Name (Print)____

(Last) **(First)**

ME 323 - Mechanics of Materials
Final Exam
Date: December 9, 2019, Time: 3:30 – 5:30 PM

Instructions:

Circle your instructor's name and your class meeting time.

Gonzalez	Kokini	Zhao	Pribe
11:30-12:20PM	12:30-1:20PM	2:30-3:20PM	4:30-5:20PM

The only authorized exam calculator is the TI-30XIIS or the TI-30Xa.

Begin each problem in the space provided on the examination sheets.

Work on **ONE SIDE** of each sheet only, with only one problem on a sheet.

Please remember that for you to obtain maximum credit for a problem, you must present your solution clearly. Accordingly,

- coordinate systems must be clearly identified,
- free body diagrams must be shown,
- units must be stated,
- write down clarifying remarks,
- state your assumptions, etc.

If your solution cannot be followed, **it will be assumed that it is in error**.

When handing in the test, make sure that ALL SHEETS are in the correct sequential order.

Please review and sign the following statement:

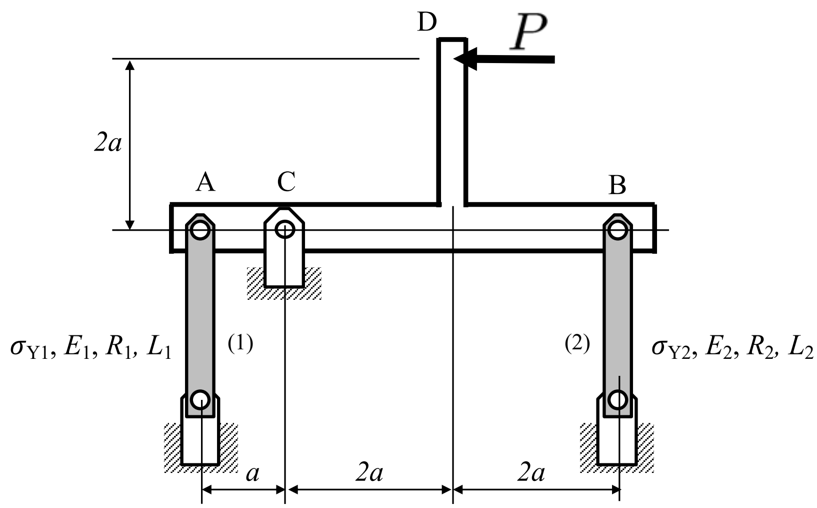
Purdue Honor Pledge – "As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – We are Purdue."

Signature: ________________________________

ME 323 Examination # 3

Name ______________________________
(Print) (Last) (First)

PROBLEM #1 (20 Points):



Two elastic elements (1) and (2) are connected to ends A and B of a rigid inverted T-shaped bar. Each elastic member has a Young's modulus E, circular cross section of radius R, length L, and yield stress σ_Y. The rigid bar is pinned at C and a load P is applied at end D, as shown in the figure.

(a) Assuming both elastic elements are under tension, draw a free body diagram of the rigid T-shaped bar.

(b) Write the equilibrium equations for the rigid T-shaped bar and the compatibility condition(s) that relate the elongation of the elastic elements (1) and (2).

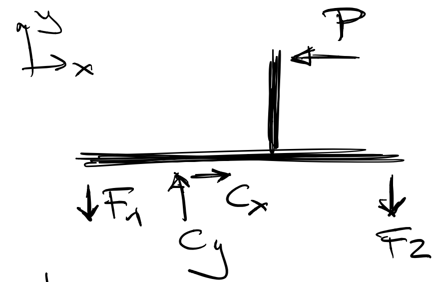
(c) Determine the axial force on elastic elements (1) and (2) as a function of material properties (E_1 and E_2), geometric parameters (R_1, R_2, L_1, L_2, a), and P.

(d) For $P > 0$, indicate whether elements (1) and (2) are under compression or tension.

(e) Determine the smallest positive force P that will induce elastic buckling on the assembly and indicate whether it will be on element (1) or (2). Express your result in terms of material properties (E_1 and E_2) and geometric parameters (R_1, R_2, L_1, L_2, a).

(f) Determine the smallest negative force P that will induce ductile failure on the assembly and indicate whether it will be on element (1) or (2). Express your result in terms of material properties (E_1, E_2, σ_{Y1}, σ_{Y2}) and geometric parameters (R_1, R_2, L_1, L_2, a).
Note: Use the maximum-distortion-energy theory.

* F.B.D. & Equilibrium



$$\sum F_y = 0 \Rightarrow C_y = F_1 + F_2$$

$$\sum F_x = 0 \Rightarrow C_x = P$$

$$\left(\sum M\right)_c = 0 \Rightarrow P \cdot 2a + F_1 \cdot a = F_2\, 4a$$

Assuming both members under tension

* Compatibility conditions.

$$e_1/a = -e_2/4a \Rightarrow 4e_1 = -e_2$$

$$e_1 = \frac{F_1 L_1}{E_1 A_1} \; ; \; e_2 = \frac{F_2 L_2}{E_2 A_2}$$

$$I_{zz} = \frac{\pi R^4}{4}$$

* Axial forces:

$$\frac{4F_1 L_1}{E_1 A_1} = -\frac{F_2 L_2}{E_2 A_2} \Rightarrow 4F_1 = -F_2 \text{ \& equilibrium} \Rightarrow 2P + F_1 = -16F_1$$

$$F_1 = -2P/17 < 0 \Rightarrow F_2 = 8P/17 > 0$$

compression   tension

* (1) under compression ⇒ buckling

Member (1)

$$\text{Pin-pin} \Rightarrow P_{cr} = \pi^2 E_1 I_1 / L_1^2 \Rightarrow \frac{\pi^2 E_1 \pi R^4}{4 L_1^2} = 2P/17 \Rightarrow P = \frac{17\pi^3 E R^4}{8L^2}$$

* The member with the largest force will yield ⇒ Member (2)

$$\sigma_M = \sigma_Y \text{ with } \sigma_M = \frac{F_2}{\pi R^2} \text{ then } \sigma_Y = \frac{8P}{17\pi R^2} \Rightarrow P = \frac{17\pi R^2 \sigma_Y}{8}$$

PROBLEM #2 (25 points)

An angled wrench ABC is fixed to the ground at A. The wrench is aligned along the *xz* plane as shown in the Figure 2A, such that the AB is along the *x* axis and BC is along the *z* axis. Two point-forces of the same magnitude *P*, **one in the negative *x* direction and the other in the positive *y* direction**, are applied at the end C. The segment AB has a circular cross section of the radius *r* as shown in Figure 2B. The length of AB and BC are *a* and *b*, respectively.

a) Determine the resultant load (forces and moments) on the cross section at the ground due to the applied forces at C.

b) Determine the state of stress at point M located on the cross section at the ground. Show the non-zero stresses on the given stress element.

c) Determine the state of stress at point N located on the cross section at the ground. Show the non-zero stresses on the given stress element.

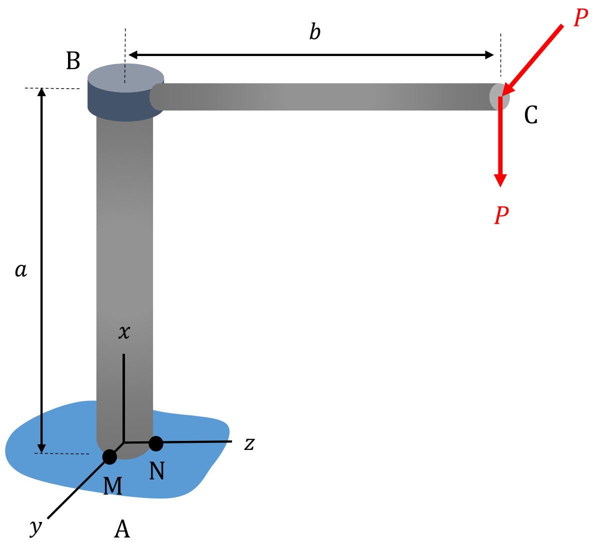


Figure 2A

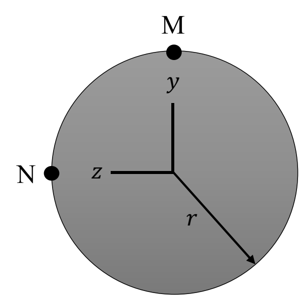


Figure 2B

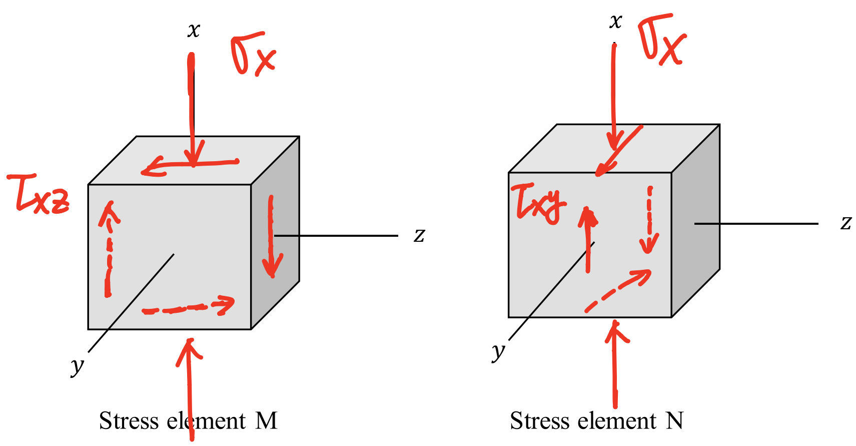


Figure 2C

Resultant load on the cross section at ground (+x face):

$$\vec{F} = -P\hat{i} + P\hat{j} + 0\hat{k}$$

$$\vec{M} = \vec{r} \times \vec{F} = (a\hat{i} + b\hat{j}) \times (-P\hat{i} + P\hat{j})$$

$$= Pa\hat{k} - Pb\hat{j} - Pb\hat{i}$$

Load	Stress at M	Stress at N
$F_x = -P$	$\sigma_x = -P/\pi r^2$	$\sigma_x = -P/\pi r^2$
$F_y = P$	$\tau_{xy} = 0$	$\tau_{xy} = \frac{4P}{3\pi r^2}$
$F_z = 0$	$\tau_{xz} = 0$	$\tau_{xz} = 0$
$M_x = -Pb$ (torque)	$\tau_{xz} = -\frac{Pb \cdot r}{\pi r^4/2} = -\frac{2Pb}{\pi r^3}$	$\tau_{xy} = +\frac{Pb \cdot r}{\pi r^4/2} = \frac{2Pb}{\pi r^3}$
$M_y = -Pb$	$\sigma_x = 0$	$\sigma_x = -\frac{Pb \cdot r}{\pi r^4/4} = -\frac{4Pb}{\pi r^3}$
$M_z = Pa$	$\sigma_x = -\frac{Pa \cdot r}{\pi r^4/4} = -\frac{4Pa}{\pi r^3}$	$\sigma_x = 0$

M: $\sigma_x = -\frac{P}{\pi r^2} - \frac{4Pa}{\pi r^3}$, $\tau_{xz} = -\frac{2Pb}{\pi r^3}$

N: $\sigma_x = -\frac{P}{\pi r^2} - \frac{4Pb}{\pi r^3}$, $\tau_{xy} = \frac{2Pb}{\pi r^3} + \frac{4P}{3\pi r^2}$

Name (Print) SOLUTION (Last) (First)

PROBLEM #3 (25 points)

A point A on the structure in Figure 3A is subjected to in-plane stresses as shown in Figure 3B.

(a) Use the stress element in Figure 3B to draw the **Mohr's circle** on the attached graph paper.

(b) Use the **Mohr's circle** to calculate:

i. The principal stresses in the X-Y plane.
ii. The maximum in-plane shear stress.
iii. The absolute maximum shear stress.
iv. The angle of rotation from the X-axis to the direction of the in-plane principal stress σ_{p1}.
v. Draw a stress element to show the in-plane principal stresses correctly orientated with respect to the X axis

(c) Determine the normal and shear stresses in the X'-Y' directions, draw a stress element to show the calculated stresses, and mark the state of stress in the X'-Y' directions on the Mohr's circle. Note: The X' axis is oriented at 45° from the X axis as shown in Figure 3A.

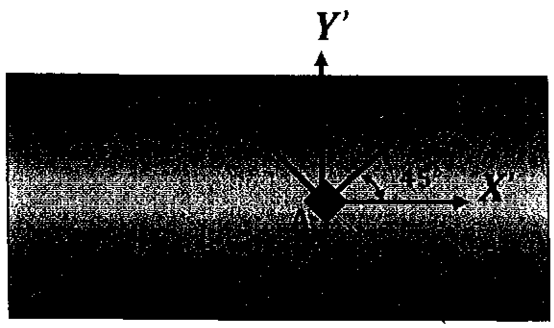


Figure 3A: Structure with element A

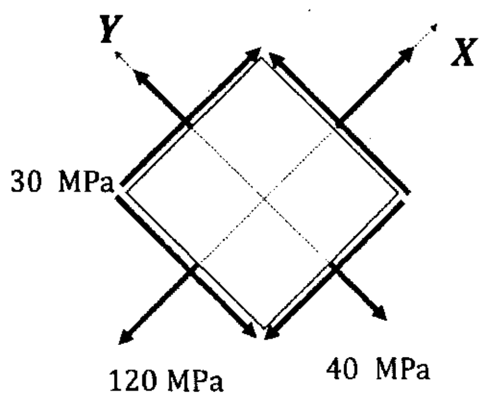
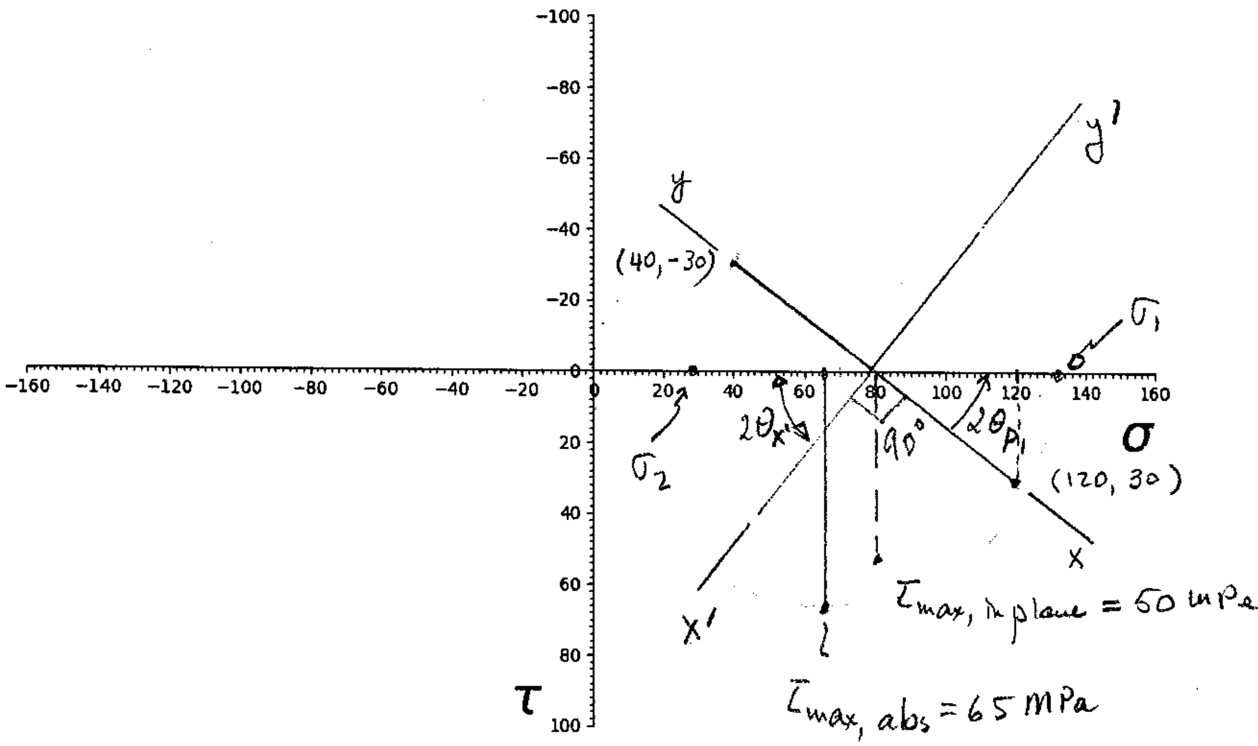


Figure 3B: Stress element A

SOLUTION



$$\sigma_{av} = \frac{\sigma_x + \sigma_y}{2} = \frac{120 + 40}{2} = 80 \text{ MPa}$$

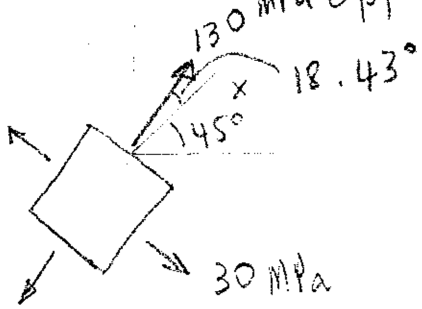
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{120 - 40}{2}\right)^2 + (30)^2} = 50 \text{ MPa}$$

$$\sigma_1 = \sigma_{av} + R = 80 + 50 = 130 \text{ MPa}$$
$$\sigma_2 = \sigma_{av} - R = 80 - 50 = 30 \text{ MPa}$$

$$\tau_{max,\,in\,plane} = R = 50 \text{ MPa}$$

$$\tau_{max,\,abs} = \frac{\sigma_1}{2} = \frac{130}{2} = 65 \text{ MPa}$$

$$\tan 2\theta_{p_1} = \frac{30}{40} \Rightarrow 2\theta_{p_1} = 36.87^\circ$$
$$\theta_{p_1} = 18.43^\circ$$



(c) $2\theta_{x'} = 90^\circ - 2\theta_{p_1}$

$= 90^\circ - 36.87^\circ = 53.13^\circ$

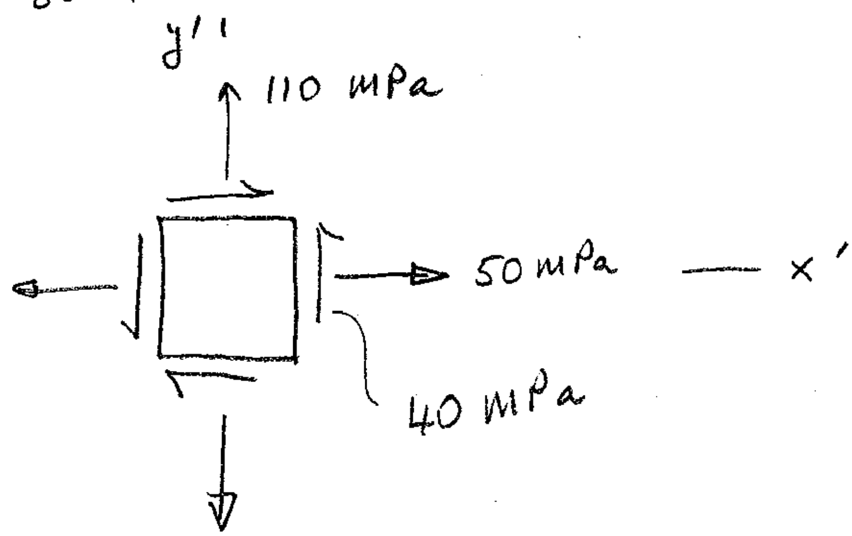
$\sigma_{x'} = \sigma_{av} - R\cos 53.13^\circ$

$= 80 - 50\cos 53.13^\circ = 50 \text{ MPa}$

$\tau_{x'y'} = R\sin 53.13^\circ = 50\sin 53.13^\circ = 40 \text{ MPa}$

$\sigma_{y'} = \sigma_{av} + R\cos 53.13^\circ$

$= 80 + 50\cos 53.13^\circ = 110 \text{ MPa}$



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(Print) (Last) (First)

PROBLEM #4 (25 Points):

PART A – 5 points

(i) For the state of stress shown below, $\boldsymbol{\sigma_x = \sigma_y = 0}$ and $\boldsymbol{\tau_{xy} = \tau}$. Let $\boldsymbol{\tau_{MSS}}$ and $\boldsymbol{\tau_{MDE}}$ be the values of $\boldsymbol{\tau}$ required to cause yielding based on the maximum shear stress and maximum distortional energy theories, respectively. The yield strength of the ductile material is $\boldsymbol{\sigma_Y}$.

Circle the answer that best describes the relative sizes of $\boldsymbol{\tau_{MSS}}$ and $\boldsymbol{\tau_{MDE}}$.

(a) $\boldsymbol{\tau_{MSS} > \tau_{MDE}}$

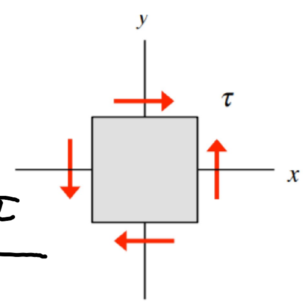
(b) $\boldsymbol{\tau_{MSS} = \tau_{MDE}}$

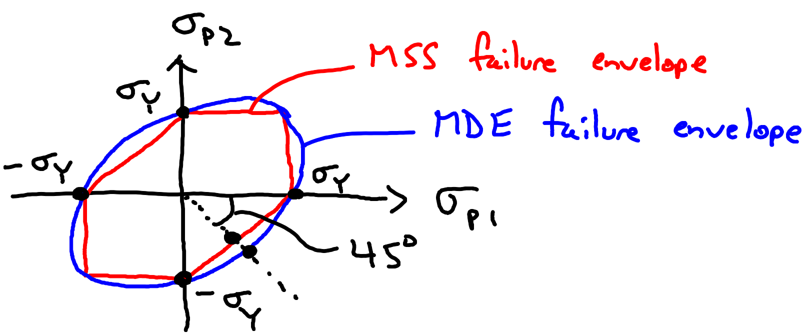
(c) $\boldsymbol{\tau_{MSS} < \tau_{MDE}}$

$\sigma_{avg} = 0$

$R = \sqrt{0^2 + \tau^2} = \tau$

$\Rightarrow \sigma_{p1} = \tau, \ \sigma_{p2} = -\tau$





Since $|\sigma_{p1}| = |\sigma_{p2}|$ and $\sigma_{p1} > 0 > \sigma_{p2}$, we are on a 45° line in the 4th quadrant (see plot to the left)

$\Rightarrow \tau_{MSS} < \tau_{MDE}$

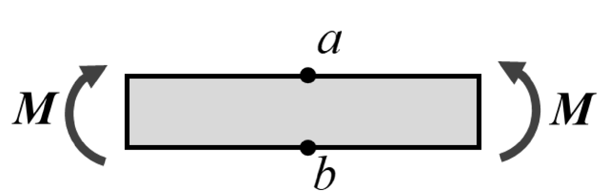
(ii) The beam shown below has a square cross section and is made of a brittle material where the ultimate compressive strength is ***larger*** than the ultimate tensile strength. The beam is subjected to a bending moment $\boldsymbol{M > 0}$ as shown below. Let $\boldsymbol{M_a}$ and $\boldsymbol{M_b}$ be the values of the bending moment required to cause brittle failure at points a and b, respectively, based on **Mohr's failure criterion**.

Circle the answer that best describes the relative sizes of $\boldsymbol{M_a}$ and $\boldsymbol{M_b}$.

(a) $\boldsymbol{M_a > M_b}$

(b) $\boldsymbol{M_a = M_b}$

(c) $\boldsymbol{M_a < M_b}$



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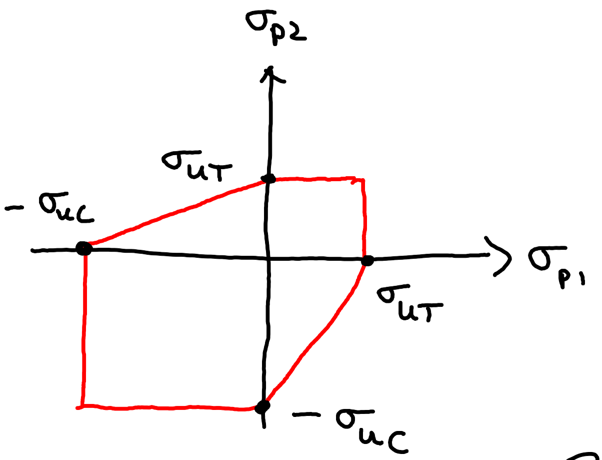
(ii) $\sigma_{x,a} < 0$, $\sigma_{x,b} > 0$, and $|\sigma_{x,a}| = |\sigma_{x,b}|$



at a: $\sigma_{p1} = 0$, $\sigma_{p2} = \sigma_{x,a} < 0$

$\Rightarrow$ failure at a when $\underline{\sigma_{x,a} = -\sigma_{UC}}$

at b: $\sigma_{p1} = \sigma_{x,b} > 0$, $\sigma_{p2} = 0$

$\Rightarrow$ failure at b when $\underline{\sigma_{x,b} = \sigma_{UT}}$

Since $\sigma_{UC} > \sigma_{UT}$, $\boxed{M_a > M_b}$

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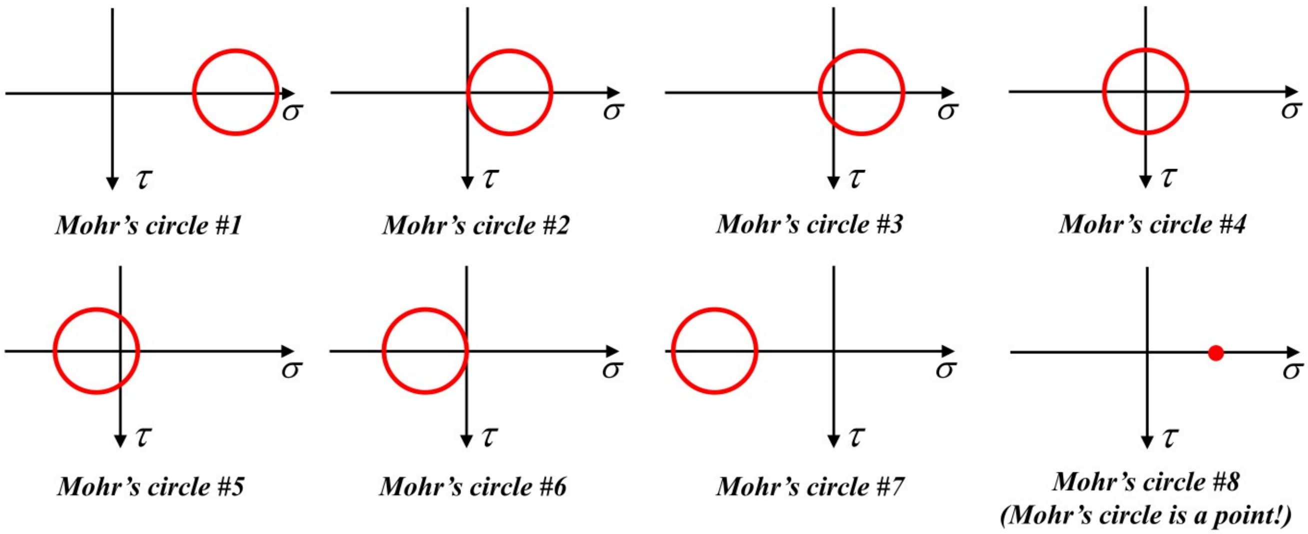
PROBLEM #4 (cont.):

PART B – 10 points

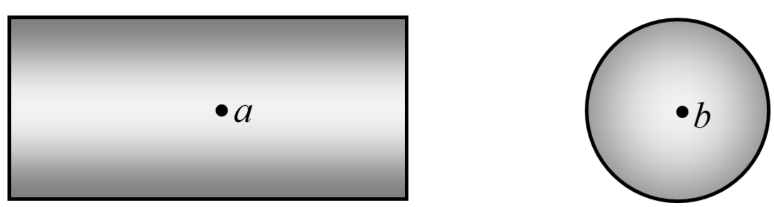
Questions (i) and (ii) in Part B refer to the eight Mohr's circles shown below.



Mohr's circle #1 ***Mohr's circle #2*** ***Mohr's circle #3*** ***Mohr's circle #4***

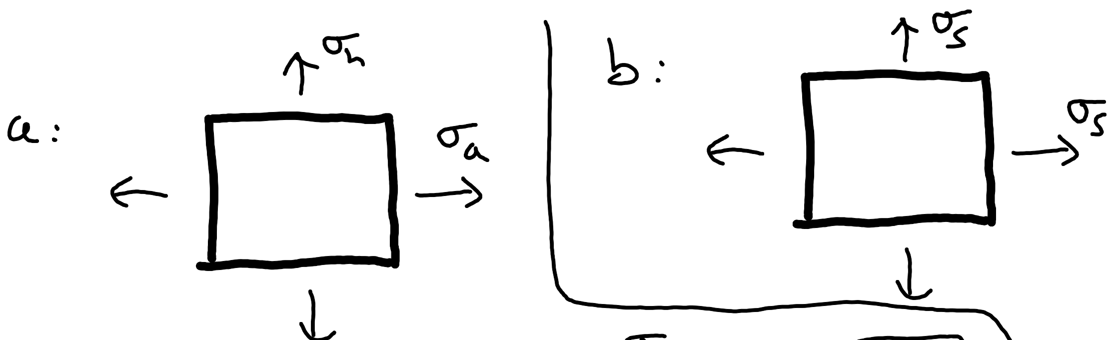
Mohr's circle #5 ***Mohr's circle #6*** ***Mohr's circle #7*** ***Mohr's circle #8 (Mohr's circle is a point!)***

(i) The cylindrical and spherical thin-walled pressure vessels are each subjected to an internal pressure. Point a is on the surface of the cylindrical pressure vessel. Point b is on the surface of the spherical pressure vessel.



Circle the number of the **correct in-plane Mohr's circle** for the state of stress at:

- Point a: #1 #2 #3 #4 #5 #6 #7 #8
- Point b: #1 #2 #3 #4 #5 #6 #7 #8



$\sigma_{p1} = \sigma_{p2} = \sigma_s$

$\Rightarrow \boxed{\#8}$

$\sigma_{p1} = \sigma_h > 0 \quad \sigma_{p2} = \sigma_a = \frac{\sigma_h}{2} > 0 \Rightarrow \boxed{\#1}$

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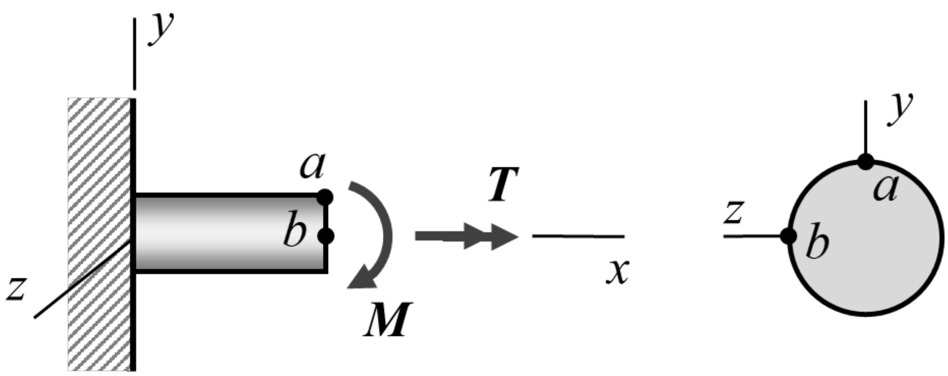
PROBLEM #4 (cont.):

PART B (cont.)

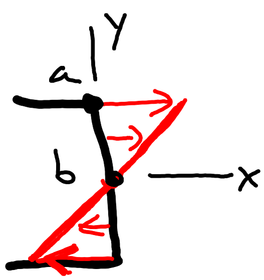
(ii) At a cut in the circular rod shown below, the internal resultant loads are determined to be a bending moment ***M*** about the negative z-axis and a torque ***T*** about the positive x-axis.



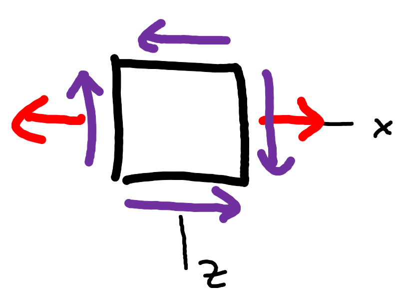
Circle the number of the **correct in-plane Mohr's circle** for the state of stress at:

- Point a: #1 #2 #3 #4 #5 #6 #7 #8
- Point b: #1 #2 #3 #4 #5 #6 #7 #8

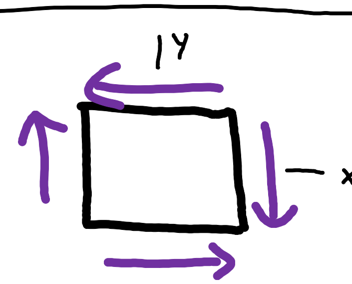
Normal stress due to M:



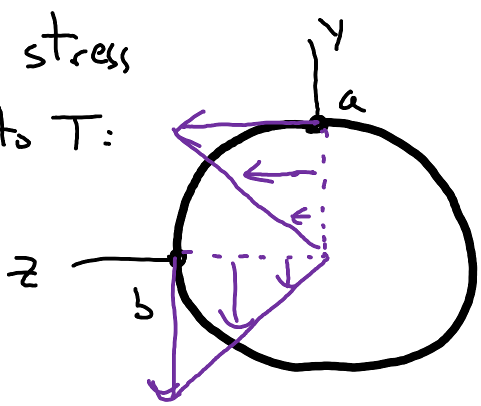
Stress element at a



Stress element at b



Shear stress due to T:

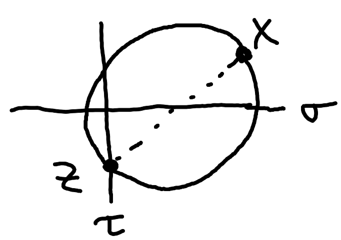


$\sigma_{x,a} > 0$

$\tau_{xz,a} > 0$

$\sigma_{z,a} = 0$

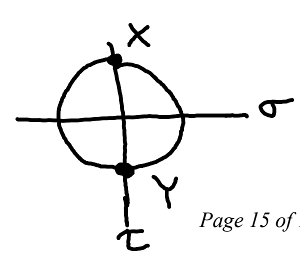
$\Rightarrow \boxed{\#3}$



$\sigma_{x,b} = \sigma_{y,b} = 0$

$\tau_{xy,b} < 0$

$\Rightarrow \boxed{\#4}$



Name SOLUTION
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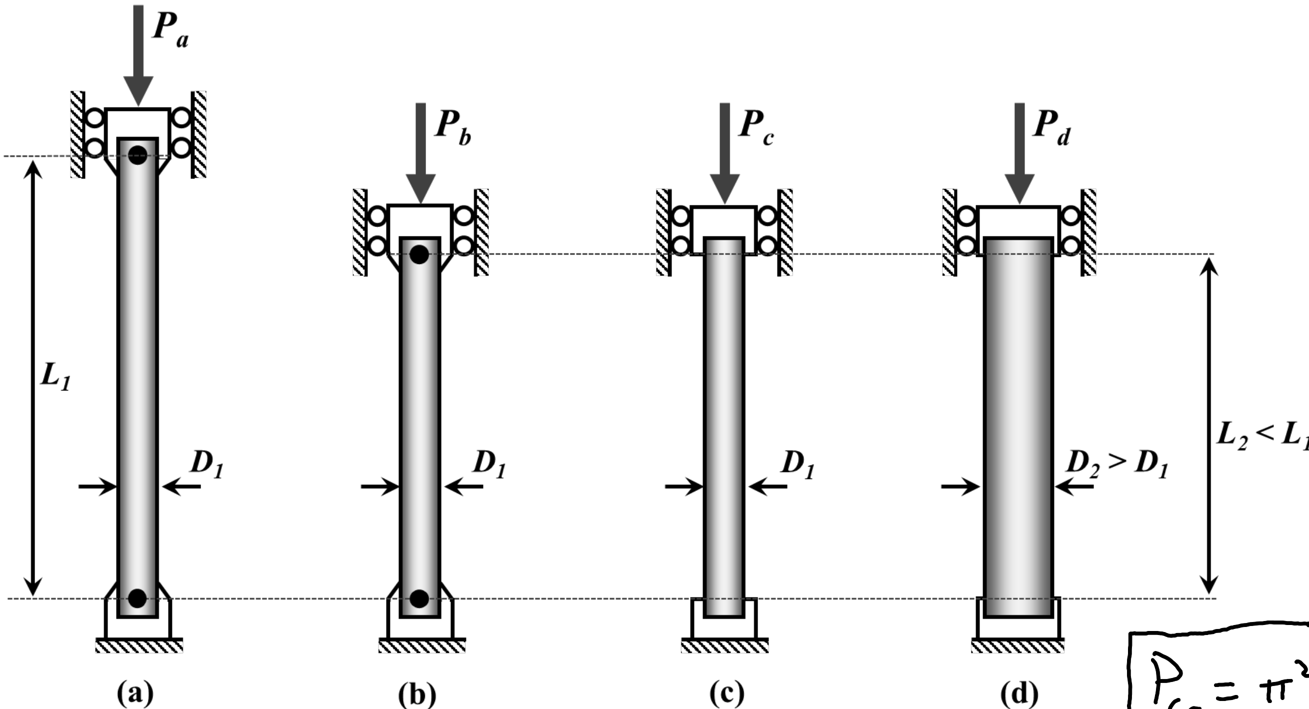
PART C – 6 points



$$P_{cr} = \pi^2 \frac{EI}{L_e^2}$$

Solid cylindrical columns (a), (b), (c), and (d) are made of the same material with Young's modulus E. A compressive axial load is applied to each column. Let $P_{a,cr}$, $P_{b,cr}$, $P_{c,cr}$, and $P_{d,cr}$ represent the critical buckling loads for columns (a), (b), (c), and (d), respectively, according to Euler's buckling theory.

Rank order the critical buckling loads for each column from 1 to 4, where 1 represents the *largest* critical buckling load, and 4 represents the *smallest* critical buckling load, on the lines below.

$P_{a,cr}$ 4 $\qquad L_{a,e} = L_1$

$P_{b,cr}$ 3 $\qquad L_{b,e} = L_2 < L_1$

$P_{c,cr}$ 2 $\qquad L_{c,e} = 0.5 L_2 < L_2$

$P_{d,cr}$ 1 $\qquad L_{d,e} = 0.5 L_2 < L_2$

$I = I_a = I_b = I_c$

$I_d > I$

d has smallest Le + largest I $\Rightarrow$ $P_{d,cr}$ is the largest

a has largest Le + smallest I $\Rightarrow$ $P_{a,cr}$ is the smallest

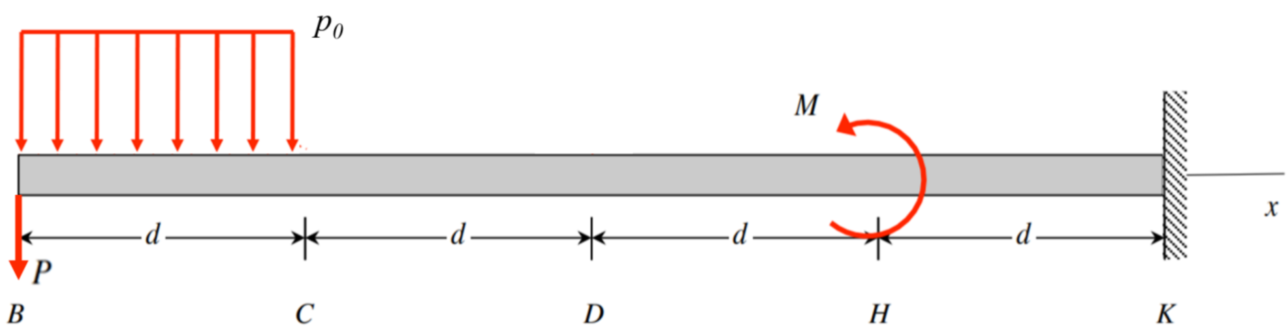
$L_{b,e} > L_{c,e} \Rightarrow P_{b,cr} < P_{c,cr}$

Name SOLUTION
(Print) (Last) (First)

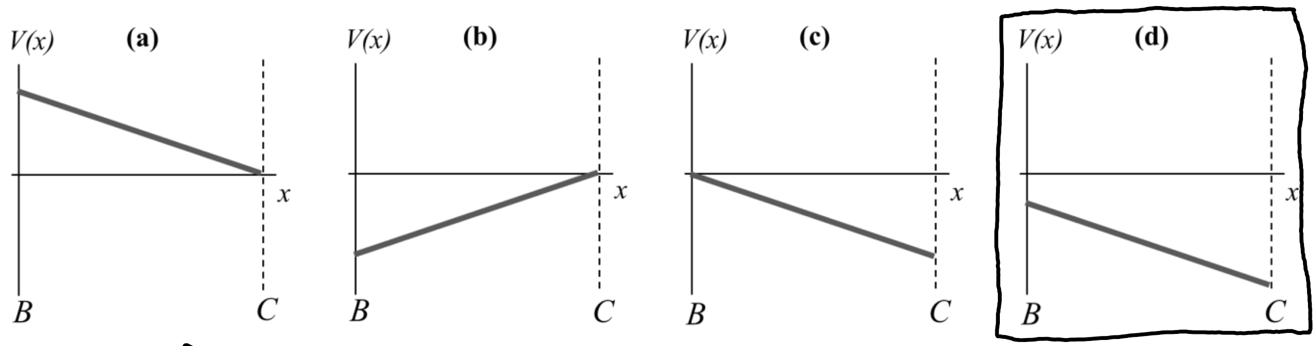
PROBLEM #4 (cont.):

PART D – 4 points

A cantilevered beam is loaded with a force ***P***, a distributed load $\boldsymbol{p_0}$, and a moment ***M***.

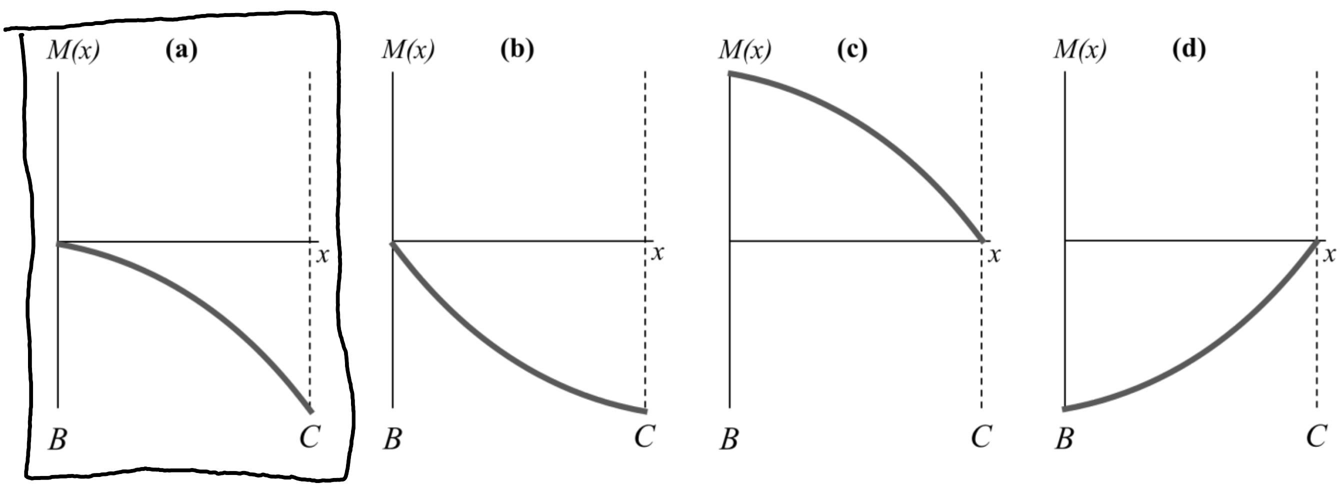


(i) Circle the answer that most accurately describes the internal **shear force** between points *B* and *C*.



$V(0^+) = -P \quad V(x) = -P - p_0 x$ between $B + C \Rightarrow$ $\boxed{\text{(d) is correct}}$

(ii) Circle the answer that most accurately describes the internal **bending moment** between points B and C.



$M(0^+) = 0, \quad M(x) = -Px - \frac{1}{2} p_0 x^2$ between $B + C \Rightarrow$ (c) + (d) are wrong

M is decreasing + concave down between $B + C \Rightarrow$ $\boxed{\text{(a) is correct}}$