Name: ___Solution___________________ Instructor: Siegmund Sarkar Susilo
(Print) (Last) (First) (Circle one)

ME 323 FINAL EXAM
FALL SEMESTER 2010
7:00 PM – 9:00 PM
Dec. 16, 2010

Instructions

1. Begin each problem in the space provided on the examination sheets. If additional space is required, use the paper provided. Work on one side of each sheet only, with only one problem on a sheet.

2. Each problem is of value as indicated below.

3. To obtain maximum credit for a problem, you must present your solution clearly. Accordingly:
   a. Identify coordinate systems
   b. Sketch free body diagrams
   c. State units explicitly
   d. Clarify your approach to the problem including assumptions

4. **If your solution cannot be followed, it will be assumed that it is in error**.

Prob. 1 _(25)________________________

Prob. 2 _(25)________________________

Prob. 3 _(25)________________________

Prob. 4 _(25)________________________

Total (100)

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$\sigma = F_n / A \quad \tau_{avg} = V / A \quad F.S. = F_{fail} / F_{allow} \quad F.S. = \sigma_{fail} / \sigma_{allow} \quad F.S. = \tau_{fail} / \tau_{allow}$

$\varepsilon_{avg} = (\Delta s' - \Delta s) / \Delta s = \delta / L_0 \quad \gamma = (\pi / 2) - \theta' \quad \sigma = E\varepsilon \quad \nu = -\varepsilon_{lat} / \varepsilon_{long} \quad \tau = G\gamma$

$$\delta_{th} = \alpha(\Delta T)L \quad \delta_{th} = \int_0^L \alpha(\Delta T)dx$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T \,, \varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T \,, \gamma_{xy} = \frac{1}{G}\tau_{xy} \quad , \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad , \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\delta = \frac{F_i L_0}{EA} \quad \delta = \int_0^L \frac{F_i(x)}{E(x)A(x)}dx \quad u_B = u_A + \delta_{AB}$$

$$\phi\rho = \gamma L \quad \tau = Gc\frac{\phi}{L} \quad \tau = \frac{T_i\rho}{J} \quad \phi = \frac{T_i L}{GJ} \quad \phi = \int_0^L \frac{T_i(x)}{G(x)J(x)}dx \quad \phi_B = \phi_A + \phi_{AB}$$

$$J = \frac{\pi c^4}{2} \ldots bar \quad J = \frac{\pi(c_o^{\,4} - c_i^{\,4})}{2} \quad \cdots tube$$

$$\frac{dV}{dx} = p(x) \qquad \left\langle x-a \right\rangle^n = \begin{cases} 0 & for\ x < a \\ (x-a)^n & for\ x \ge a \end{cases} \quad n = 0,1,2,3$$

$$\frac{dM}{dx} = V(x) \qquad EIv'' = M, \quad (EIv'')' = V, \quad (EIv'')'' = p$$

$$\Delta V = P \qquad \Delta M = -M_0$$

$$\int \left\langle x-a \right\rangle^n dx = \begin{cases} \left\langle x-a \right\rangle^{n+1} & for\ n \le 0 \\ \frac{1}{n+1}\left\langle x-a \right\rangle^{n+1} & for\ n > 0 \end{cases}$$

$$\sigma(x,y) = \frac{-E(x)y}{\rho(x)} = \frac{-M(x)y}{I_z},$$

rectangle: $I = (bh^3)/12$, circle: $I = (\pi r^4)/4$,

semicircle : $I = (\pi r^4)/8$, centroid at $(4r)/(3\pi)$ from diameter

$$\bar{y} = (\sum_i \bar{y}_i A_i)/(\sum_i A_i) \qquad I = \sum_i I_{0,i} + d_i^{\,2} A_i$$

$$\tau(x,y) = \frac{V(x)Q(y)}{I_z b}, \quad q = \frac{VQ}{I}, \quad Q(y) = \int_{A'} \eta dA = \bar{y}' A'$$

spherical PV : $\sigma_s = \frac{pr}{2t}$; cylindrical PV : $\sigma_h = \frac{pr}{t}$; $\sigma_a = \frac{pr}{2t}$;

$$u = (1/2)(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz})$$

$$U_{elas}(truss) = \sum \frac{N_i^2 L_i}{2E_i A_i}, U_{elas}(torsion\ bar) = \sum \frac{T_i^2 L_i}{2G_i J_i}, U_{elas}(bending) = \int_L \frac{[M(x)]^2}{2EI}dx$$

$$(1/2)P_{ext}\Delta = U_{elas} \quad (1/2)M_{ext}\theta = U_{elas}$$

$$1\cdot\Delta = \sum \frac{n_i N_i L_i}{E_i A_i}, 1\cdot\theta = \sum \frac{t_i T_i L_i}{G_i J_i}, 1\cdot\Delta = \int_L \frac{m(x)M(x)}{EI}dx, 1\cdot\theta = \int_L \frac{m_\theta(x)M(x)}{EI}dx$$

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$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta_{xx'} + \tau_{xy}\sin 2\theta_{xx'} \quad \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta_{xx'} - \tau_{xy}\sin 2\theta_{xx'}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta_{xx'} + \tau_{xy}\cos 2\theta_{xx'} \quad \sigma_1 = \sigma_{avg} + R, \ \sigma_2 = \sigma_{avg} - R, \quad \tau_{\max} = R$$

$$\sigma_{avg} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}, \ \tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)} \quad \tan 2\theta_s = \frac{-\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\tau_{xy}}$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta_{xx'} + \frac{\gamma_{xy}}{2}\sin 2\theta_{xx'} \quad \varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta_{xx'} - \frac{\gamma_{xy}}{2}\sin 2\theta_{xx'}$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta_{xx'} + \frac{\gamma_{xy}}{2}\cos 2\theta_{xx'} \quad \varepsilon_1 = \varepsilon_{avg} + R, \ \varepsilon_2 = \varepsilon_{avg} - R, \quad \frac{\gamma_{in-plane}^{\max}}{2} = R$$

$$\varepsilon_{avg} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) \quad R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}, \ \tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \quad \tan 2\theta_s = -\left(\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}}\right)$$

Tresca: $\tau_{\max}^{abs} = \frac{\sigma_{yield}}{2}$, von Mises: $\sigma_{yield}^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2$, Brittle Solids: $|\sigma_1| = \sigma_{ultimate}, |\sigma_2| = \sigma_{ultimate}$

$$P_{crit} = \frac{\pi^2 EI}{(L_{eff})^2}$$

PROBLEM #1 (25 points)

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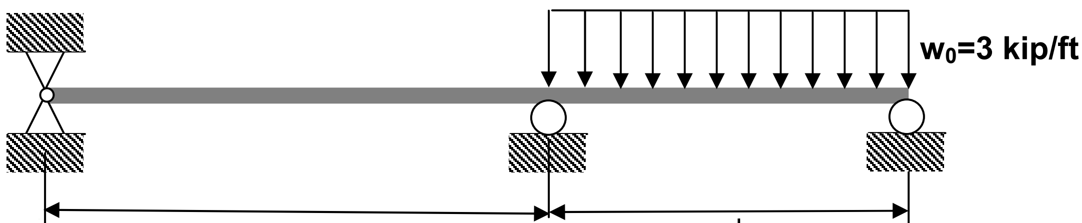
The beam (length **a+b**) shown below is loaded by a distributed load $\mathbf{w_0}$ and is supported by three simple supports. **EI** is constant.

- Determine the reactions at the three supports
- Sketch the shear force and bending moment diagrams.



$\Sigma F_x = 0$

$\Sigma F_y = 0 \quad A_y + B_y + C_y - 3\,\text{kip/ft} \cdot 8\,\text{ft} = 0$

$\Sigma M_A = 0 \quad 18C_y + 8B_y - 24 \cdot (4) = 0$ (1)

$p(x) = C_y\langle x-0\rangle^{-1} + B_y\langle x-10\rangle^{-1} - 3\langle x-10\rangle^{0}$

$V(x) = C_y\langle x\rangle^{0} + B_y\langle x-10\rangle^{0} - 3\langle x-10\rangle^{1}$

$M(x) = C_y\langle x\rangle^{1} + B_y\langle x-10\rangle^{1} - \frac{3}{2}\langle x-10\rangle^{2}$

$EI v'' = M(x) = C_y\langle x\rangle^{1} + B_y\langle x-10\rangle^{1} - \frac{3}{2}\langle x-10\rangle^{2}$

$EI v' = \frac{C_y}{2}\langle x\rangle^{2} + \frac{B_y}{2}\langle x-10\rangle^{2} - \frac{3}{6}\langle x-10\rangle^{3} + C_1\langle x\rangle^{0}$

$EI v = \frac{C_y}{6}\langle x\rangle^{3} + \frac{B_y}{6}\langle x-10\rangle^{3} - \frac{3}{24}\langle x-10\rangle^{4} + C_1\langle x\rangle^{1} + C_2\langle x\rangle^{0}$

Boundary Conditions

$v(0) = 0 \Rightarrow C_2 = 0$

$v(10) = 0$

$EI(0) = \frac{C_y}{6} 10^3 + \frac{B_y}{6} \langle 0 \rangle^3 - \frac{1}{8}(0)^4 + C_1(10)$

(2) $= 166.67\, C_y + 10\, C_1 = 0$

$v(18) = 0$

$EI(0) = \frac{C_y}{6}(18)^3 + \frac{B_y}{6}(8)^3 - \frac{1}{8}(8)^3 + C_1(18)$

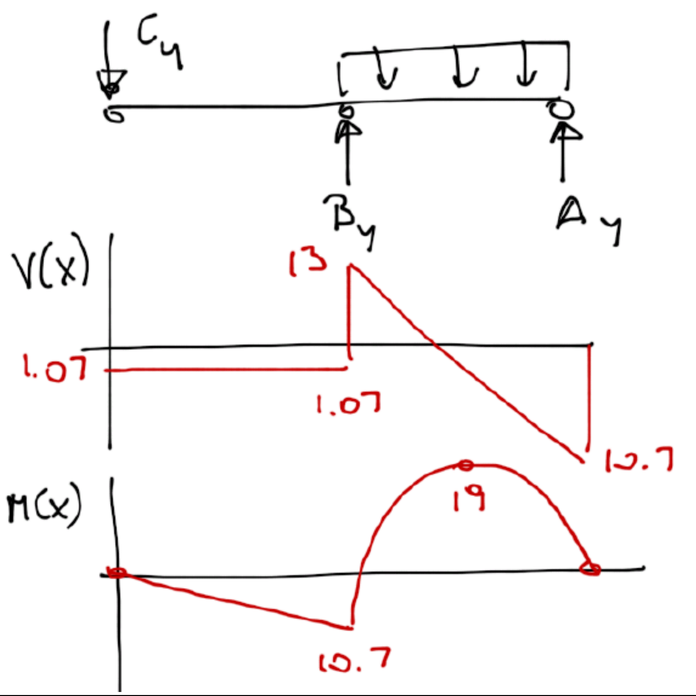
(3) $972\, C_y + 85.3\, B_y - 512 + 18\, C_1 = 0$

Solve (1) + (2) + (3)

$B_y = 14.4$ Kip $\quad A_y = 10.7$ Kip

$C_y = -1.07$ Kip

$C_1 = 17.78$ and



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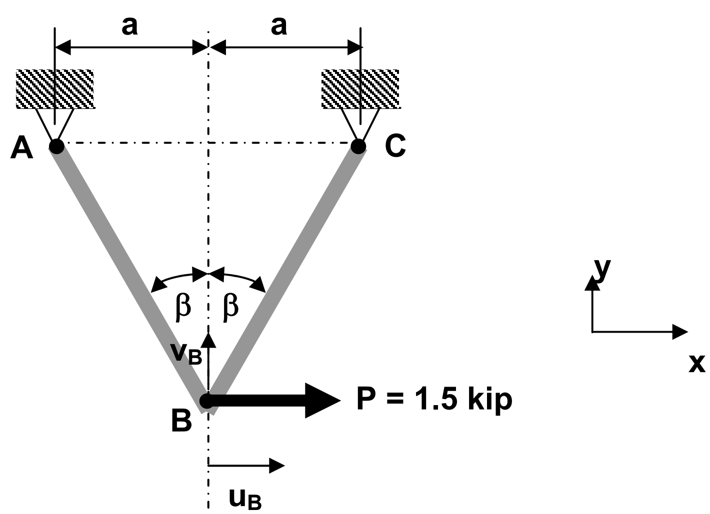
PROBLEM #2 (25 points)

The truss structure shown below is loaded by a force **P=1.5 kip** acting in the horizontal direction. The elements AB and BC possess cross section area **A=1.0 in^2**, and elastic modulus **E=30x10^3 ksi**. The distance AC is **2a=48 in**. The angle between the two elements is **2β=60°**.

Using work-energy principle and/or the principle of virtual work:

- Determine the horizontal displacement of point B, $\mathbf{u_B}$.
- Determine the vertical displacement of point B, $\mathbf{v_B}$.



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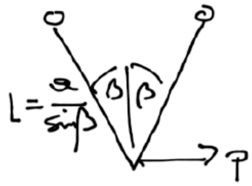
Problem #2



$$\sum F_y = 0$$

$$F_1 \cos\beta + F_2 \cos\beta = 0$$

$$F_1 = -F_2$$

$$\sum F_x = 0$$

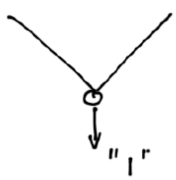
$$-F_1 \sin\beta + F_2 \sin\beta + P = 0$$

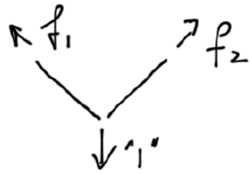
$$\Rightarrow F_2 = -\frac{P}{2\sin\beta} \qquad F_1 = \frac{P}{2\sin\beta}$$

$$U = \frac{F_1^2 L_1}{2E_1A_1} + \frac{F_2^2 L_2}{2E_2A_2} =$$

$$= \left(\frac{P^2}{4\sin^2\beta} \cdot \frac{a}{\sin\beta} \cdot \frac{1}{2EA}\right) \times 2$$

$$= \frac{P^2 a}{4\sin^3\beta} \frac{1}{EA} = \frac{1}{2} P u \quad \Rightarrow \boxed{u = 0.0048 \text{ in}}$$





$$\sum F_y = 0 \quad f_1 \cos\beta + f_2 \cos\beta - 1 = 0$$

$$\sum F_x = 0 \quad -f_1 \sin\beta + f_2 \sin\beta = 0$$

$$f_1 = f_2$$

$$2 f_1 \cos\beta = 1$$

$$f_1 = \frac{1}{2\cos\beta} = \frac{1}{\sqrt{3}} = f_2 = 0.577$$

$$\text{"1"}\, v = f_1 \frac{F_1 L}{EA} + f_2 \frac{F_2 L}{EA}$$

$$= \frac{1}{\sqrt{3}} \frac{Pa}{2\sin^2\beta} \frac{1}{EA} + \frac{1}{\sqrt{3}} \left(\frac{-Pa}{2\sin^2\beta} \frac{1}{EA}\right)$$

$$\Rightarrow \boxed{v = 0}$$

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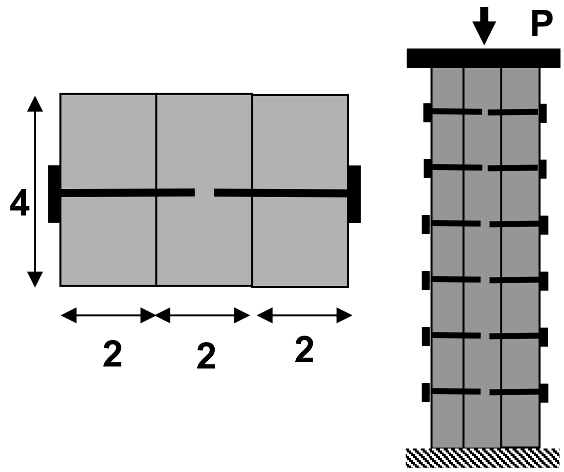
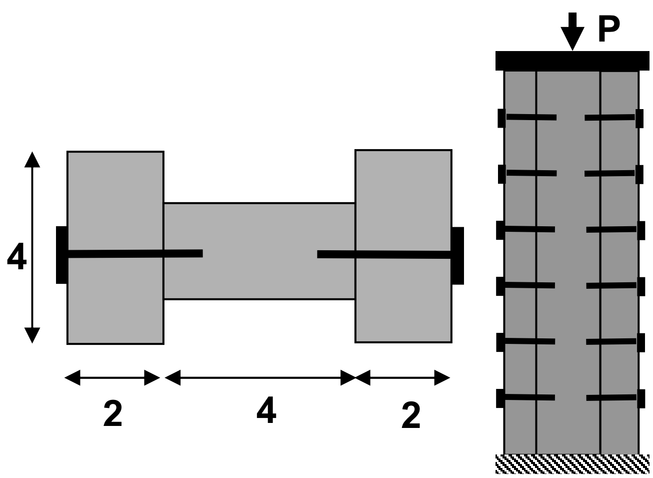
PROBLEM #3 (25 points)
Consider that three wood planks of cross section **a=2 in.** by **b=4 in.** and length **L=6 ft** are available to construct a column structure to carry an axial load. Given the three planks of wood and a few nails you can construction the two columns shown below. Assume that the ends of the column act as pin supports, and that wood possesses an elastic modulus of **E=1,000 ksi**.

Given the two options which one should be selected such that the largest load **P** can be sustained. Your answers need to be based on calculated numbers.

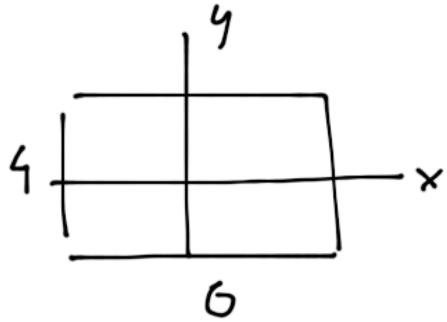




Problem #3

$$P = \frac{\pi^2 E I}{L_{eff}^2}$$

Case A:



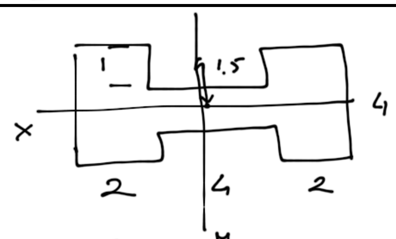
$$I_x = \frac{6 \cdot 4^3}{12} = 32 \text{ in}^4$$

$$I_y = \frac{4 \cdot 6^3}{12} = 72 \text{ in}^4$$

$$P = \frac{\pi^2 E I_x}{L^2} = 60 \text{ Kip}$$

$$P = \frac{\pi^2 E I_y}{L^2} = 137 \text{ Kip}$$

Case B



$$I_x = \frac{8 \cdot 4^3}{12} - \left[\frac{4 \cdot 1^3}{12} + 1.5^2 (1 \cdot 4)\right] \cdot 2 = 25 \text{ in}^4$$

$$I_y = \frac{4 \cdot 8^3}{12} - \left[\frac{1 \times 4^3}{12}\right] \cdot 2 = 160 \text{ in}^4$$

$$P = \frac{\pi^2 E I_x}{L^2} = 45 \text{ Kip}$$

$$P = \frac{\pi^2 E I_y}{L^2} = 304 \text{ Kip}$$

$\Rightarrow$ Largest load is <u>60 Kip</u>

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PROBLEM #4a (10 points)

A state of plane stress is given as: $\sigma_x = 10MPa, \sigma_y = 2MPa, \tau_{xy} = 2MPa$.

Check the correct answer

If the material is a ductile metal with yield strength of $\sigma_{Yield} = 10.0MPa$:

(3 pts) Following the Mises criterion, yielding **occurs** ☐ or **does not occur** ☐**.**
(3 pts) Following the Tresa criteriaon, yielding **occurs** ☐ or **does not occur** ☐.

If the material is a brittle ceramic with ultimate strength of $\sigma_{Ultimate} = 12.0MPa$:

(3 pts) Fracture of **occurs** ☐ or **does not occur** ☐.

Problem 4(a)

$\sigma_x = 10$ MPa $\quad \tau_{xy} = 2$ MPa
$\sigma_y = 2$ MPa

$\sigma_{avg} = 6$ MPa $\quad R = 4.4$ MPa
$\sigma_1 = 10.4$ MPa
$\sigma_2 = 1.5$ MPa

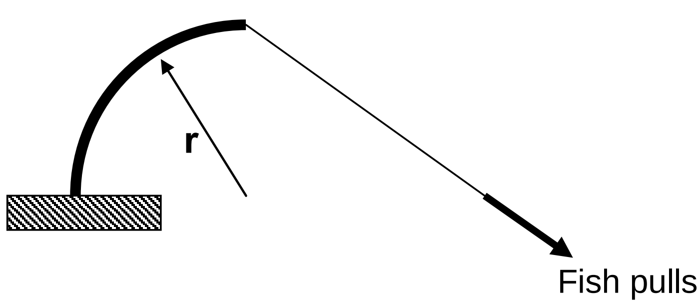
$\sigma_M^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2$
$\sigma_M = 9.7$ MPa $< \sigma_y \Rightarrow$ no yield

$\sigma_1 > 0 \quad \sigma_2 > 0 \Rightarrow \tau_{max}^{abs} = \frac{\sigma_1}{2} = 5.2$ MPa
5.2 MPa $> \sigma_y/2 \Rightarrow$ yield

PROBLEM #4b (5 points) Multiple Choice, no partial credit

While hooking a fish, a fishing rod is bend to a quarter circle of radius r=2.0 m. Consider that the fishing rod possesses a diameter of 5.0 mm and an elastic modulus of E=100 GPa.

The maximum value of flexural stress is the rod is 50 MPa ☐**.**
The maximum value of flexural stress is the rod is 125 MPa ☑**.**
The maximum value of flexural stress is neither of the above ☐**.**
The flexural stresses cannot be determined based on the information provided ☐**.**



PROBLEM #4c (5 points) Multiple Choice, no partial credit

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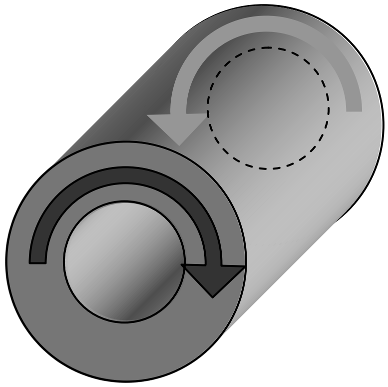
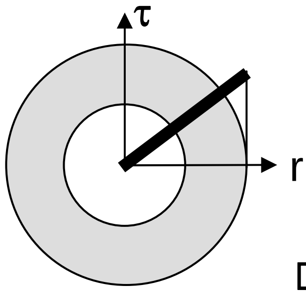
A tube is subjected to torsion. From the options shown below, select the correct answer for the stress distribution in a tube cross section.





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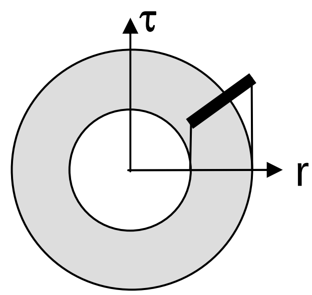

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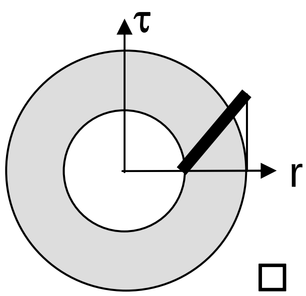

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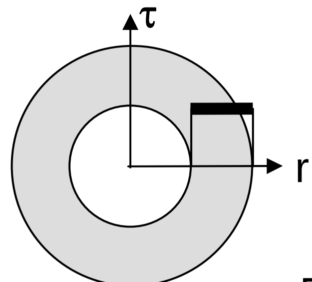

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PROBLEM #4d (6 points) Multiple Choice, no partial credit

A bar is clamped between two walls. The bar is initially free of stress. Then, the system is heated to a temperature $\mathbf{T_2}$. The stresses that develop in the bar depend on:
(check all that apply)

The coefficient of thermal expansion α of the material the bar is made of. ☑
The elastic modulus **E** of the material the bar is made of. ☑
The bar's length **L**. ☐
The bar's cross section area **A**. ☐
The initial temperature $\mathbf{T_1}$ of the bar. ☑