Name (Print)____

(Last) (First)

ME 323 - Mechanics of Materials
Exam # 3
Date: December 9, 2013 Time: 7:00 – 9:00 PM
Location: EE 129 & EE170

Instructions:

Circle your lecturer's name and your class meeting time.

Krousgrill	Sadeghi	Bilal
7:30-8:30AM	10:30-11:30AM	3:30-4:30PM

Begin each problem in the space provided on the examination sheets. If additional space is required, use the yellow paper provided.

Work on one side of each sheet only, with only one problem on a sheet.

Please remember that for you to obtain maximum credit for a problem, you must present your solution clearly.

Accordingly,

- coordinate systems must be clearly identified,
- free body diagrams must be shown,
- units must be stated,
- write down clarifying remarks,
- state your assumptions, etc.

If your solution cannot be followed, it will be assumed that it is in error.

When handing in the test, make sure that ALL SHEETS are in the correct sequential order.

Remove the staple and restaple, if necessary.

Prob. 1 (24 points) ________________________

Prob. 2 (8 points) ________________________

Prob. 3 (30 points) ________________________

Prob. 4 (8 Points) ________________________

Prob. 5 (26 points) ________________________

Prob. 6 (4 points) ________________________

Total ____________________________

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Useful Equations

(If you do not see equations that you need, raise your hand and ask.)

$$\sigma = E\varepsilon \qquad \tau = G\gamma \qquad G = \frac{E}{2(1+\nu)}$$

$$\sigma = \frac{P}{A} \qquad u_B = u_A + \delta_{AB} \qquad \delta_{AB} = \frac{P_{AB}L}{AE} + \alpha \Delta T L$$

$$\tau = \frac{T\rho}{I_P} \qquad \theta_B = \theta_A + \phi_{AB} \qquad \phi_{AB} = \frac{T_{AB}L}{GI_p}$$

$$\frac{dV}{dx} = w(x) \qquad \frac{dM}{dx} = V \qquad EI\frac{d\theta}{dx} = M$$

$$\frac{dv}{dx} = \theta \qquad \Delta V = P \qquad \Delta M = -M_0$$

$$\sigma = -\frac{My}{I} \qquad \tau = \frac{VQ}{It} \qquad Q = A'\bar{y}'$$

$$\sigma_{x'} = \sigma_{avg} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \sigma_{avg} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_1 = \sigma_{avg} + R \qquad \sigma_2 = \sigma_{avg} - R \qquad \tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\langle x-a\rangle^n = \begin{cases} 0 & \text{for } x < a \\ (x-a)^n & \text{for } x > a \end{cases} \qquad n = 0,1,2,\ldots$$

$$\int \langle x-a\rangle^n dx = \begin{cases} \langle x-a\rangle^{n+1} & \text{for } n \le 0 \\ \dfrac{\langle x-a\rangle^{n+1}}{n+1} & \text{for } n > 0 \end{cases}$$

$$U = \frac{1}{2}\int_0^L \frac{F^2(x)}{EA}dx = \frac{1}{2}\frac{F^2L}{EA} \qquad U = \frac{1}{2}\int_0^L \frac{T^2(x)}{GI_P}dx = \frac{1}{2}\frac{T^2L}{GI_P}$$

$$U = \frac{1}{2}\int_0^L \frac{M^2(x)}{EI}dx \qquad U = \frac{1}{2}\int_0^L \frac{f_s V^2(x)}{GA}dx$$

$$\delta_i = \frac{\partial U}{\partial P_i}$$

$$\delta_i = \int_0^L \frac{M(x)\dfrac{\partial M(x)}{\partial P_i}}{EI}dx + \int_0^L \frac{F(x)\dfrac{\partial F(x)}{\partial P_i}}{EA}dx + \int_0^L \frac{T(x)\dfrac{\partial T(x)}{\partial P_i}}{GI_p}dx + \int_0^L \frac{f_s V(x)\dfrac{\partial V(x)}{\partial P_i}}{AG}dx$$

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$$\sigma_h = \frac{pR}{t} \qquad \sigma_a = \frac{pR}{2t}$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

$$\gamma_{xy} = G\gamma_{xy} \qquad \gamma_{xz} = G\gamma_{xz} \qquad \gamma_{yz} = G\gamma_{yz}$$

$$\sigma_M = \frac{\sqrt{2}}{2}\sqrt{\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_1 - \sigma_3\right)^2 + \left(\sigma_2 - \sigma_3\right)^2}$$

$$\sigma_M = \frac{\sqrt{2}}{2}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_x - \sigma_z\right)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2\right)\right]^{1/2}$$

$$FS = \frac{failure\ stress}{allowable\ stress}, \frac{yield\ strength}{state\ of\ stress}$$

$$\text{Factor of safety based on the maximum shear stress theory} = \frac{S_y / 2}{\tau_{\max_{abs}}}$$

$$I_P = \frac{\pi d^4}{32} \qquad I_{circle} = \frac{\pi d^4}{64} \qquad I_{rectangle} = \frac{bh^3}{12}$$

$$Semi-circle:\ \bar{y}' = \frac{4r}{3\pi}$$

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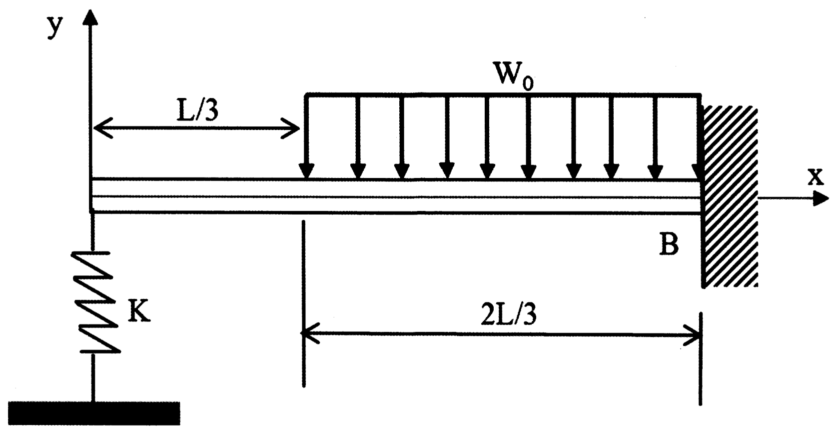
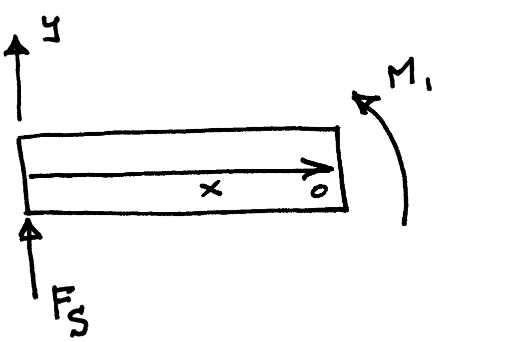
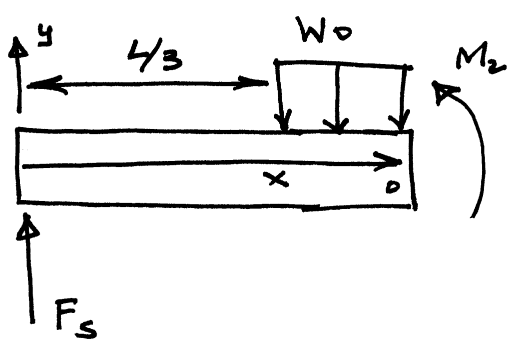
PROBLEM 1 (24 points) - The cantilever beam shown below is subject to a distributed load and supported by a spring at the left end. Using Castigliano's method, determine the force in the spring; leave your answer in terms of E, I, W_o, L, and K. **Spring constant K is known and you may neglect shear strain energy.**





$$\Sigma M_o = 0 = M_1(x) = F_s x$$

$$\frac{dM_1}{dF_s} = x$$



$$\Sigma M_o = 0 = M_2(x) = F_s x - \frac{W_o}{2}(x - L/3)^2$$

$$\frac{dM_2}{dF_s} = x$$

$$\delta_{F_s} = \int_0^{L/3} \frac{M_1(x)}{EI}\frac{dM_1}{dF_s}\,dx + \int_{L/3}^{L} \frac{M_2(x)}{EI}\frac{dM_2}{dF_s}\,dx = -\frac{F_s}{K}$$

SUB FROM ABOVE + SOLVE WE OBTAIN

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$$-\frac{F_s}{K} = \frac{1}{81}\frac{F_s L^3}{EI} - \frac{4}{27}\frac{W_0 L^4}{EI} + \frac{26}{81}\frac{F_s L^3}{EI} + \frac{26}{243}\frac{W_0 L^4}{EI}$$

SOLVE FOR F_s

$$F_s = \frac{10}{81}\frac{W_0 L^4 K}{K L^3 + 3EI}$$

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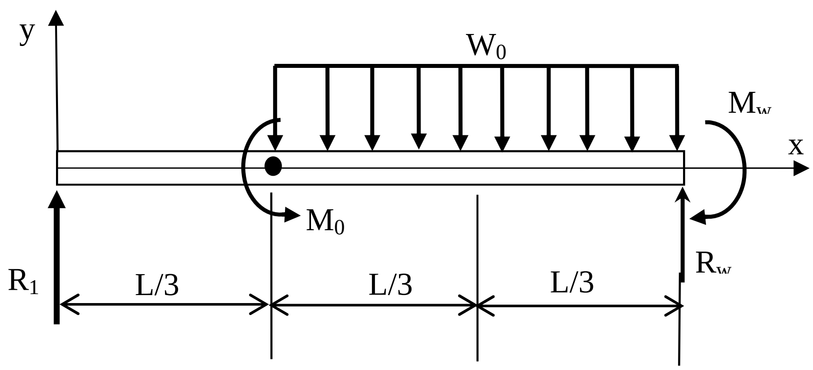
PROBLEM 2 (No partial credit. Zero credit if more than one choice is selected)

2a - (4 Points) - A beam is subject to the loading condition as shown below.



The loading function for the beam according to singularity/Macaulay function is: (choose the appropriate loading function).

a) $w(x) = R_1 \langle x \rangle^{-1} - w_0 \langle x - L/3 \rangle^{0} - M_0 \langle x - L/3 \rangle^{-2} + R_w \langle x - L \rangle^{-1} + M_w \langle x - L \rangle^{-2}$

b) $w(x) = -R_1 \langle x \rangle^{0} + w_0 \langle x - L/3 \rangle^{1} - M_0 \langle x - L/3 \rangle^{-2} + R_w \langle x - L \rangle^{-1} + M_w \langle x - L \rangle^{-2}$

c) $w(x) = R_1 \langle x \rangle^{-1} - w_0 \langle x - L/3 \rangle^{0} - M_0 \langle x - L/3 \rangle^{-1} + R_w \langle x - L \rangle^{-1} + M_w \langle x - L \rangle^{-1}$

d) $w(x) = -R_1 \langle x \rangle^{-1} + w_0 \langle x - L/3 \rangle^{1} - M_0 \langle x - L/3 \rangle^{-1} + R_w \langle x - L \rangle^{-1} + M_w \langle x - L \rangle^{-1}$

e) None of the above

The shear function for the beam according to singularity/Macaulay function is: (choose the appropriate loading function).

a) $V(x) = -R_1 \langle x \rangle^{-1} - w_0 \langle x - L/3 \rangle^{1} - M_0 \langle x - L/3 \rangle^{-1} + R_w \langle x - L \rangle^{0} + M_w \langle x - L \rangle^{-1}$

b) $V(x) = R_1 \langle x \rangle^{0} - w_0 \langle x - L/3 \rangle^{1} - M_0 \langle x - L/3 \rangle^{-1} + R_w \langle x - L \rangle^{0} + M_w \langle x - L \rangle^{-1}$

c) $V(x) = R_1 \langle x \rangle^{0} + w_0 \langle x - L/3 \rangle^{1} - M_0 \langle x - L/3 \rangle^{0} + R_w \langle x - L \rangle^{0} + M_w \langle x - L \rangle^{0}$

d) $V(x) = -R_1 \langle x \rangle^{-1} + w_0 \langle x - L/3 \rangle^{1} - M_0 \langle x - L/3 \rangle^{0} + R_w \langle x - L \rangle^{0} + M_w \langle x - L \rangle^{0}$

e) None of the above

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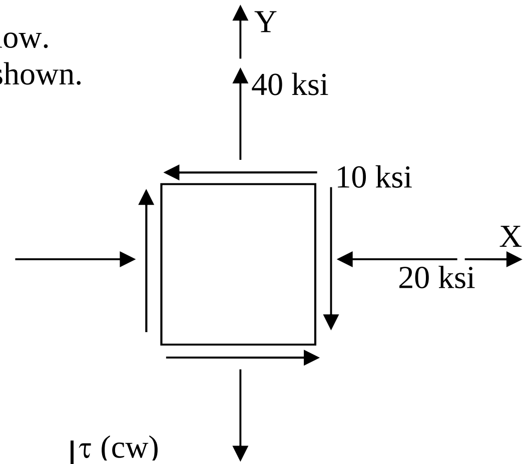
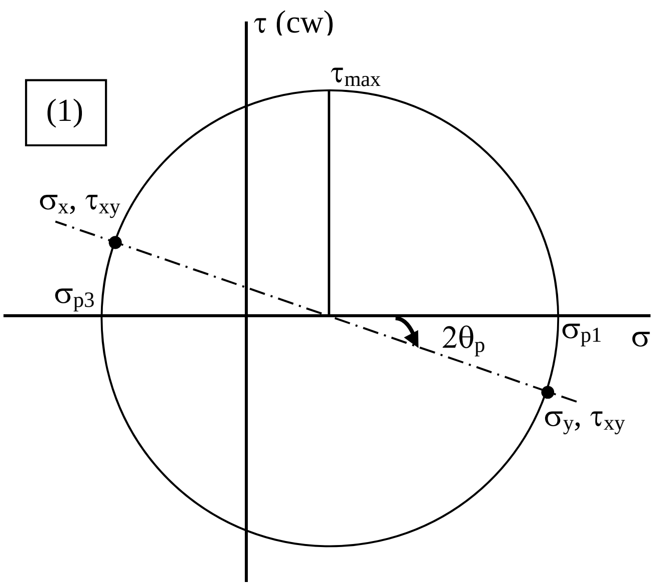
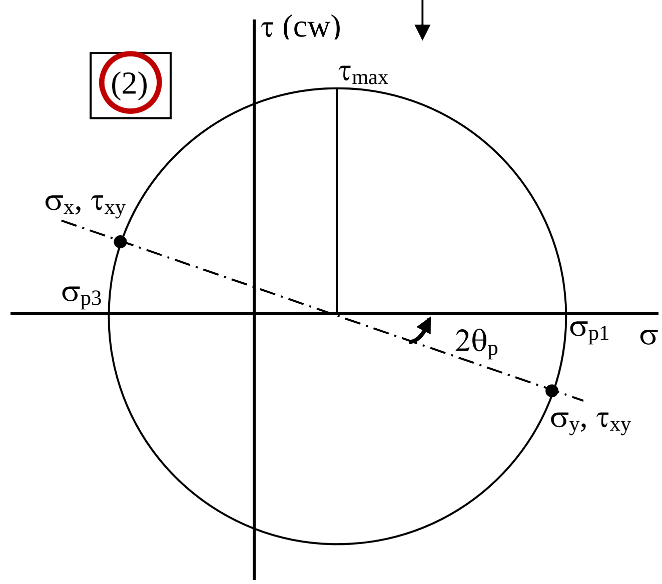
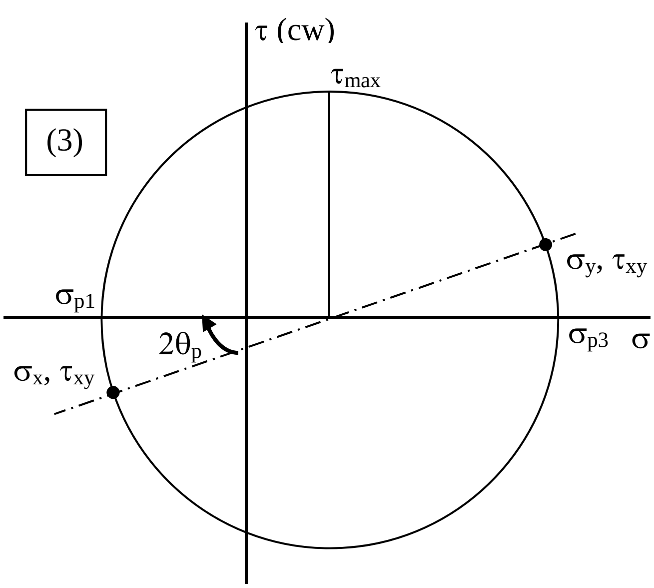
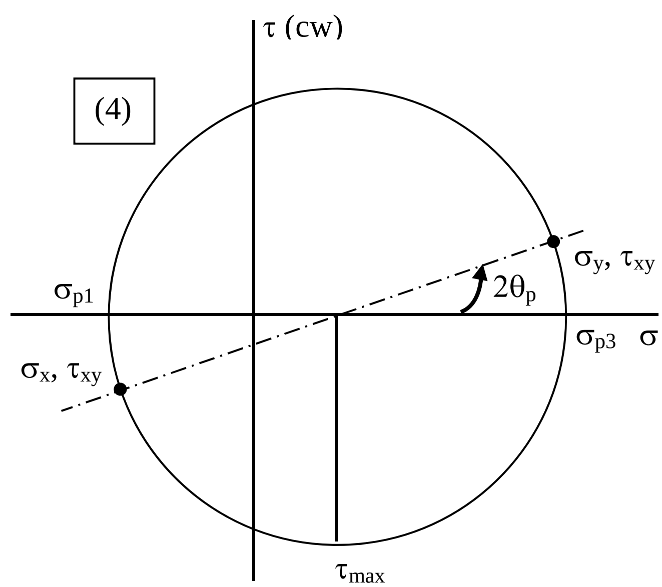
2.b - (4 Points) The state of stress at a point is given as shown below. Choose which Mohr's Circle is appropriate for the state of stress shown. The principle stresses are $\sigma_{p1} > \sigma_{p2} > \sigma_{p3}$, and $\tau_{\max}$ is the maximum in-plane shear stress.











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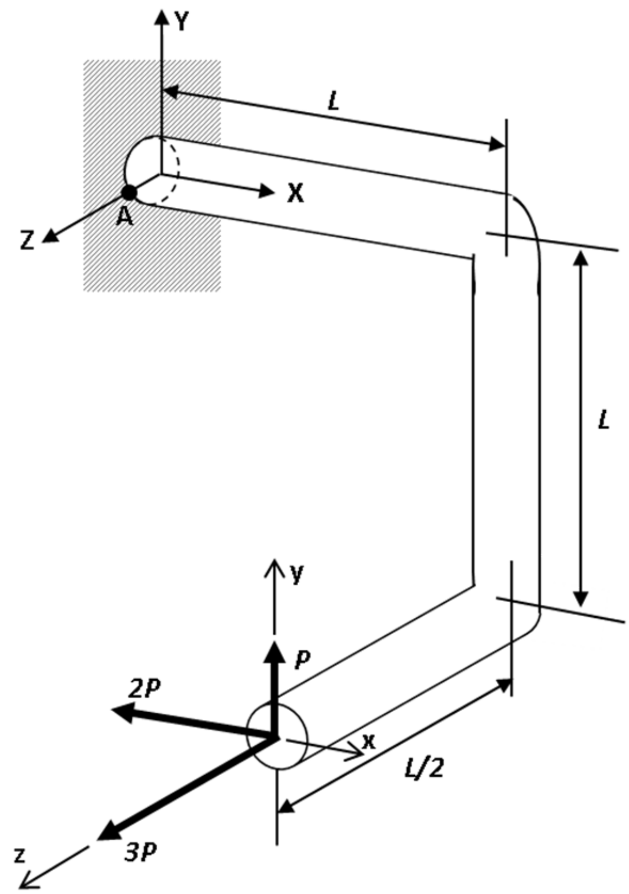
Problem 3 (30 points): - The member is fixed into a wall and subject to the loading as shown below.

1. Draw the free body diagram for the member and determine the reaction forces and moments at the wall.
2. Determine the state of stress at point A.
3. If the yield strength for the member is S_y = 40 Ksi, determine the factors of safety guarding static failure at point A, based on the maximum shear stress and distortion energy theories.

Note: P = 400 lb, L=10 in and member diameter = 2 in.



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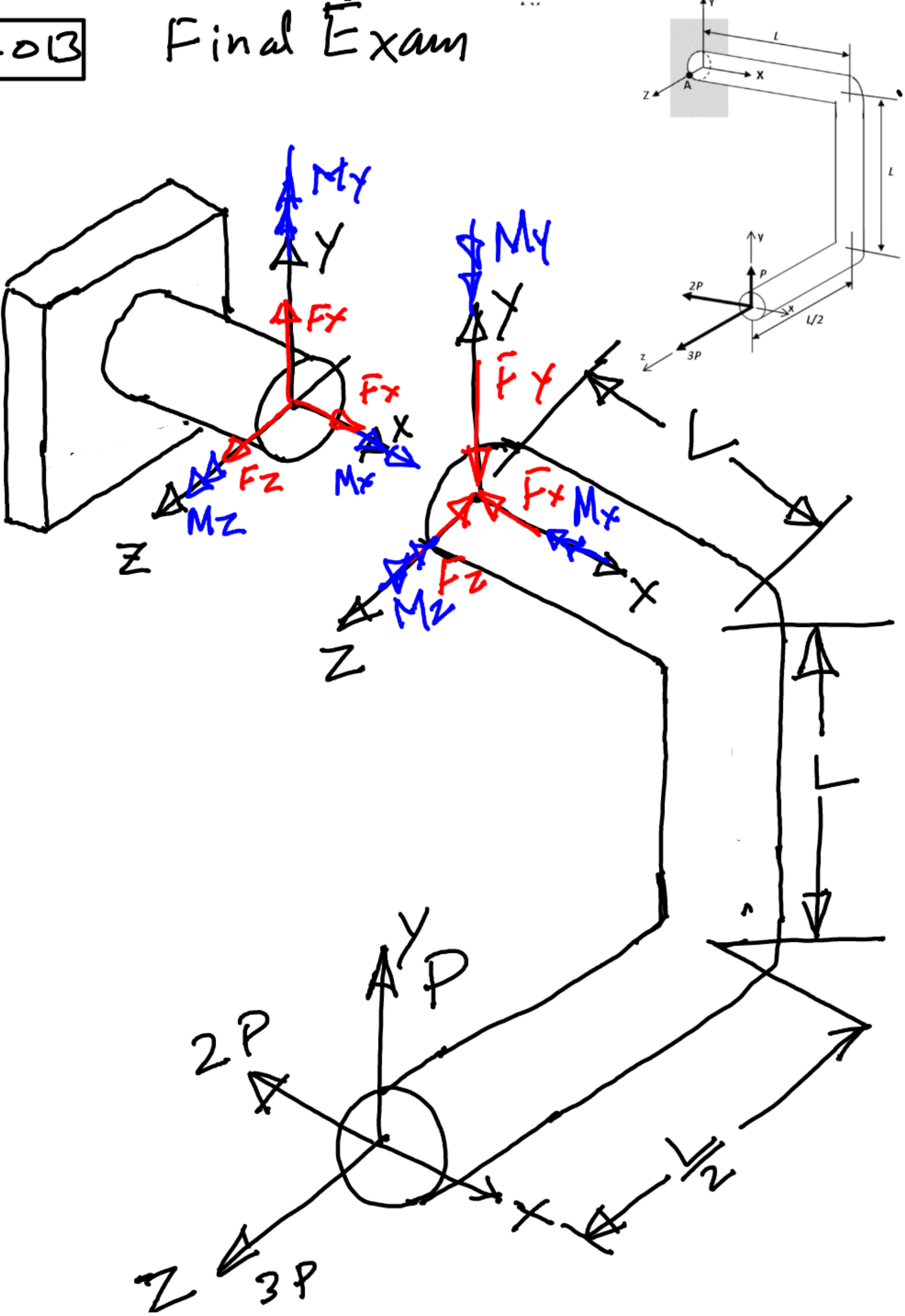
Final Exam

Given:

P = 400 lb,
L = 10 in
d = 2 in.

if S_y = 40 ksi
F.S = ?
max shear,
&
Max distortion energy.



$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(2)^2$$

$A = \pi \text{ in}^2$

$$I_{xx} = I_{yy} = \frac{\pi}{4} r^4 \text{ or } \frac{\pi}{64} d^4$$

$$\Rightarrow I_{xx} = I_{yy} = \frac{\pi}{64}(2)^4 = 0.785398 \text{ in}^4$$

$$I_P = \frac{\pi}{2} r^4 \text{ or } \frac{\pi}{32} d^4$$

$I_P = 1.5708 \text{ in}^4$

Step # 1:

Sum force in x, y, z axis.

$$\Sigma F_x \Rightarrow \quad -F_x - 2P = 0$$

$$\boxed{F_x = -2P} \quad = -2\,(400) = -800$$

$$\sigma_{A1} = \frac{F_x}{A} = \frac{-800}{\pi} = -254.648\,psi$$

$$\boxed{\sigma_{A1} = -254.648\ psi}$$

Step # 2:

Sum forces in y-direction

$$\Sigma F_y = 0 \Rightarrow$$

$$-F_y + P = 0$$

$$\Rightarrow F_y = P = 400\,lb$$

$$\therefore \boxed{F_y = 400\,lb}$$

The force in y-direction is causing a transverse shear at pt. A.

$$\tau_{A2} = \frac{VQ}{It} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{F_y}{A} = \frac{4}{3}\frac{400}{\pi}$$

$$\boxed{\tau_{A2} = 169.765\ psi}$$

Step # 3:

Sum forces in Z-direction

$$\Sigma F_z = 0$$

$$-F_z + 3P = 0$$

$$F_z = 3.(400) = 1200 \text{ lb.}$$

$$\boxed{F_z = 1200 \text{ lbs}}$$

There is no stress at point due Fz.

$$\boxed{\tau_{A3} = 0}$$

Now sum the moments about each axis.

Use cross-product to calculate the moment about each axis.

$$\boxed{M = \bar{r} \times \bar{F}}$$

$$\bar{r} = L\hat{i} - L\hat{j} + \frac{L}{2}\hat{k}$$

$$= 10\,i - 10\hat{j} + 5\,\hat{k}$$

$$F = -2P\hat{i} + P\hat{j} + 3P\hat{k}$$

$$= -2(400)\hat{i} + 400\hat{j} + 3(400)\hat{k}$$

$$\boxed{F = -800\hat{i} + 400\hat{j} + 1200\hat{k}}$$

$$M = \bar{r} \times \bar{F} = (10\,\hat{i} - 10\,\hat{j} + 5\,\hat{k}) \times (-800\hat{i} + 400\hat{j} + 1200\hat{k})$$

$$\boxed{M = -14000\,i - 16000\,\hat{j} - 4000k}$$

step #4:

The moment about x-axis causing a torque in the negative x-direction.

$$\tau_{A4} = \frac{T_x r}{J} = \frac{(-14 \times 10^3)(1)}{1.57}$$

$$\boxed{\tau_{A4} = 8.92 \times 10^3 \text{ psi}}$$ — in nogtive x-dir

Step #5

Moment about negative Y-axis is causing a bending stress at pt.

$$\sigma_{A5} = \frac{M_y (r)}{I_{YY}} = +\frac{(-16 \times 10^3)(1)}{.78539}$$

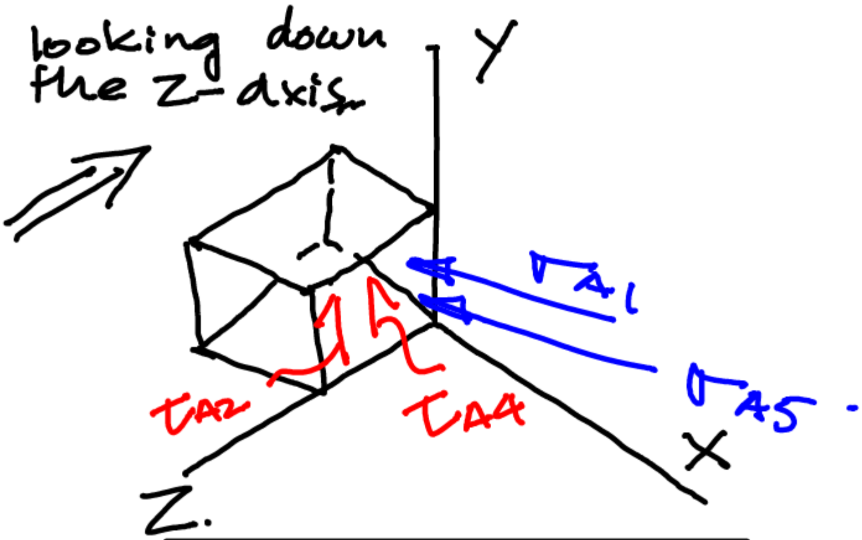
$$\boxed{\sigma_{A5} = -20.4 \times 10^3 \text{ psi}}$$ — negative y-dir. — a compressive force at A.

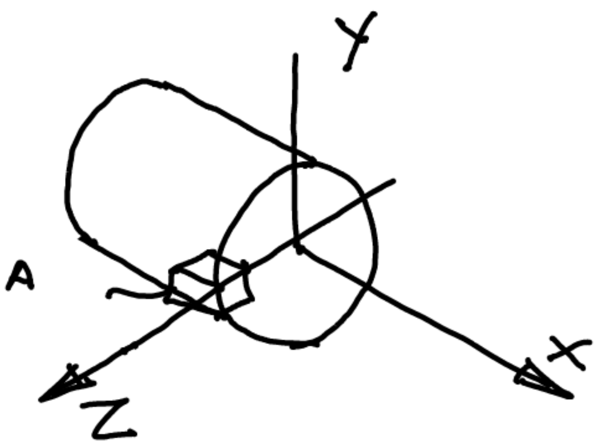
Step # 6:

The moment about the Z-axis causes doesn't any bending stress at pt. A.

$$\boxed{\sigma_{A6} = 0}$$

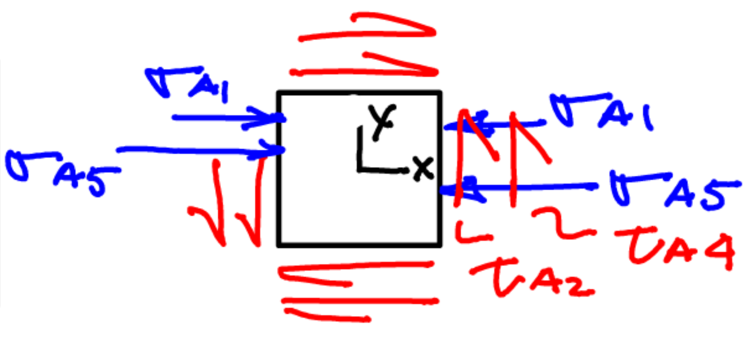
Now indicate all the stress at the stress element



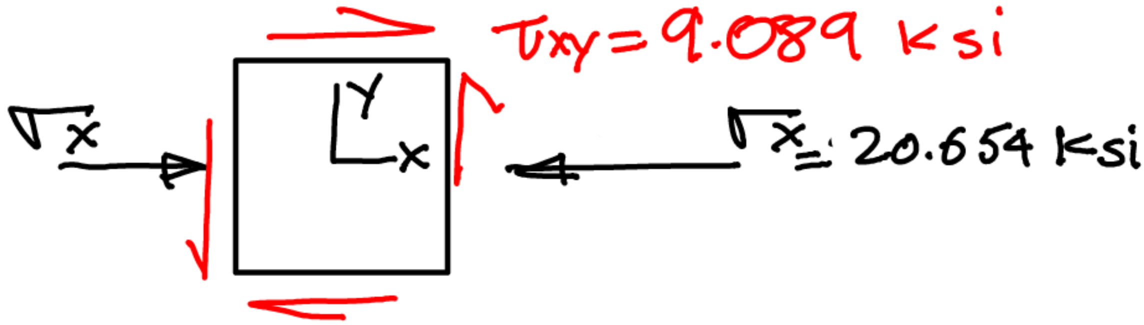


here

$\sigma_{A1} = 254.6$ psi
$\sigma_{A5} = 20.4 \times 10^3$ psi
$\tau_{A2} = 169.765$ psi
$\tau_{A4} = 8.92 \times 10^3$ psi



Now add the colinear stresses and draw a simplified stress element.



part (2).

State of stress.

$\sigma_x = 20.654$ ksi
$\sigma_y = 0$
$\tau_{xy} = 9.089$.
$\tau_{yx} = -9.089$

X (20.654, 9.089)
Y (0, -9.089)

$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$

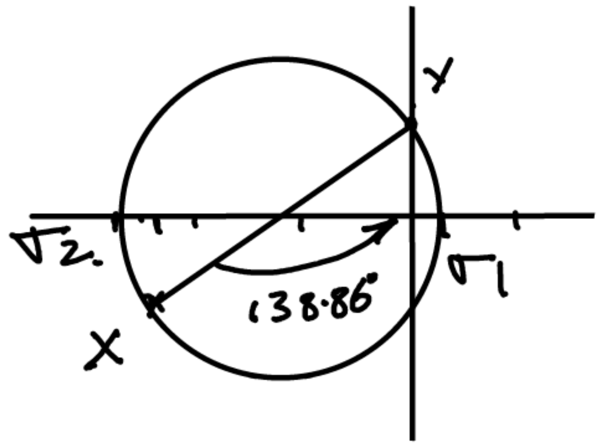
$= -10.3$ ksi

$\sigma_{ave.} = -10.3$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{((-10.3)^2 + (9.089)^2}$

$R = 13.8$ ksi



$\sigma_1 = \sigma_{ave} + R$.

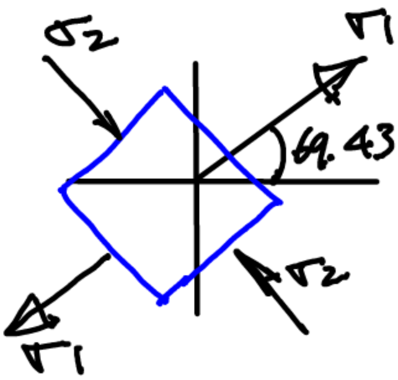
$\sigma_1 = 3.43$ ksi

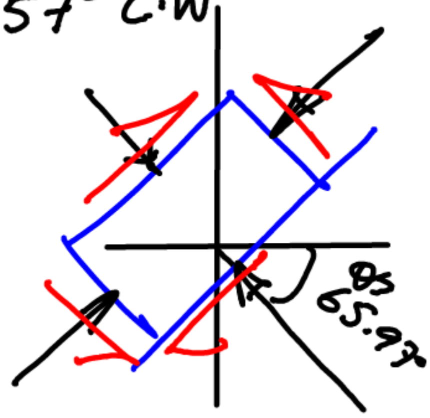
$\sigma_2 = \sigma_{ave} - R$

$\sigma_2 = -24.081$ ksi

$\theta_p = 69.43°$ CCW

$\theta_s = 65.57°$ C.W





Part (3):

Factor of safety based on max shear stress theory.

(i) $\sigma_1 > 0$, $\sigma_2 < 0$.

$$FS = \frac{\sigma_Y}{\sigma_1 - \sigma_2} = \frac{40}{3.43 + 24.081} =$$

$$\boxed{FS = 1.45}$$ fails!

(ii) $\sigma_M = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2}$

$$\sigma_M = \sqrt{(3.43)^2 + (-24.081)^2 - (3.43)(-24.081)}$$

$$\boxed{\sigma_M = 25.966 \text{ ksi}}$$

$$F.S = \frac{\sigma_Y}{\sigma_M} = \frac{40}{25.966} = 1.54$$

$$\boxed{FS = 1.54}$$

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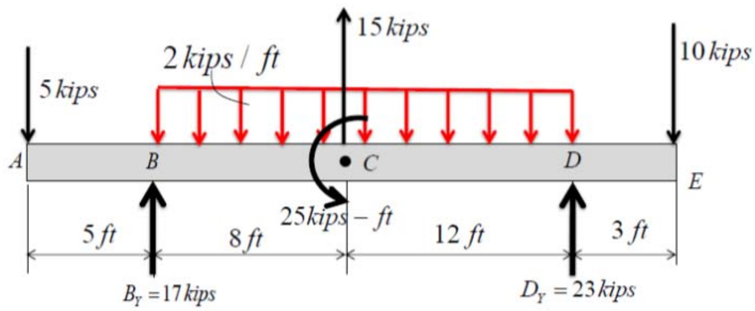
Problem 4 (8 points): - The beam is subject to the loading as shown in the figure below.

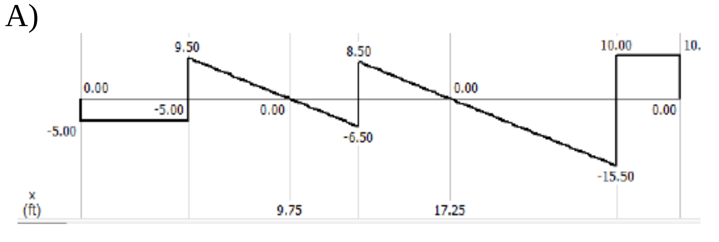
1. Choose the correct shear diagram from the column on the left.
2. Choose the correct moment diagram from the column on the right.

Note: The correct shear diagram on the left and the correct moment diagram on the right do not necessarily line up. Circle answers from the following choices.

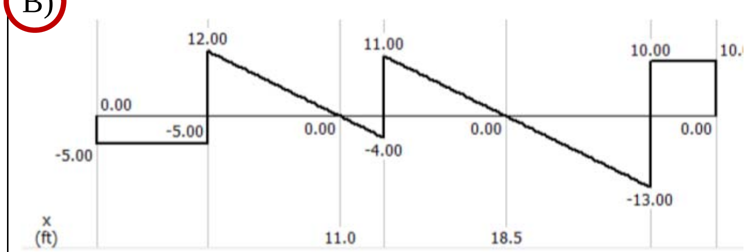


Shear Diagram

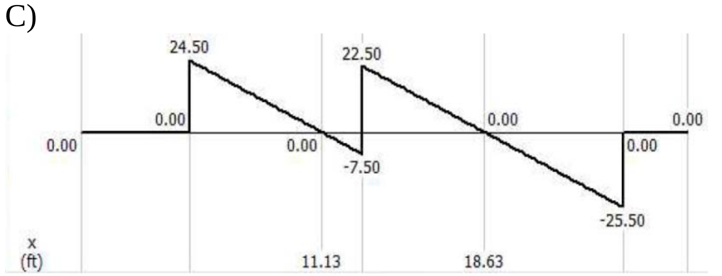
A)



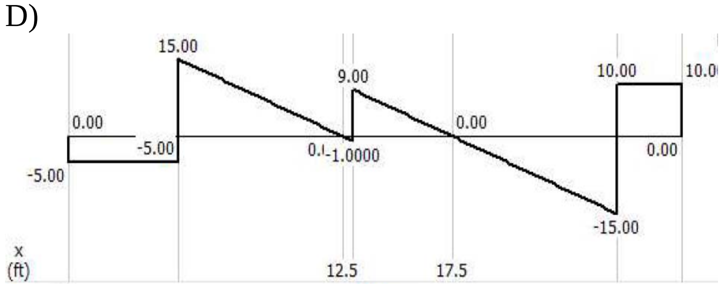
B)



C)



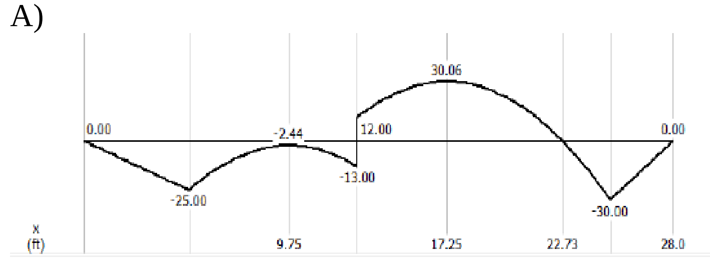
D)



E) None of the above

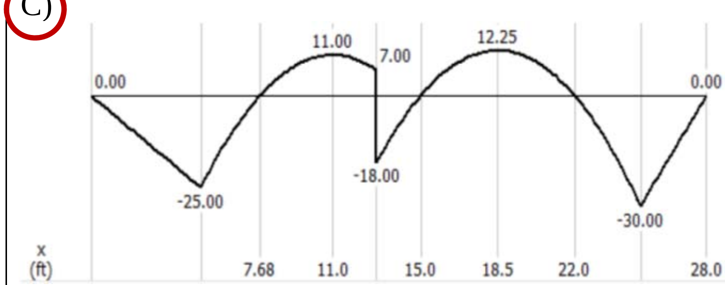
Bending Moment Diagram

A)

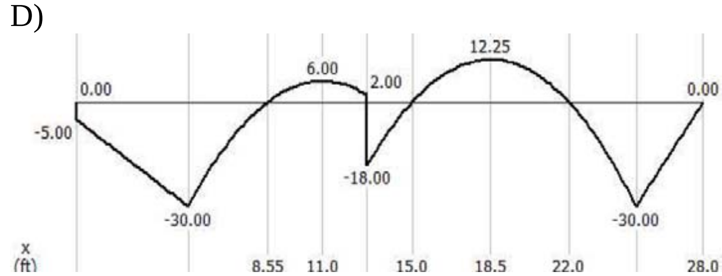


B)

C)

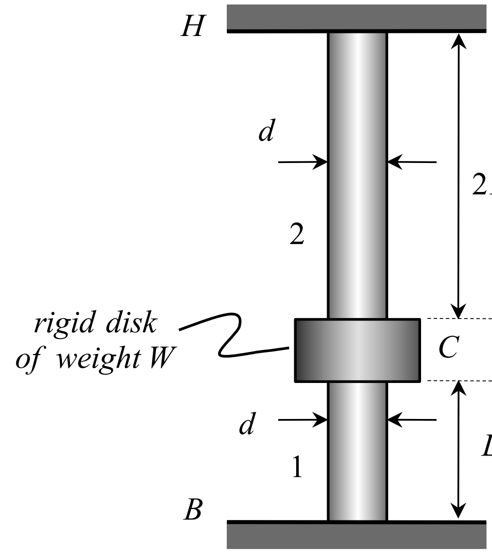


D)



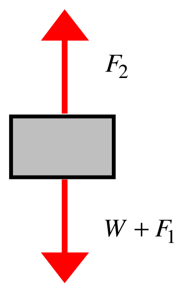
E) None of the above

Problem 5 (20 points): - Two rods (each made up of a material with a Young's modulus E, a thermal expansion coefficient α and circular cross-section of diameter d) are attached between a rigid disk C (of weight W) and two fixed supports B and H, as shown in the figure. Rod 1 has a length of L, whereas Rod 2 has a length of 2L. Rod 1 is given a temperature increase of ΔT with the temperature of Rod 2 being held constant. Ignore the weights of the rods.



a) Draw a free body diagram of the rigid disk C and write down the equilibrium equation(s) for the disk.

$$\sum F_x = F_2 - W - F_1 = 0 \quad \Rightarrow \quad F_2 = F_1 + W$$



b) Write down the load/temperature/elongation equation for each rod.

$$e_1 = \frac{F_1 L}{EA} + \alpha \Delta T\, L$$

$$e_2 = \frac{F_2 (2L)}{EA}$$

c) Write down the compatibility equation relating the elongation/compression of the two rods.

$$u_H = e_1 + e_2 = 0$$

d) Solve your equations from a), b) and c) above for the *stress* in Rod 1. Leave your answer in terms of, at most, W, ΔT , α , E, d and L.

$$0 = e_1 + e_2 = \frac{F_1 L}{EA} + \alpha \Delta T\, L + \frac{2F_2 L}{EA}$$

$$= \frac{F_1 L}{EA} + \alpha \Delta T\, L + \frac{2(F_1 + W)L}{EA} = \frac{3F_1 L}{EA} + \frac{2WL}{EA} + \alpha \Delta T\, L \quad \Rightarrow$$

$$F_1 = -\left(\frac{2W + \alpha \Delta T\, EA}{3} \right) \quad \Rightarrow$$

$$\sigma_1 = \frac{F_1}{A} = -\frac{1}{3}\left(\frac{2W}{A} + \alpha \Delta T\, E \right) = -\frac{1}{3}\left(\frac{8W}{\pi d^2} + \alpha \Delta T\, E \right) \qquad (compression)$$

ME 323 Examination #3 **Name _____________________________________**
(Print) (Last) (First)

December 9, 2013 **Instructor ________________________________**

<u>Problem 6 (4 points):</u> - Consider two situations. One, where the beam below is made from steel, having a Young's modulus of E_{st}. Two, where the beam is made from aluminum, having a Young's modulus of E_{al}, where $E_{st} > E_{al}$. Let $(\sigma_{max})_{st}$ and $(\sigma_{max})_{al}$ denote the absolute values of the maximum normal stress on the cross section at location B for the steel and aluminum beams, respectively. Circle the correct statement below related to the relative sizes of $(\sigma_{max})_{st}$ and $(\sigma_{max})_{al}$:

a) $(\sigma_{max})_{st} > (\sigma_{max})_{al}$

b) $(\sigma_{max})_{st} = (\sigma_{max})_{al}$

c) $(\sigma_{max})_{st} < (\sigma_{max})_{al}$