Cover sheet and Problem 1

Name: ______________, ________________ Instructor: Susilo Koslowski Raman
(Print) (Last) (First) (Circle one)

ME 323 FINAL EXAM

FALL SEMESTER 2011

Time allowed: 2 hours

Instructions

1. There are 4 problems, each problem is of equal value. The first problem consists of three smaller sub-problems
2. Begin each problem in the space provided on the examination sheets. If additional space is required, use the yellow paper provided. Work on one side of each sheet only, with only one problem on a sheet.
3. To obtain maximum credit for a problem, you must present your solution clearly. Accordingly:
    a. Identify coordinate systems
    b. Sketch free body diagrams
    c. State units explicitly
    d. Clarify your approach to the problem including assumptions
4. If your solution cannot be followed, it will be assumed that it is in error.
5. When handing in the test, make sure to place it under the name of your instructor and separate them by problem number.

Prob. 1 ________________________

Prob. 2 ________________________

Prob. 3 ________________________

Prob. 4________________________

Total __________________________

Equation Sheet

Hooke's law:

$\varepsilon = (L_f - L_i)/L_i$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu(\varepsilon_y + \varepsilon_z)\right]$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_y + \nu(\varepsilon_z + \varepsilon_x)\right]$$

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_z + \nu(\varepsilon_x + \varepsilon_y)\right]$$

Stress transformation and Mohr's circle:

$$\sigma_{x'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta_{xx'} + \tau_{xy}\ \mathrm{s}$$

$$\sigma_{y'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta_{xx'} - \tau_{xy}\ \mathrm{s}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta_{xx'} + \tau_{xy}\cos 2\theta_{xx'}$$

$$\sigma_{\mathrm{p1}} = \sigma_{\mathrm{avg}} + \mathrm{R} \qquad \sigma_{\mathrm{p2}} = \sigma_{\mathrm{avg}} - \mathrm{R}$$

$$\tau_{\max} = R$$

$$\sigma_{avg} = \left(\frac{\sigma_x + \sigma_y}{2}\right)$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

Axial deformation, thermal expansion:

$$e = \frac{FL}{EA} + L\alpha\Delta T$$

$$F = \frac{EA}{L}(e - L\alpha\Delta T)$$

$$F = K(e - L\alpha\Delta T)$$

$$e = u\cos(\theta) + v\sin(\theta)$$

Torsion:

$$\tau = Gr\frac{\phi}{L}$$

$$\tau = \frac{Tr}{I_p}$$

$$\phi = \frac{TL}{GI_p}$$

$$K = \frac{GI_p}{L}$$

$$T = K\phi$$

$$I_{p\_Circular\_Cross\_Section} = \frac{\pi d^4}{32}$$

$$I_{p\_Hollow\_Circular\_Cross\_Section} = \frac{\pi(d_o{}^4 - d_i{}^4)}{32}$$

Stress due to bending moment:

$$\sigma(x,y) = \frac{-E(x)y}{\rho(x)} = \frac{-M(x)y}{I_{zz}}$$

$$I_{zz} = \frac{bh^3}{12}\ \text{(rectangle)} \qquad I_{zz} = \pi\frac{d^4}{64}\ \text{(circle)}$$

Stress due to shear force:

$$\tau(x,y) = \frac{V(x)Q(y)}{I_z b} \qquad Q(y) = \int_{A'}\eta\, dA = A'\bar{y}'$$

Equation Sheet

Failure criteria, factor of safety:

von Mises Equivalent Stress:

$$\sigma_M = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_{p_1}-\sigma_{p_2}\right)^2+\left(\sigma_{p_2}-\sigma_{p_3}\right)^2+\left(\sigma_{p_3}-\sigma_{p_1}\right)^2}$$

von Mises Stress (Plane Stress):

$$\sigma_M = \sqrt{\sigma_{p_1}^2 - \sigma_{p_1}\,\sigma_{p_2} + \sigma_{p_2}^2}$$

$$FS = \frac{Failure\ Stress}{Allowable\ Stress}, \frac{Yield\ \ Strength}{State\ of\ \ Stress}$$

Buckling:

Critical buckling load for a pinned-pinned beam

$$P_{cr} = EI\,\frac{\pi^2}{L^2}$$

Critical buckling for fixed-fixed beam

$$P_{cr} = EI\,\frac{\pi^2}{(0.5L)^2}$$

$$\left(\tau_{max}\right)_{N.A.} = \frac{3V}{2A}\text{(rectangle)} \quad \left(\tau_{max}\right)_{N.A.} = \frac{4V}{3A}\text{(circle)}$$

Stress in pressure vessels:

$$\sigma_{spherical} = \frac{pr}{2t};\quad \sigma_h = \frac{pr}{t};\quad \sigma_a = \frac{pr}{2t}$$

Integration rules for discontinuity and Macaulay functions

$$\left\langle x-a\right\rangle^n = \begin{cases} 0 & for\ x<a \\ (x-a)^n & for\ x\ge a \end{cases} \qquad n = 0,1,2,3$$

$$\int \left\langle x-a\right\rangle^n dx = \begin{cases} \left\langle x-a\right\rangle^{n+1} & for\ n\le 0 \\ \dfrac{1}{n+1}\left\langle x-a\right\rangle^{n+1} & for\ n>0 \end{cases}$$

$$EIv'' = M$$

$$(EIv'')' = V$$

$$(EIv'')'' = p$$

Energy methods:

Strain energy: $U_{tot} = \int \frac{M^2}{2EI}dx + \int \frac{F^2}{2AE}dx + \int \frac{T^2}{2GI_p}dx + \int \frac{f_s V^2}{2AG}dx$ where f_s is the shape factor

- Castigliano's theorem for deflection Δ at a point in the direction of the force $\Delta = \partial U_{tot} / \partial P$ where P is the point force
- Castigliano's theorem for slope θ_C or angle of twist ϕ_C in the direction of an applied moment M or applied torque T is given by $\theta_C = \partial U_{tot} / \partial M$ $or\ \phi_C = \partial U_{tot} / \partial T$
- Work-energy theorem states that the deflection Δ at the point of application of a force P in the direction of applied force can be calculated by equating $\frac{1}{2}P\Delta = U_{tot}$
- Work-energy theorem states that the slope θ_C or angle of twist ϕ_C in the direction of an applied moment M or applied torque T or can be calculated by equating $\frac{1}{2}M\theta_C = U_{tot}\ or\ \frac{1}{2}T\phi_C = U_{tot}$

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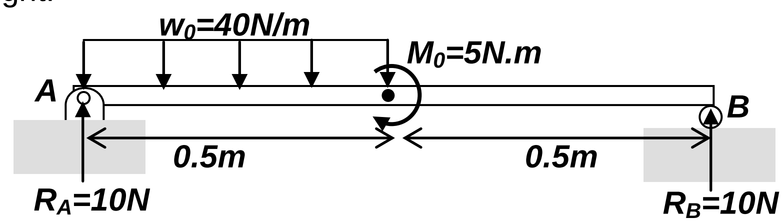
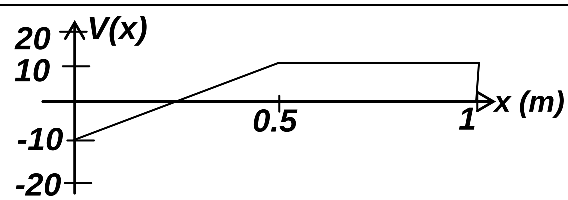
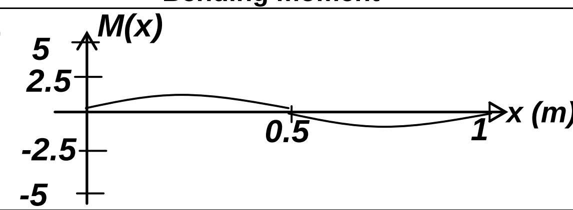
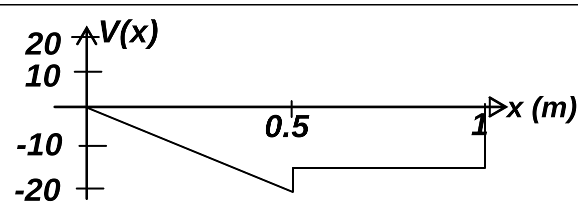
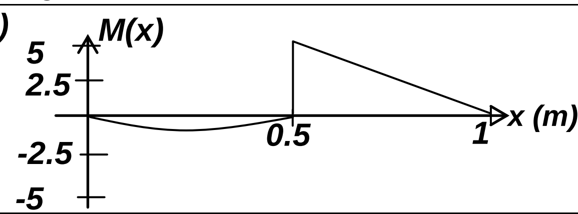
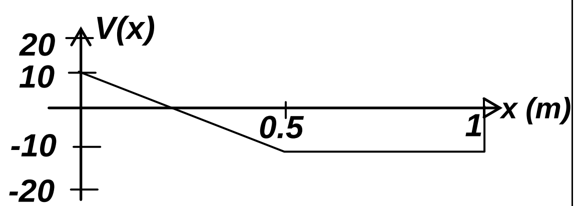
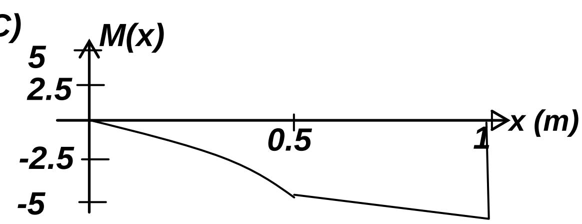
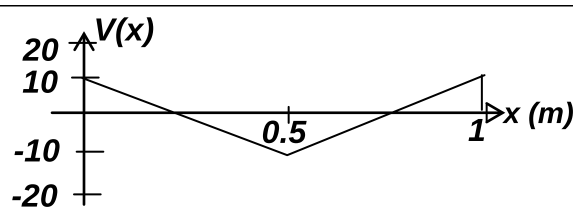
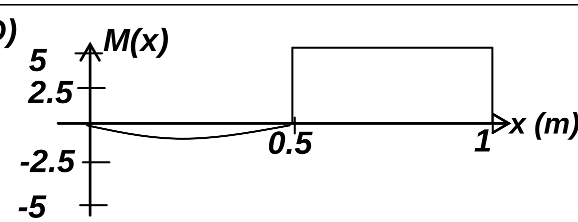
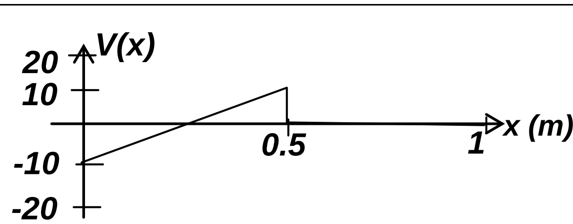
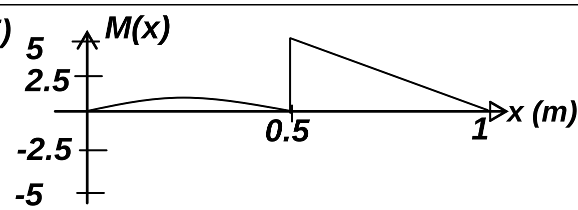
(Print) (Last) (First) (Circle one)

Problem 1.1 (8 Points, short problem)

For the beam loaded as shown below, choose the correct shear force and bending moment diagrams from below (*you need to correctly select **both** the correct shear and the correct moment diagram*). The diagrams on the left do not correspond to the diagrams on the right!



Shear Force	Bending moment
(A) 	(A) 
(B) 	(B) 
(C) 	(C) 
(D) 	(D) 
(E) 	(E) 
(F) None of the above	(F) None of the above

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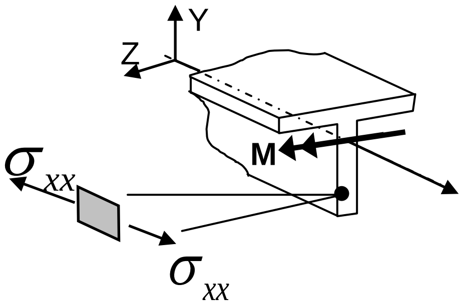
Problem 1.2 (8 Points, short problem)

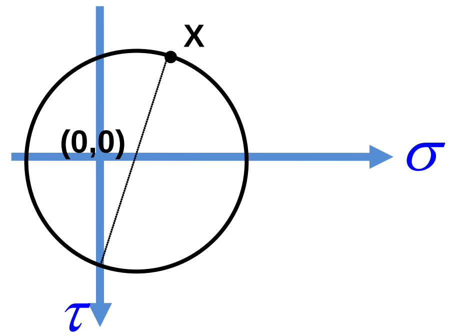
Match the stress element in the left column with the correct Mohr's circle on the right

T-section beam with internal moment at a section

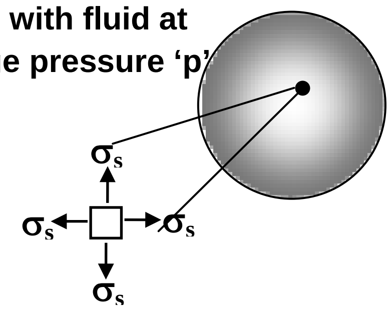


A

A

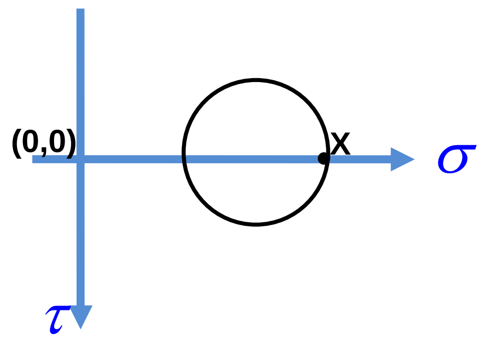


Thin spherical vessel filled with fluid at gauge pressure 'p'

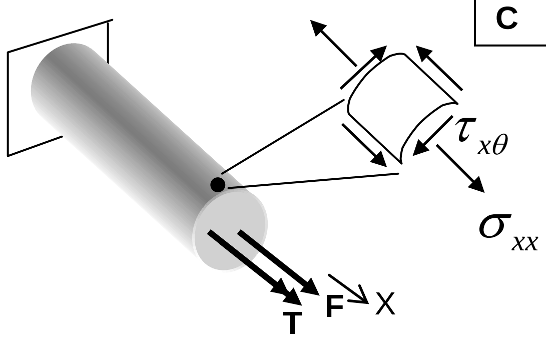


B

B

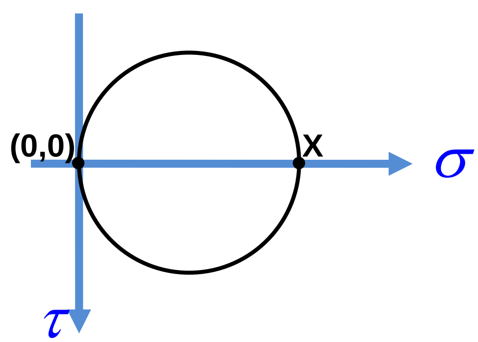


Rod subject to end torque and axial force



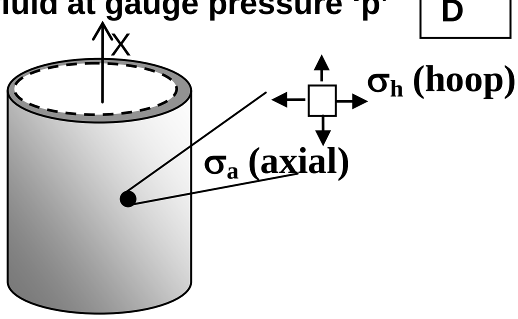
C

C

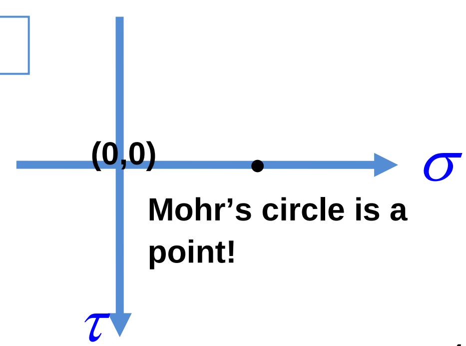


Thin cylindrical vessel filled with fluid at gauge pressure 'p'

D



D



Mohr's circle is a point!

Cover sheet and Problem 1

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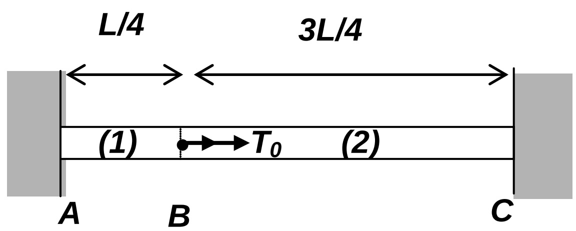
Problem 1.3 (9 Points, short problem)

A solid cylindrical rod of length $\boldsymbol{L}$, shear modulus $\boldsymbol{G}$, and polar area moment $\boldsymbol{I_p}$ is clamped between two walls as shown. A known torque $\boldsymbol{T_0}$ is applied as shown. State whether the following statements are true or false.



Circle either "True" or "False" for each of the statements A, B, C, D below.

A. Reaction torques at A and C must be equal in magnitude *True* *False*

B. $\left|(\tau_{max})_1\right| = 3\left|(\tau_{max})_2\right|$ (magnitude of max. shear stress in (1) is three times larger than (2)) *True* *False*

C. $\left|(\tau_{max})_2\right| = 3\left|(\tau_{max})_1\right|$ (magnitude of max. shear stress in (2) is three times larger than (1)) *True* *False*

D. $\left|\phi_2\right| = \left|\phi_1\right|$ (magnitudes of twist angles in (1) and (2) are the same) *True* *False*

E. $T_1 = T_2$ (internal torques in (1) and (2) are identical) *True* *False*

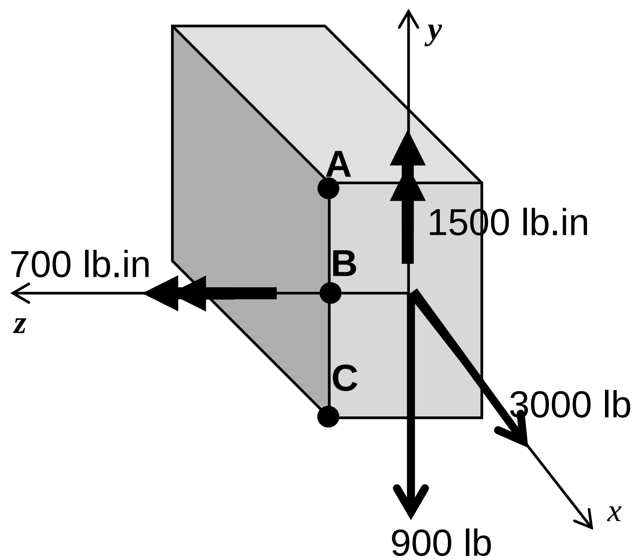
Problem 2

Name: _______________, _________________ **Instructor: Susilo Koslowski Raman**

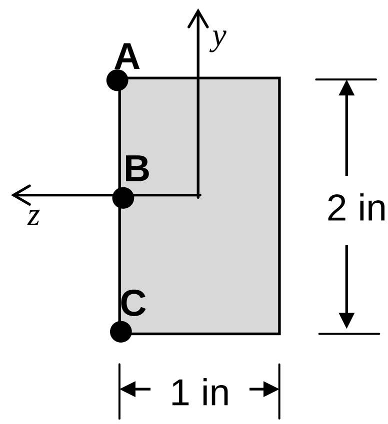
(Print) (Last) (First) (Circle one)

(25 Points) The internal resultants of a particular cross section of a structure are:

- Axial force in +x direction of magnitude 3000 lb
- Shear force in –y direction of magnitude 900 lb
- Bending moment in +y direction of magnitude 1500 lb-in
- Bending moment in +z direction of magnitude **7**00 lb-in

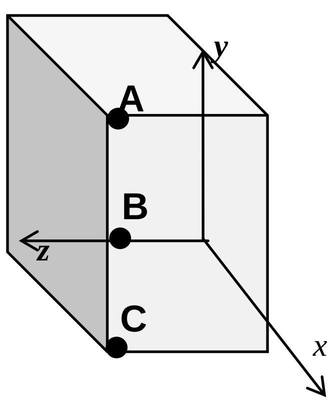


Cross-Sectional Area:

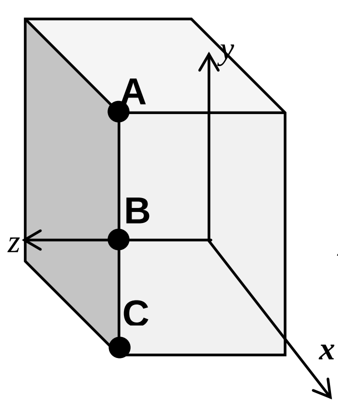


a. Clearly sketch the <u>stress distribution</u> on the cross section that results from each internal resultant on the diagram below. *If applicable, indicate the neutral axis and maximum and minimum stress location.*

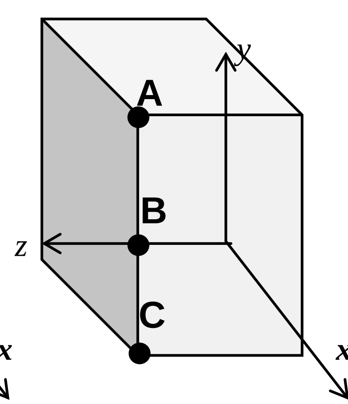
Shear Force in -y



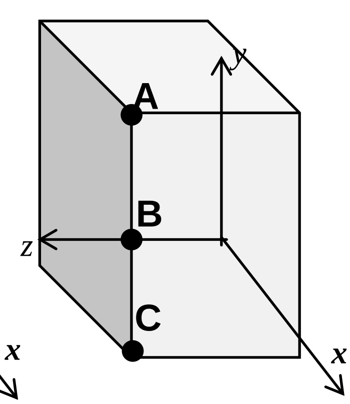
Axial Force in +x



Bending Moment in +y

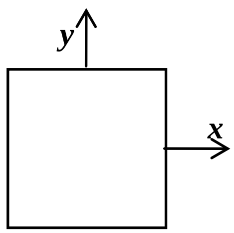


Bending Moment in +z

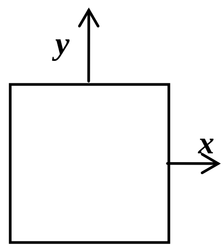


b. Determine the <u>state of stress</u> at points **A**, **B**, and **C and sketch it on the stress elements below**. *Indicate stress magnitudes on the stress element.*

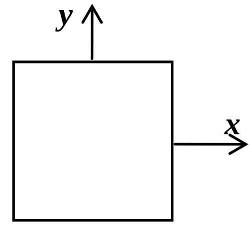
State of Stress at A:



State of Stress at B:



State of Stress at C:



Problem 3

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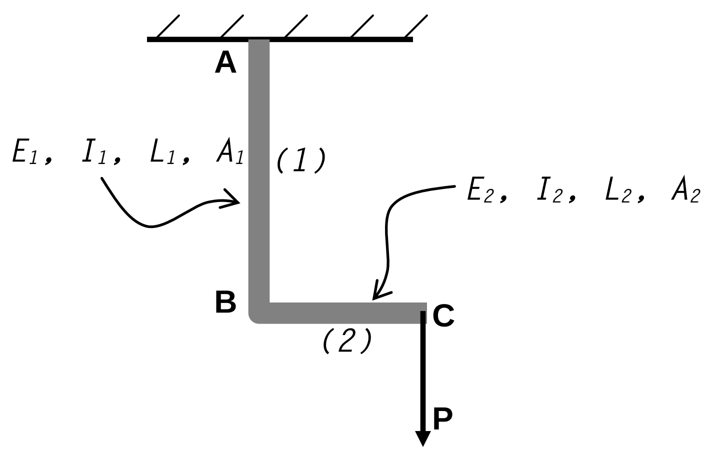
(25 Points)

The slender L-shaped beam consisting of two members (1) and (2) is welded to the wall (cantilevered) and is subject to a vertical force **P** as shown. Neglecting gravity and the contribution of shear strain to the total strain energy, derive an expression for the downward deflection Δ_C at the point the force is applied at.

The final answer for Δ_C needs to have all the integrals worked out and the answer should be given in terms of the applied load *P* as well as E_1, I_1, L_1, A_1 and E_2, I_2, L_2, A_2 which are respectively the Young's modulus, Area moment, length, and cross sectional areas of the two members.

You can either use the work-energy theorem or Castigliano's theorem.



Problem 4

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(25 Points) The figure shows a double-walled carbon nanotube (DWCNT) that consists of two concentric tubes with external diameters d_1 and d_2 with the same wall thickness, **t**. The dimensions, Young modulus and thermal expansion coefficient are **L=100 10^{-9} m, t=0.05 10^{-9} m, d_1=2.0 10^{-9} m, d_2=1.2 10^{-9} m, E=10^{12}Pa and α=1.6 10^{-6}/C.** The double-walled carbon nanotube has rigid stoppers at the ends (fixed-fixed boundary conditions).

Each tube can be regarded as a thin beam with hollow cross section with the following areas and area moments of inertia (subscript 2 is for inner tube, subscript 1 is for outer tube):

$$A_1 = \frac{\pi}{4}(d_1^2 - (d_1 - 2t)^2) = 0.30 \cdot 10^{-18} m^2$$

$$A_2 = \frac{\pi}{4}(d_2^2 - (d_2 - 2t)^2) = 0.18 \cdot 10^{-18} m^2$$

$$I_1 = \frac{\pi}{64}(d_1^4 - (d_1 - 2t)^4) = 0.15 \cdot 10^{-36} m^4$$

$$I_2 = \frac{\pi}{64}(d_2^4 - (d_2 - 2t)^4) = 0.030 \cdot 10^{-36} m^4$$

(a) Calculate the force P applied to these stoppers that prevents elongation of the tubes when the temperature is increased by **ΔT=200C.**
(b) Under the conditions of (a), calculate the (normal) axial stress in each tube.
(c) Under the conditions of (a), check if the system would fail by buckling. Which tube would buckle first?