ME 323 Final Examination **Name** ________________________________

December 12, 2017 ***Instructor***________________________________

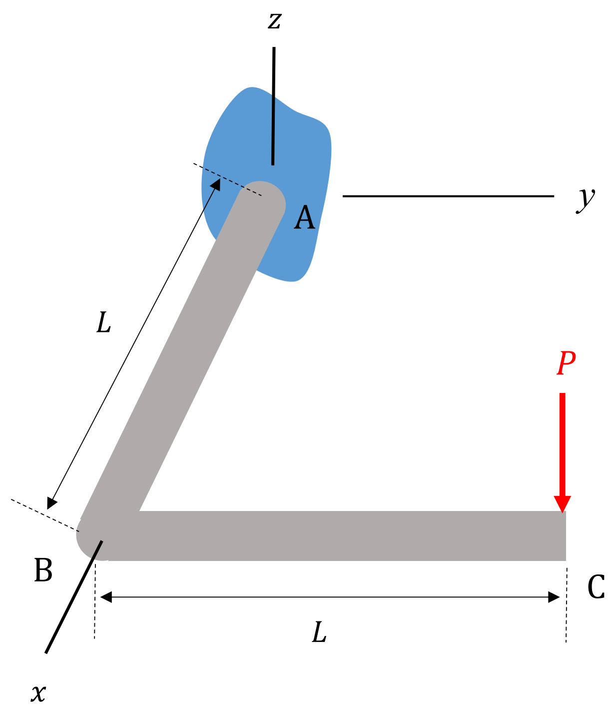
PROBLEM NO. 1 – 25 points max.

The right-angle bar ABC has a circular cross section of radius *r*. It is fixed at wall at A, and is subject to a concentrated force *P* at location C. The elastic modulus of the bar is *E* and shear modulus is *G*. Using Castigliano's theorem:

a) Determine the deflection of the bar in the *z*-direction at C.

b) Determine the deflection of the bar in the *z*-direction at B.

Ignore the shear energy due to bending. Express your answers in terms of *P*, *L*, *E*, *G*, and *r*.

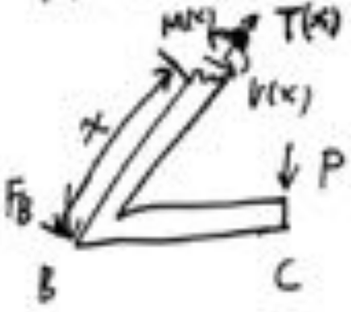


P1: solution:

Apply a dummy force at B: F_B

$$U_{total} = U_{AB} + U_{BC}$$

For AB section:



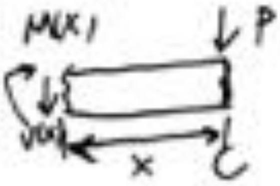
$$T(x) = -PL$$

$$M(x) = (P+F_B)\,x$$

$$U_{AB} = \frac{1}{2}\int_0^L \frac{T^2(x)}{GI_p}\,dx + \frac{1}{2}\int_0^L \frac{M^2(x)}{EI}\,dx$$

$$= \frac{1}{2}\int_0^L \frac{(PL)^2}{GI_p}\,dx + \frac{1}{2}\int_0^L \frac{[(P+F_B)x]^2}{EI}\,dx$$

For BC section:



$$M(x) = -Px$$

$$U_{BC} = \frac{1}{2}\int_0^L \frac{M^2(x)}{EI}\,dx = \frac{1}{2}\int_0^L \frac{(Px)^2}{EI}\,dX$$

$$\Delta_C = \left.\frac{\partial U_{total}}{\partial P}\right|_{F_B=0} = \frac{PL^3}{GI_p} + \frac{\frac{1}{3}PL^3}{EI} + \frac{1}{3}\frac{PL^3}{EI} = \frac{PL^3}{GI_p} + \frac{2}{3}\frac{PL^3}{EI}$$

$$\Delta_B = \left.\frac{\partial U_{total}}{\partial F_B}\right|_{F_B=0} = \frac{1}{3}\frac{PL^3}{EI}$$

$$I_p = \frac{\pi r^4}{2}, \quad I = \frac{\pi r^4}{4}$$

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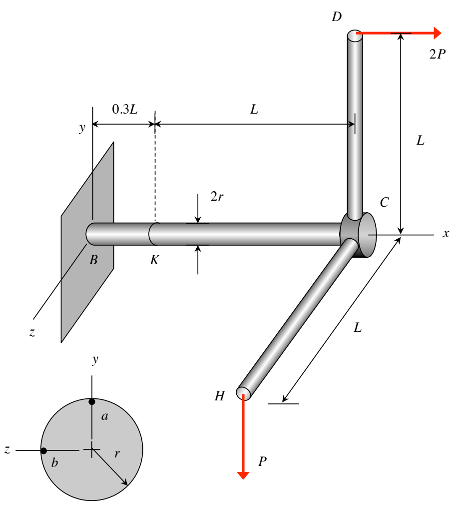
PROBLEM NO. 2 – 25 points max.

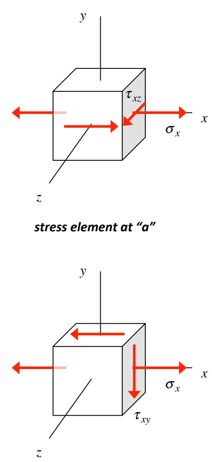
A bracket is made up of components BC, DC and HC, all joined together at C, with BC, DC and HC aligned with the x-, y- and z-axes, respectively. Loads of P and 2P act at ends H and D in the negative y-direction and positive x-direction, respectively. Component BC has a circular cross section with an outer radius of r.

a) Determine the xyz components of stress at points "a" and "b" on the cross section K of component BC. Express your answers in terms of, at most, L, r and P.

b) Show these components of stress on the stress elements provided below for points "a" and "b".

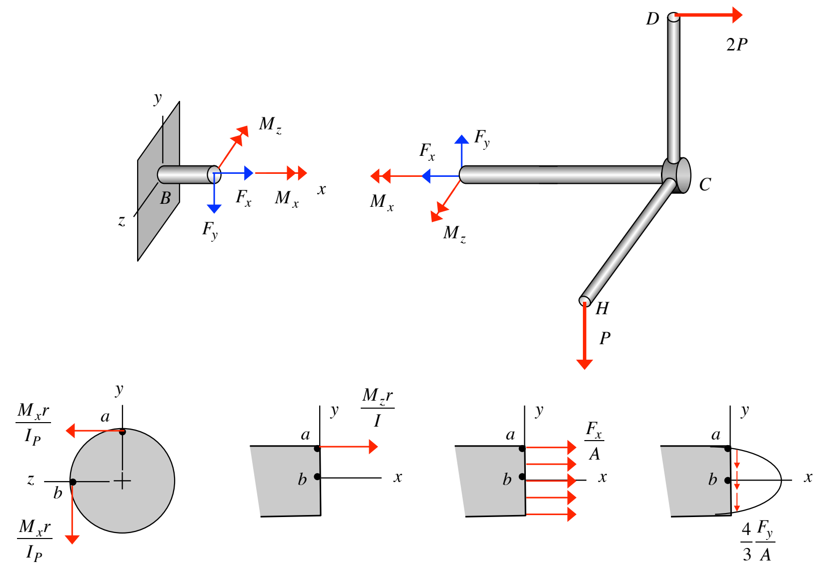


cross section of member BC at location "K"



stress element at "a"

stress element at "b"



Equilibrium – FBD above

$$\sum F_x = 2P - F_x = 0 \ \Rightarrow \ F_x = 2P$$

$$\sum F_y = -P + F_y = 0 \ \Rightarrow \ F_y = P$$

$$\sum \vec{M}_K = \left(L\hat{i} + L\hat{k}\right)\times\left(-P\hat{j}\right) + \left(L\hat{i} + L\hat{j}\right)\times\left(2P\hat{i}\right) - M_x\hat{i} - M_z\hat{k} = \left(PL - M_x\right)\hat{i} + \left(-PL - 2PL + M_z\right)\hat{k} = \vec{0} \ \Rightarrow$$

$$\hat{i}: \ M_x = PL$$

$$\hat{k}: \ M_z = 3PL$$

Using: $I = \frac{\pi}{4}r^4, I_P = \frac{\pi}{2}r^4 \ and \ A = \pi r^2$, and the stress distributions above, we have:

Point "a"

$$\sigma_x = \frac{M_z r}{I} + \frac{F_x}{A} = \frac{(3PL)r}{\pi r^4/4} + \frac{2P}{\pi r^2} = \frac{2P}{\pi r^2}\left[1 + 6\frac{L}{r}\right]$$

$$\tau_{xz} = \frac{M_x r}{I_P} = \frac{(PL)r}{\pi r^4/2} = \frac{2P}{\pi r^2}\left(\frac{L}{r}\right)$$

Point "b"

$$\sigma_x = \frac{F_x}{A} = \frac{2P}{\pi r^2}$$

$$\tau_{xy} = -\frac{M_x r}{I_P} - \frac{4}{3}\frac{F_y}{A} = -\frac{(PL)r}{\pi r^4/2} - \frac{4}{3}\frac{P}{\pi r^2} = -\frac{P}{\pi r^2}\left(2\frac{L}{r} + \frac{4}{3}\right)$$

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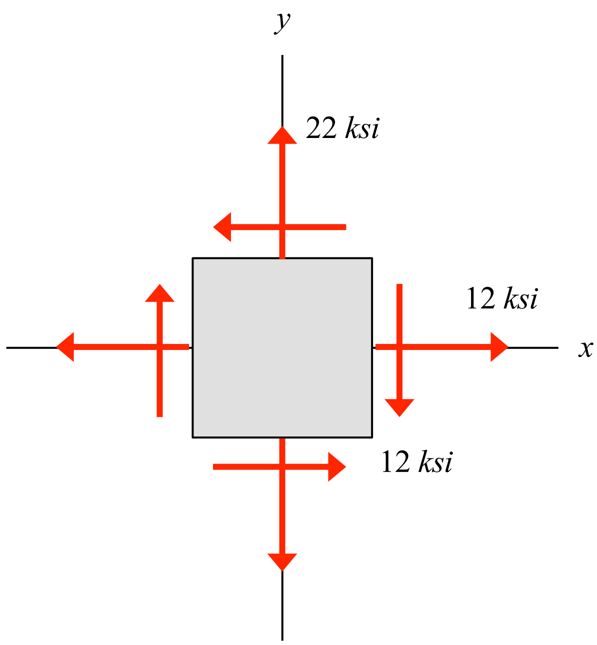
PROBLEM NO. 3A – 15 points max.

The x-y components for a state of plane stress in a machine component are shown below. The component is made up from a ductile material having a yield strength of $\sigma_Y = 36\ ksi$. For this state of stress:

a) Determine if the *maximum-shear stress theory* predicts a failure of the material. If the material has not failed, what is the factor of safety predicted by this theory?

b) Determine if the *maximum-distortional-energy theory* predicts a failure of the material. If the material has not failed, what is the factor of safety predicted by this theory?



$$\sigma_{ave} = \frac{12+22}{2} = 17\ ksi$$

$$R = \sqrt{\left(\frac{12-22}{2}\right)^2 + (-12)^2} = 13\ ksi$$

$$\therefore \begin{cases} \sigma_{p1} = \sigma_{ave} + R = 30\ ksi \\ \sigma_{p2} = \sigma_{ave} - R = 4\ ksi \end{cases}$$

Since both σ_{p1} & σ_{p2} are positive $\Rightarrow$

$$|\tau|_{max,abs} = \frac{\sigma_{p1}}{2} = 15\ ksi$$

(a) $FS_{MSS} = \dfrac{\sigma_Y/2}{|\tau|_{max,abs}} = \dfrac{36/2}{15} = 1.20$

(b) $\sigma_M = \sqrt{\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2}$

$= \sqrt{30^2 - (30)(4) + 4^2}$

$= 28.2$ ksi

$\therefore \quad FS_{MDE} = \dfrac{\sigma_Y}{\sigma_M} = \dfrac{36}{28.2} = 1.28$

(Note: $FS_{MDE} > FS_{MSS}$, as expected)

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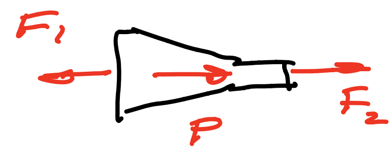
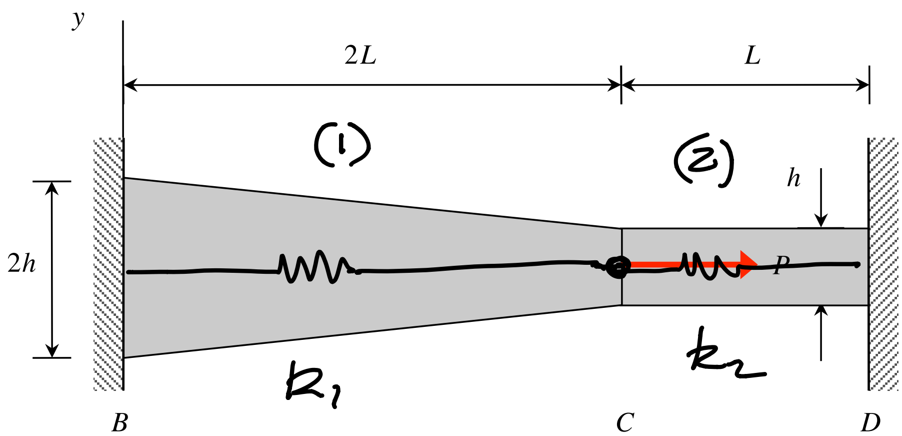
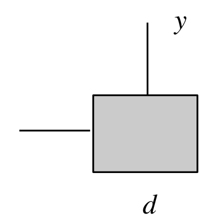
PROBLEM NO. 3B - 10 points max.

A rectangular cross-section rod is made up of sections BC and CD, with each section having a constant depth dimension d as shown in the figures below and being made up of a material having a Young's modulus of E. Section BC has a thickness that varies linearly from $2h$ on its left end to h on its right end at C, whereas section CD has a constant thickness h throughout its length. Ends B and D of the rod are connected to rigid walls. An axial load P acts the point C where the two sections are joined. Use a two-element *finite element model* to determine the *stress experienced in section CD of the rod*. Express your answer in terms of, at most, the variables h, d, L and E.







rod cross section
(showing dimension
d into the page)

$$k_1 = \frac{E}{2L} A_{1,ave} = \frac{E}{2L} \frac{1}{2}(2hd + hd) = \frac{3}{4}\frac{Ehd}{L}$$

$$k_2 = \frac{E}{L} A_{2,ave} = \frac{Ehd}{L}$$

$$[K] = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & -\frac{3}{4} & 0 \\ -\frac{3}{4} & \frac{7}{4} & -1 \\ 0 & -1 & 1 \end{bmatrix} \frac{Ehd}{L}$$

$$\{F\} = \begin{Bmatrix} -F_1 \\ P \\ F_2 \end{Bmatrix}$$

$$[K]\{u\} = \{F\}$$

To enforce BC's, remove 1st/3rd columns and rows of [K] and the 1st/3rd rows of {F}:

$$\left(\frac{7}{4}\frac{Ehd}{L}\right)u_2 = P \Rightarrow u_2 = \frac{4PL}{7Ehd}$$

From 3rd equation prior to enforcing BCs:

$$-\frac{Ehd}{L}u_2 + \frac{Ehd}{L}\cancel{u_3}^{\,0} = F_2$$

$$\hookrightarrow F_2 = -\frac{Ehd}{\cancel{L}}\left(\frac{4P\cancel{L}}{7\cancel{Ehd}}\right) = -\frac{4}{7}P$$

$$\therefore \quad \sigma_2 = \frac{F_2}{A_2} = \frac{-\frac{4}{7}P}{hd} = -\frac{4}{7}\frac{P}{hd}$$

compression

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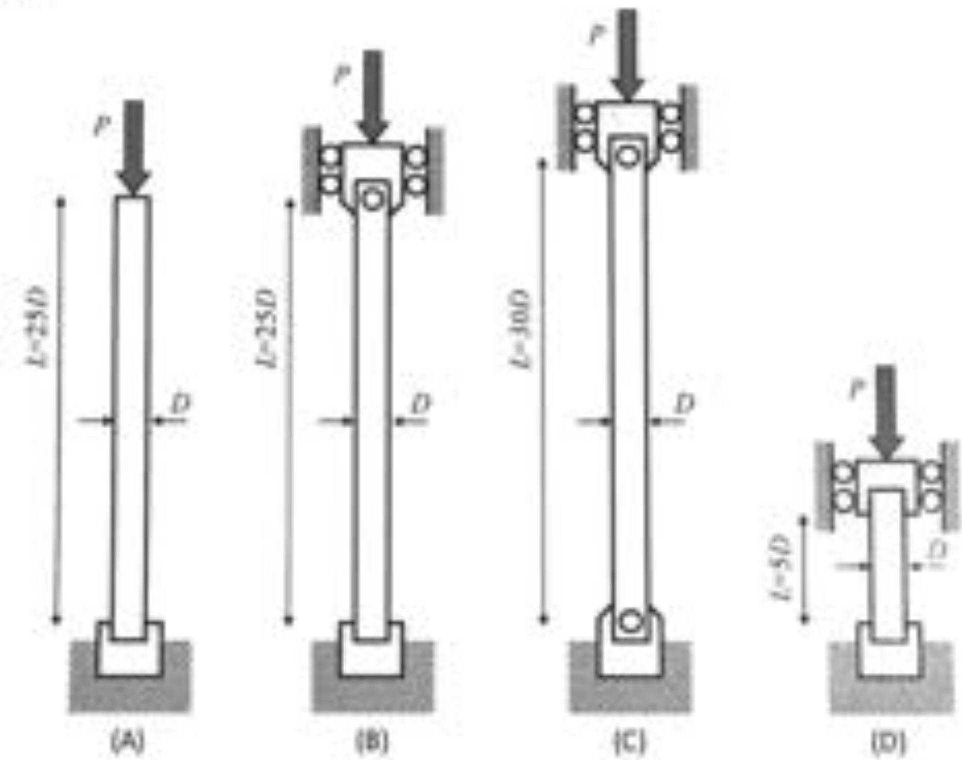
PROBLEM NO. 4A - 5 points max. (partial credit, please show your work here and on the next page):



Cylindrical columns A, B, C and D shown above are made of the same material (Young's modulus of E and yield stress of $\sigma_Y = E/100$) and have the same circular cross-section of diameter D. A compressive axial load P is applied to each column.

Let P_{cr}^A, P_{cr}^B, P_{cr}^C, and P_{cr}^D represent the critical buckling loads for columns A, B, C, and D, respectively. <u>Rank order</u> (from largest to smallest, 1 to 5) the following: P_{cr}^A, P_{cr}^B, P_{cr}^C, P_{cr}^D, and $\pi R^2\sigma_Y/2$ below.

P_{cr}^A 5

P_{cr}^B 3

P_{cr}^C 4

P_{cr}^D 1

$\dfrac{\pi R^2\sigma_Y}{2}$ 2

Ⓐ $l_{eff} = 2L$

$$S_r = \frac{8L}{D} = 200 > (S_r)_{cr} \Rightarrow \text{Euler's column}$$

$$P_{cr}^{A} = \frac{\pi EI}{l_{eff}^2} = \frac{\pi EI}{200^2}$$

$$S_r = \frac{l_{eff}}{\sqrt{I/A}} = \frac{4 l_{eff}}{D}$$

$$(S_r)_{cr} = \pi \sqrt{\frac{2E}{\sigma_Y}} = 44.4$$

Ⓑ $l_{eff} = 0.7L$

$$\Rightarrow S_r = \frac{2.8L}{D} = 70 > (S_r)_{cr} \Rightarrow \text{Euler's column}$$

$$P_{cr}^{B} = \frac{\pi EI}{l_{eff}^2} = \frac{\pi EI}{70^2}$$

Ⓒ $l_{eff} = L$

$$\Rightarrow S_r = \frac{4L}{D} = 120 > (S_r)_{cr} \Rightarrow \text{Euler's column}$$

$$P_{cr}^{C} = \frac{\pi EI}{l_{eff}^2} = \frac{\pi EI}{120^2}$$

Ⓓ $l_{eff} = 0.7L$

$$\Rightarrow S_r = \frac{2.8L}{D} = 14 < (S_r)_{cr} \Rightarrow \text{Jhonson's column.}$$

$$P_{cr}^{D} = \sigma_Y A\left[1 - \frac{\sigma_Y S_r^2}{4\pi^2 E}\right] = 0.95\, \sigma_Y A > \sigma_Y \frac{\pi R^2}{2}$$

Also, for given E and σ_Y, any Euler's column will have a buckling load smaller than the buckling load of a Jhonson's column.

$[\text{at } (S_r)_{cr}\ P_{cr} = \sigma_Y A/2]$

$$P_{cr}^{D} > \sigma_Y \frac{\pi R^2}{2}$$

$$\Rightarrow \left.\begin{array}{l} P_{cr}^{D} > P_{cr}^{A} \\ P_{cr}^{D} > P_{cr}^{B} \\ P_{cr}^{D} > P_{cr}^{C} \end{array}\right\}$$

ORDER:

$$P_{cr}^{D} > \frac{\sigma_Y \pi R^2}{2} > P_{cr}^{B} > P_{cr}^{C} > P_{cr}^{A}$$

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The schematics presented below correspond to beams with span L and loaded with either a couple at the left end or with a force per unit of length with different distributions (i.e., linear, constant or quadratic).

Problem 4B - 7 points max. (partial credit, please show your work on next page)

Which schematic corresponds to a beam whose bending moment is equal to

$$M(x) = \frac{x\, w_0 L}{10} - \frac{w_0\, x^3}{6L}$$

Circle the correct answer (a) (b) (c) (d) (e) (f) (g) (h) (i) (j)

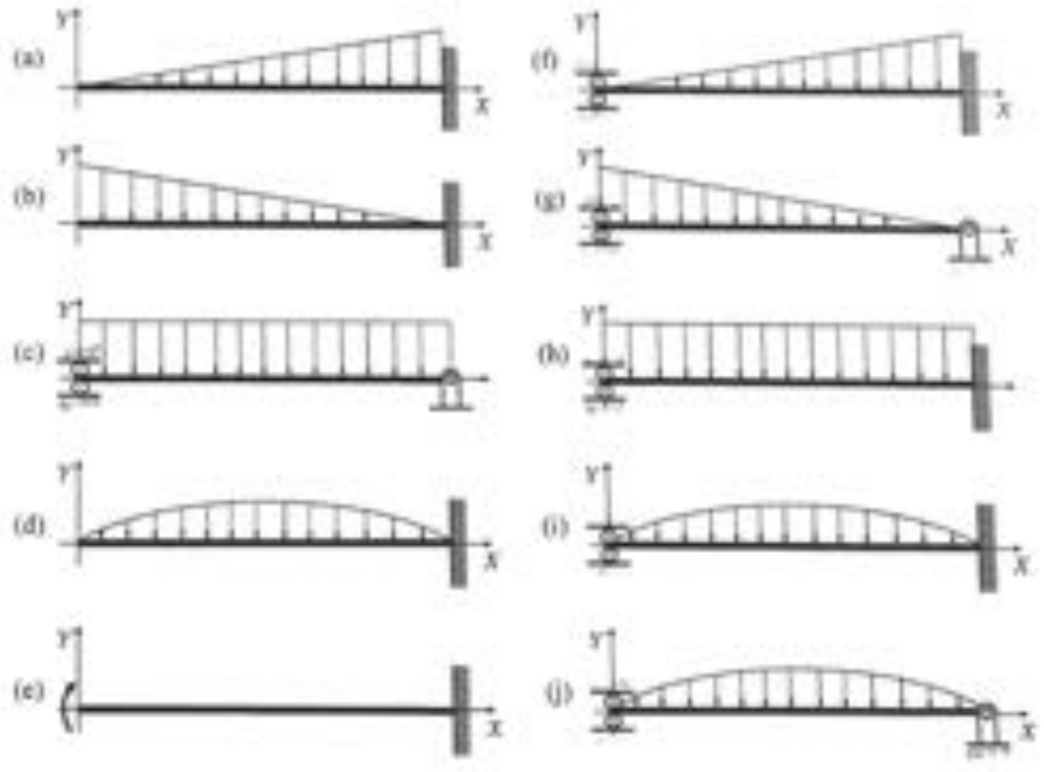
Problem 4C - 7 points max. (partial credit, please show your work on next page)

Which schematic corresponds to a beam whose bending moment is equal to

$$M(x) = \frac{w_0\, x^4}{12} - \frac{w_0\, L\, x^3}{6}$$

Circle the correct answer (a) (b) (c) (d) (e) (f) (g) (h) (i) (j)

HINT: Identify which supports and loads are compatible with the given bending moments.



P4B:

$$V(x) = \frac{dM}{dx} = \frac{w_0 L}{10} - \frac{w_0 x^2}{2L}$$

$$p(x) = \frac{dV}{dx} = \frac{-w_0 x}{L} \Rightarrow \textbf{linear load}$$

$@x = 0$

$$M(0) = 0\ V(0) = \frac{w_0 L}{10} \neq 0$$

Simply supported at $x = 0$

$@x = L$

$$M(L) = \frac{w_0 L^2}{10} - \frac{w_0 L^2}{6} \neq 0$$

$$V(L) = \frac{w_0 L}{10} - \frac{w_0 L}{2} \neq 0$$

Fixed end at $x = L$

P4C:

$$V(x) = \frac{dM}{dx} = \frac{w_0 x^3}{3} - \frac{w_0 L x^2}{2}$$

$$p(x) = \frac{dV}{dx} = w_0 x^2 - w_0 L x \Rightarrow \textbf{quadratic load} \text{ with } p = 0 \text{ at } x = 0 \text{ and at } x = L$$

$@x = 0$

$$M(0) = 0\ V(0) = 0$$

Free end at $x = 0$

$@x = L$

$$M(L) = \frac{w_0 L^4}{12} - \frac{w_0 L^4}{6} \neq 0$$

$$V(L) = \frac{w_0 L^3}{3} - \frac{w_0 L^3}{2} \neq 0$$

Fixed end at $x = L$

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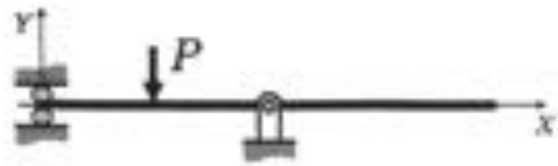
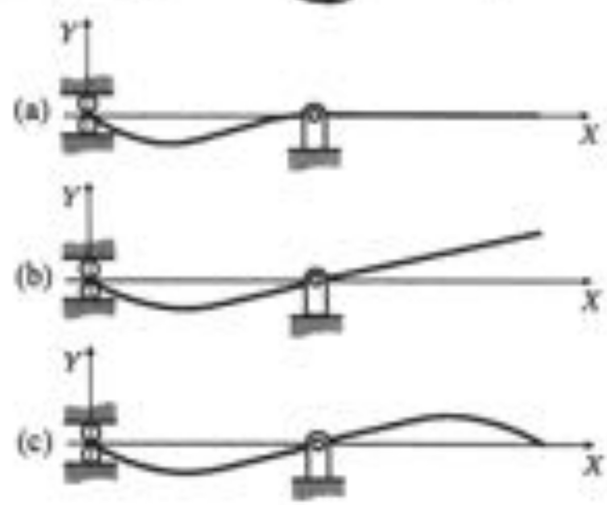
PROBLEM NO. 4D – 3 points max. (no partial credit)

Indicate which of the schematics presented below depicts the deflection curve of the following beam:



Circle the correct answer (a) (b) (c) (d)



(d) None of the above

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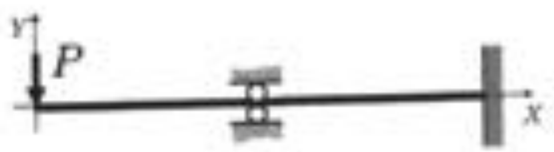
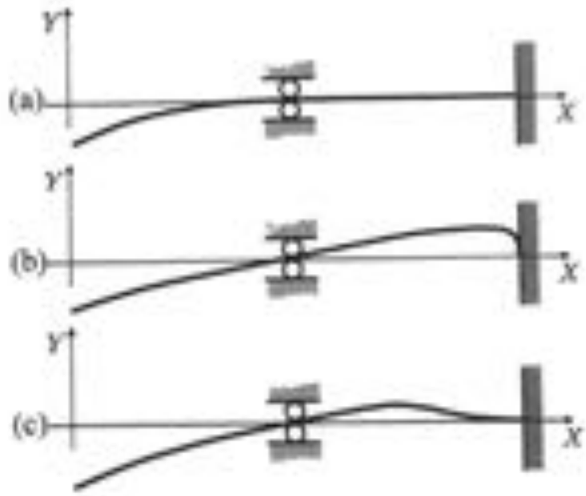
PROBLEM NO. 4E – 3 points max. (no partial credit)

Indicate which of the schematics presented below depicts the deflection curve of the following beam:



Circle the correct answer (a) (b) (c) (d)



(d) None of the above