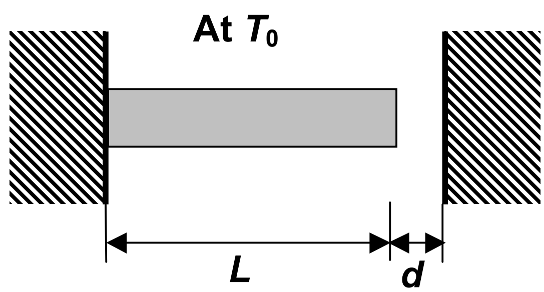
PROBLEM #1.1 (4 + 4 points, no partial credit)

A thermal switch consists of a copper bar which under elevation of temperature closes a gap and closes an electrical circuit. The copper bar possesses a length ***L***, modulus ***E***, and coefficient of thermal expansion a. At room temperature T_0 the gap is ***d***.

(1) The gap is closed at a temperature T_C calculated from equation **(circle one)**:



(a) $d = \alpha(T_C - T_0)$

(b) $d = L\alpha(T_C)$

(c) $d = L\alpha(T_C - T_0)$*.....correct*

(d) $d = E\alpha(T_C - T_0)$

(2) As the temperature is increased beyond T_C, the there exist stress in the bar equal to **(circle one)**:

(a) $\sigma = E\alpha(T - T_0)$

(b) $\sigma = -E\alpha(T - T_C)$*....correct*

(c) $\sigma = -E\alpha(T - T_0)$

(d) $\sigma = EL\alpha(T - T_0)$

PROBLEM #1.2 (4 + 4 points, no partial credit)

(1) A cylindrical pressure vessel with diameter ***D***=2.5 m and wall thickness ***t***=10 mm is filled with gas at an internal pressure of ***p***=2400 kPa.

The absolute maximum shear stress in the vessel is **(circle one)**

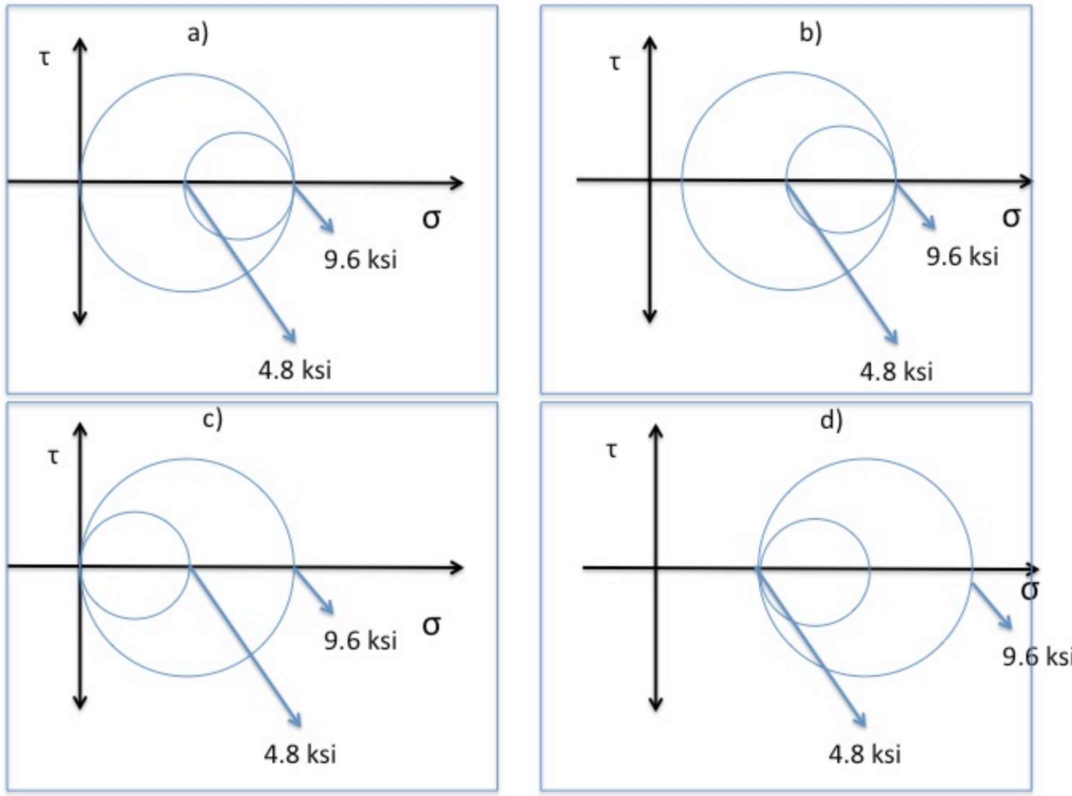
(a) 300 MPa

(b) 150 MPa …. correct

(c) 75 MPa

(d) 37.5 MPa

(2) A propane tank with shape of a cylindrical pressure vessel has a diameter ***D***=12 in. and a wall thickness of ***t***=1/8 in. The tank is pressurized to ***p***=200 psi. Choose the Mohr's circle diagram that corresponds to the state of stress **(circle one)**. (*a) is correct*



PROBLEM #1.3 (4 + 4 points, no partial credit)

A circular torsion bar consists of a core material (diameter $\boldsymbol{D}_{core}$, shear modulus $\boldsymbol{G}_{core}$) and a sleeve material (outer diameter $\boldsymbol{D}_{sleeve}$, $\boldsymbol{G}_{sleeve}$) bonded firmly together at the interface. An external torque $\boldsymbol{T}$ is applied to the composite shaft.

(1) In the analysis of this problem one needs to consider that **(circle one):**

(a) The stress distribution is possesses a jump (i.e. is discontinuous) at the interface. Correct

(b) The strain distribution is possesses a jump (i.e. is discontinuous) at the interface.

(c) Both stress and strain possess a jump (i.e. are discontinuous) at the interface.

(d) Neither stress and strain possess a jump (i.e. are discontinuous) at the interface.

(2) Which ones of the following statements are true regarding the maximum shear stress τ_{max} and its location in the bar? **(circle one)**

(a) The maximum shear stress τ_{max} always occurs in the sleeve at the outer surface of the sleeve.

(b) If the cross sectional area of the sleeve is greater than that of the core, then the maximum stress τ_{max} occurs on the outer surface of the sleeve; and if the cross sectional area of the core is greater than that of the sleeve then τ_{max} occurs on the outer surface of the core.

(c) If $G_{sleeve}D_{sleeve} > G_{core}D_{core}$ then τ_{max} occurs on the outer surface of the sleeve; and if $G_{sleeve}D_{sleeve} < G_{core}D_{core}$ then τ_{max} occurs on the outer surface of the core. Correct

(d) If $G_{sleeve}J_{sleeve} > G_{core}J_{core}$ then τ_{max} occurs on the outer surface of the sleeve; and if $G_{sleeve}J_{sleeve} < G_{core}J_{core}$ then τ_{max} occurs on the outer surface of the core.

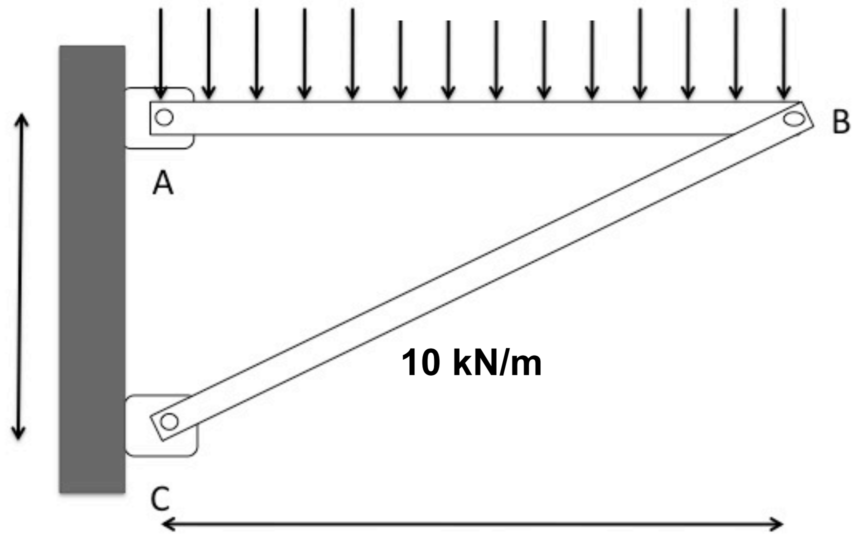
(e) The maximum shear stress τ_{max} always occurs at the outer surface of the core.

(f) None of the above.

<u>PROBLEM #2 (25 points)</u>

A truss structure consists of two members AB and BC which are connected by pin joints. The **rigid** member AB is subjected to a distributed load of magnitude 10kN/m. The **deformable** member BC possesses a circular cross section of diameter **D**.

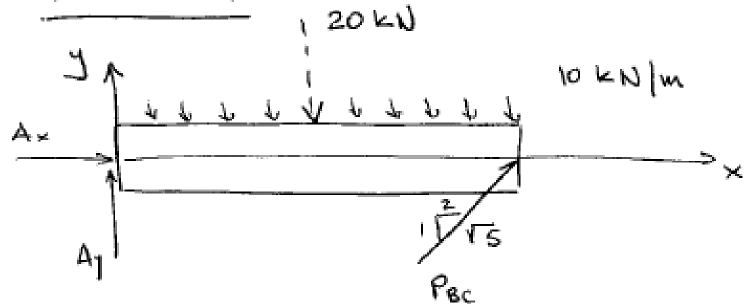
Determine the diameter of the truss BC in order to prevent buckling of the member BC with a factor of safety ***FS***=2.9. Consider that the modulus of the material used for the truss BC is ***E***=210GPa.



1 m

2 m

Problem 2



$$\sum M_A = 0 \qquad 20 \text{ kN} \cdot 1\text{m} - P_{BC} \frac{2}{\sqrt{5}} = 0$$

$$P_{BC} = 10\sqrt{5} \text{ kN}$$
$$= 22.36 \text{ kN}$$

$$P_{BC}^{critic} = \frac{\pi^2 E I}{L^2} \qquad I = \frac{\pi}{64} d^4$$

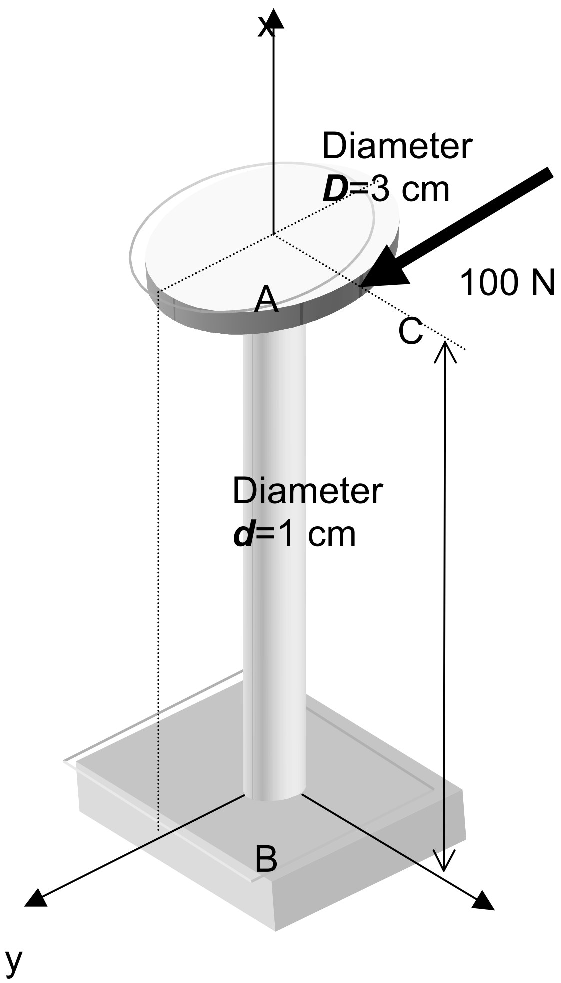
$$L^2 = 5 \text{m}^2$$

$$= \frac{\pi^3 \cdot 210 \cdot 10^9 \; d^4}{64 \cdot 5}$$

$$= 20.35 \; 10^9 \; d^4 \text{ N}$$

$$F.S = 2.9 = \frac{P_{BC}^{crit}}{P_{BC}} = \frac{20.35 \, d^4 \, 10^9}{22.36 \, 10^3} \Rightarrow d = 0.042 \text{ m}$$

PROBLEM #3 (25 points)



A force is applied at point C to a (very thin) circular **rigid** plate with diameter ***D***=3cm. The plate is attached to an **elastic shaft** (AB) of diameter ***d***=1cm and length ***L***=10cm. All the strain energy is stored in the shaft (none in the plate since it is rigid). Also assume that strain energy stored due to transverse shear is negligible. The material is Aluminum (***E***= 73 GPa, ***G***=27 GPa)

Using the work-energy principle determine the deflection of point C in the direction of the applied force (i.e. along +y). Both torsion and bending are to be considered.

$$\frac{1}{2}P\delta = U_{el}$$

$$R = D/2$$

$$U_{el} = \frac{T^2 L}{2GJ} + \int_0^L \frac{M^2}{2EI}dx = \frac{(P\cdot R)^2 L}{2GJ} + \int_0^L \frac{(P\cdot x)^2}{2EI}dx =$$

$$U_{el} = \frac{(P\cdot R)^2 L}{2GJ} + \frac{P^2}{2EI}\int_0^L x^2\,dx = \frac{(P\cdot R)^2 L}{2GJ} + \frac{P^2}{2EI}\frac{L^3}{3} = \frac{P^2 R^2 L}{2GJ} + \frac{P^2 L^3}{6EI}$$

$$\frac{1}{2}P\delta = \frac{P^2 R^2 L}{2GJ} + \frac{P^2 L^3}{6EI} \Rightarrow \delta = \frac{PR^2 L}{GJ} + \frac{PL^3}{3EI}$$

$$\sigma_M = \frac{\sqrt{2}}{2}\sqrt{[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2 + 6(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)]}$$

For A and C

$$\sigma_M = \frac{\sqrt{2}}{2}\sqrt{[(\sigma_x)^2 + (\sigma_x)^2 + 6\tau_{xz}^2]}$$

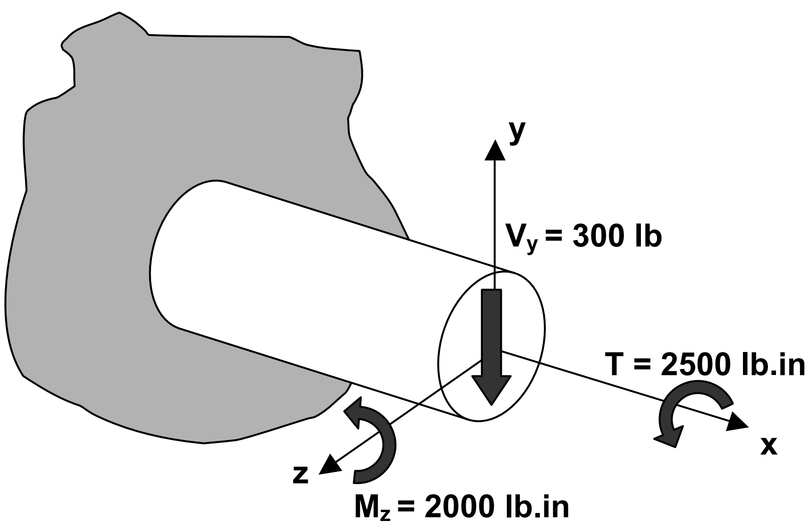
For B and D

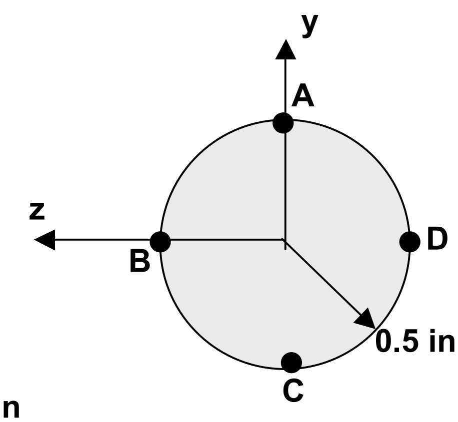
$$\sigma_M = \frac{\sqrt{2}}{2}\sqrt{[(\sigma_x)^2 + (\sigma_x)^2 + 6\tau_{xy}^2]}$$

PROBLEM #4 (26 points)

At a particular cross section of a shaft the internal resultants were determined to consist of a torque (**T = 2500 lb.in**), a bending moment (**M_z = 2000 lb.in**) and a shear force (**V_y = 300 lb**). The shaft possesses a circular cross-section with radius **r = 0.5 in.**

- Determine the states of stress at locations A,B,C,D on the cross-section. Document your answer by drawing the respective material elements at each location with the stress components clearly indicated by vectors and stress magnitudes.
- Determine the factor of safety using the vom Mises criterion, considering that the yield strength is **30 ksi.**

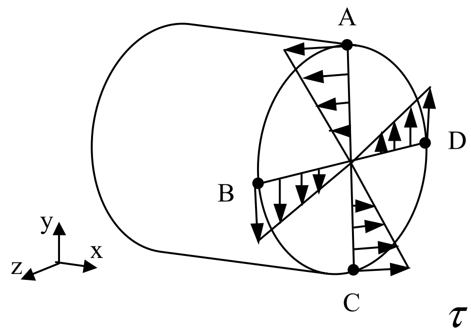




The stresses at points A, B, C, and D will be due to

- torsion
- bending moment
- shear force

Torsion



$$\tau = \frac{T_x \rho}{J_{xx}}$$

$T = T_x = 2500 \text{ lb.in}$

$J_{xx} = \frac{\pi}{2} c^4 = 98.17 \times 10^{-3} \text{ in}^4$

$\tau = \frac{T_x c}{J_{xx}} = \frac{(2500 \text{ lb.in})(0.5 \text{ in})}{(98.17 \text{ in}^4)} = 12.73 \text{ ksi (CCW)}$

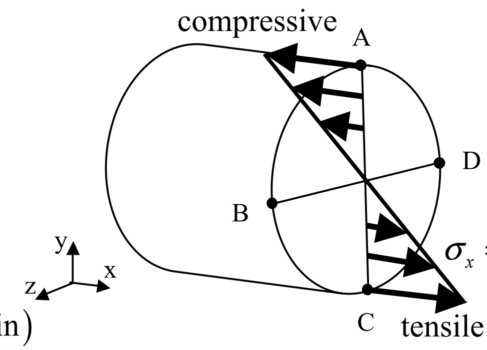
@ A, $\tau_{xz} = \tau = 12.73$ ksi (towards +Z)

@ B, $\tau_{xy} = \tau = 12.73$ ksi (towards -Y)

@ C, $\tau_{xz} = \tau = 12.73$ ksi (towards -Z)

@ D, $\tau_{xy} = \tau = 12.73$ ksi (towards +Y)

Bending moment



$$\sigma_x = \frac{-M_z y}{I_{zz}}$$

$M_z = 2000 \text{ lb.in}$

$I_{zz} = \frac{\pi}{4} c^4 = 49.09 \times 10^{-3} \text{ in}^4$

@A, $\sigma_x = \frac{-M_z y}{I_{zz}} = \frac{-(2500 \text{ lb.in})(+0.5 \text{ in})}{(49.09 \times 10^{-3} \text{ in}^4)}$

$= -20.37$ ksi (compressive)

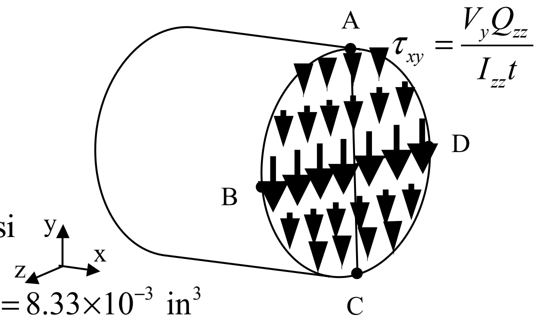
@B, $\sigma_x = \frac{-M_z y}{I_{zz}} = \frac{-(2500 \text{ lb.in})(0 \text{ in})}{(49.09 \times 10^{-3} \text{ in}^4)} = 0 \text{ ksi}$

@C, $\sigma_x = \frac{-M_z y}{I_{zz}} = \frac{-(2500 \text{ lb.in})(-0.5 \text{ in})}{(49.09 \times 10^{-3} \text{ in}^4)}$

$= +20.37$ ksi (tensile)

@B, $\sigma_x = \frac{-M_z y}{I_{zz}} = \frac{-(2500 \text{ lb.in})(0 \text{ in})}{(49.09 \times 10^{-3} \text{ in}^4)} = 0 \text{ ksi}$

Shear force



$V_y = 300 \text{ lb}$

$I_{zz} = \frac{\pi}{4} c^4 = 49.09 \times 10^{-3} \text{ in}^4$

@A & C, $\left. Q_{zz} \right|_{y=\pm 0.5 \text{ in}} = 0 \text{ in}^3 \quad \Rightarrow \quad \tau_{xy} = 0 \text{ ksi}$

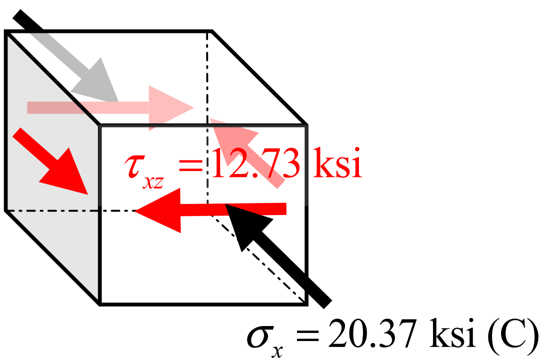
@B & D, $\left. Q_{zz} \right|_{y=0 \text{ in}} = \sum \bar{y}_i' A_i' = \left(\frac{4r}{3\pi} \right) \left(\frac{\pi}{2} r^2 \right) = 8.33 \times 10^{-3} \text{ in}^3$

$$\Rightarrow \quad \tau_{xy} = \frac{V_y Q_{zz}}{I_{zz} t} = \frac{(300 \text{ lb})(8.33 \times 10^{-3} \text{ in}^3)}{(49.09 \times 10^{-3} \text{ in}^4)(2 \times 0.5 \text{ in})}$$

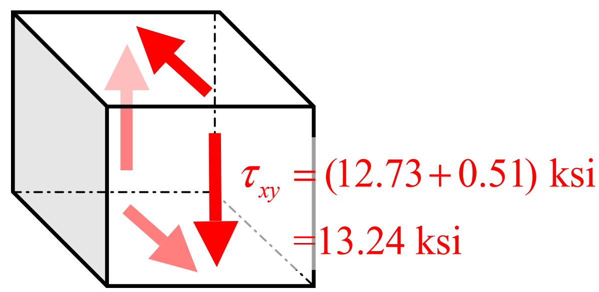
$$= 0.51 \text{ ksi (downwards)}$$

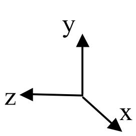
The combined state of stress at points A, B, C, and D will be the resultant **linear superposition** of the individual stress values due to torsion, bending, and shear force.

Point element A

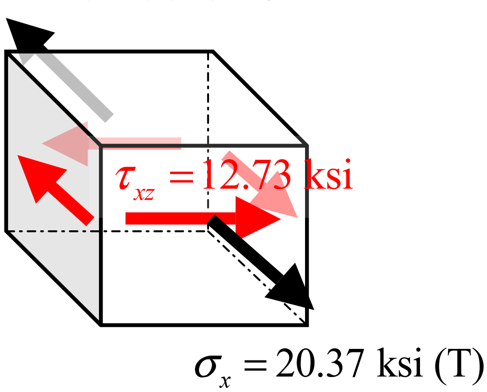


Point element B

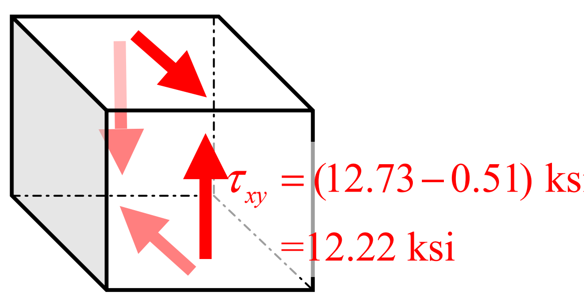




Point element C



Point element D



$$\sigma_M = \frac{\sqrt{2}}{2}\sqrt{[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2 + 6(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)]}$$

For A and C

$$\sigma_M = \frac{\sqrt{2}}{2}\sqrt{[2\sigma_x^2 + 6\tau_{xz}^2]} \Rightarrow \sigma_M(A) = \sigma_M(C) = 42.5ksi \quad FS = \sigma_{yield} / \sigma_M$$

For B and D

$$\sigma_M = \frac{\sqrt{2}}{2}\sqrt{[6\tau_{xy}^2]} \Rightarrow \sigma_M(B) = 32ksi \quad \sigma_M(C) = 30ksi \quad FS = \sigma_{yield} / \sigma_M$$