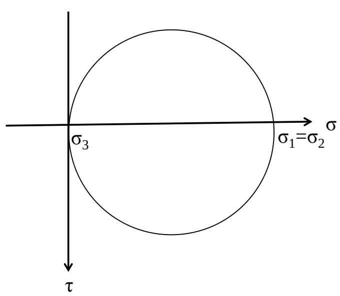
A B C D E



A B C D E



A B C D E



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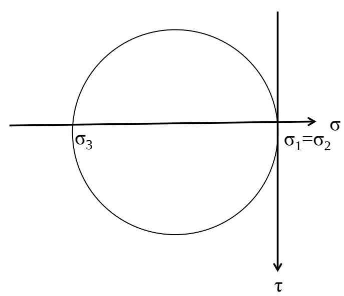

A B C D E

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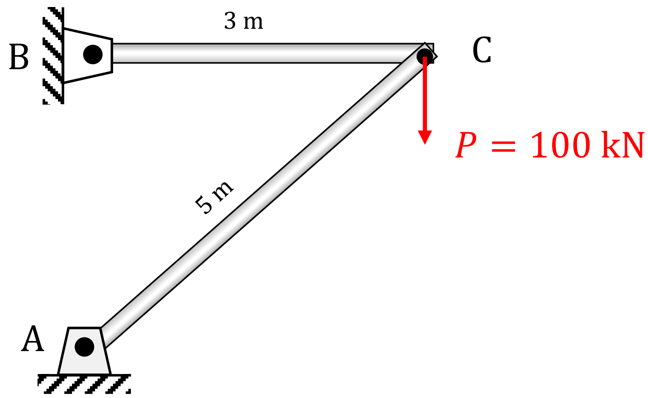
4.2 (9 Points)

Two columns AC and BC have a circular cross section of radius 5 cm, yield strength σ_Y=100 MPa, Young's modulus E=100 GPa. The columns are pin connected and a vertical load P is applied at the joint C. Considering the pin-pin boundary condition and in-plane buckling for both columns,

(a) will BC buckle? Justify your answer.

(b) will AC buckle? Justify your answer.



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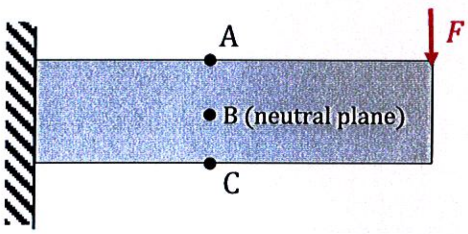
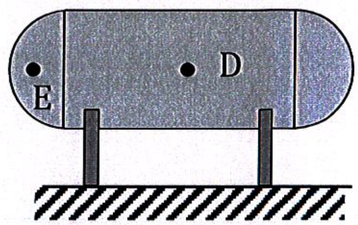
PROBLEM #4 (24 Points):
4.1. (15 Points)





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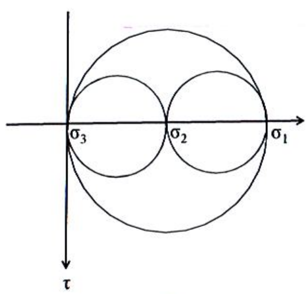
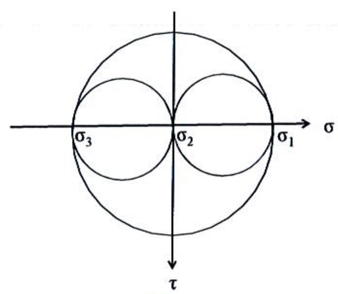


A B C D E



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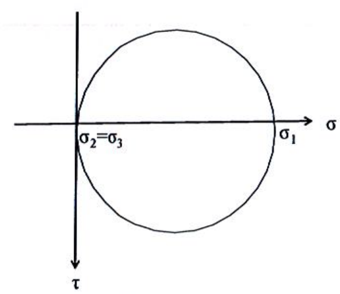
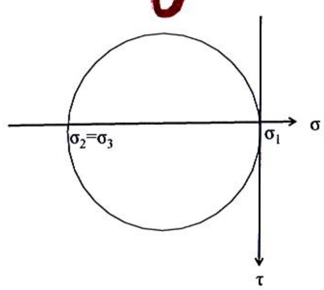


A B C D E



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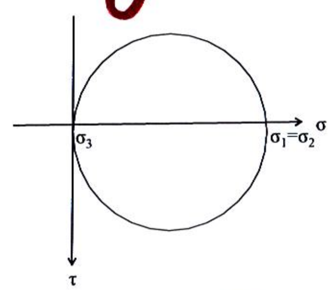
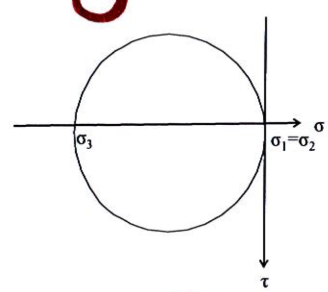


A B C D E



A B C D E

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4.2 (9 Points)

Two columns AC and BC have a circular cross section of radius 5 cm, yield strength σ_Y=100 MPa, Young's modulus E=100 GPa. The columns are pin connected and a vertical load P is applied at the joint C. Considering the pin-pin boundary condition and in-plane buckling for both columns,

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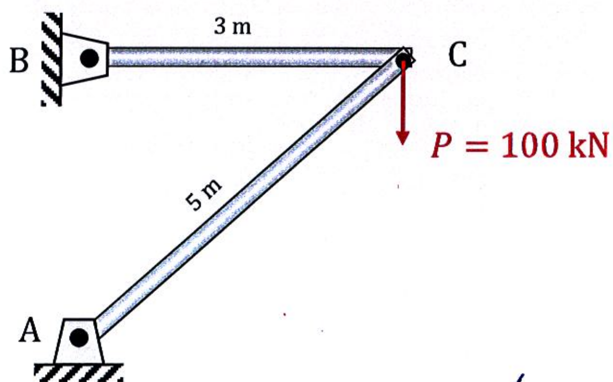

FBD: F_{BC}, C, F_{AC}, $P = 100$ kN

Equilibrium: $F_{AC} \cdot \frac{4}{5} + 100 = 0$

$F_{AC} \cdot \frac{3}{5} + F_{BC} = 0$

$\Rightarrow F_{AC} = -125$ kN, $F_{BC} = 75$ kN

(a): BC will Not buckle because BC is in tension

(b): $\sigma_{AC} = \frac{F_{AC}}{A} = \frac{-125 \times 10^3}{\pi \cdot 25 \times 10^{-4}} = -15.9$ MPa

$\frac{L_{eff}}{r} = \frac{5}{I/A} = \frac{5}{R/2} = 200$, $\left(\frac{L_{eff}}{r}\right)_c = \sqrt{\frac{\pi^2 E}{0.5\sigma_Y}} = 140$

$\frac{L_{eff}}{r} > \left(\frac{L_{eff}}{r}\right)_c$, AC is Euler column

$\sigma_{cr} = \frac{\pi^2 E}{(L_{eff}/r)^2} = \frac{\pi^2 \cdot 100 \times 10^9}{4 \times 10^4} = 24.6$ MPa

$|\sigma_{AC}| < |\sigma_{cr}|$, AC will Not buckle #