sample exam / SOLUTION

Name: ______________________________ **Instructor:**
(Print) (Last) (First) **(Circle one)**

Answers at the end, look at them only after you have put in a genuine attempt to solve the problem first, ideally timing yourself in the process

ME 323 FINAL EXAM
FALL SEMESTER 2009
7:00 PM – 9:00 PM
Dec. 15, 2009

Instructions

1. Begin each problem in the space provided on the examination sheets. If additional space is required, use the paper provided. Work on one side of each sheet only, with only one problem on a sheet.

2. Each problem is of value as indicated below.

3. To obtain maximum credit for a problem, you must present your solution clearly. Accordingly:
   a. Identify coordinate systems
   b. Sketch free body diagrams
   c. State units explicitly
   d. Clarify your approach to the problem including assumptions

4. **If your solution cannot be followed, it will be assumed that it is in error.**

Prob. 1 _(26)________________

Prob. 2 _(25)________________

Prob. 3 _(25)________________

Prob. 4 _(24)________________

Total ________________

1

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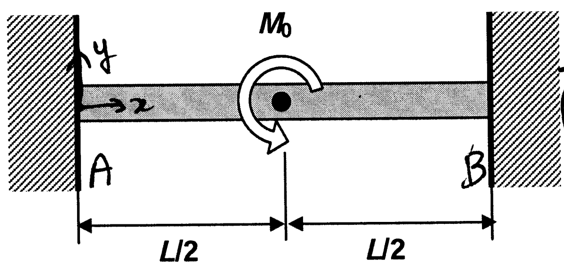
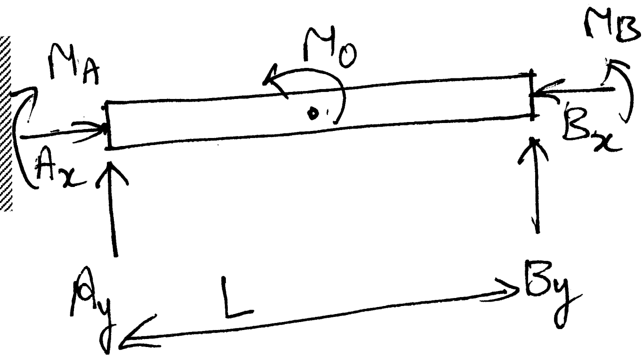
PROBLEM #1 (26 points)

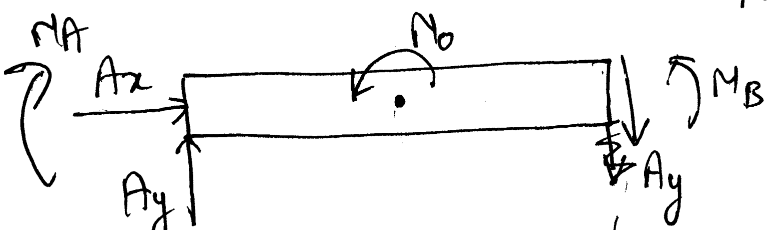
A beam possesses length L, modulus E and cross section property I_z. It supported with a cantilever support at both ends. Midway along the beam ($x=L/2$) a moment M_0 is applied.

- **You are asked to determine the bending moment diagram for this beam.**
- Your answer must emerge from a solution of the differential equation for beam deflection. The solution can make use of discontinuity functions if you like

Hint: At ($x=L/2$) the deflection is $v(x=L/2)=0$.





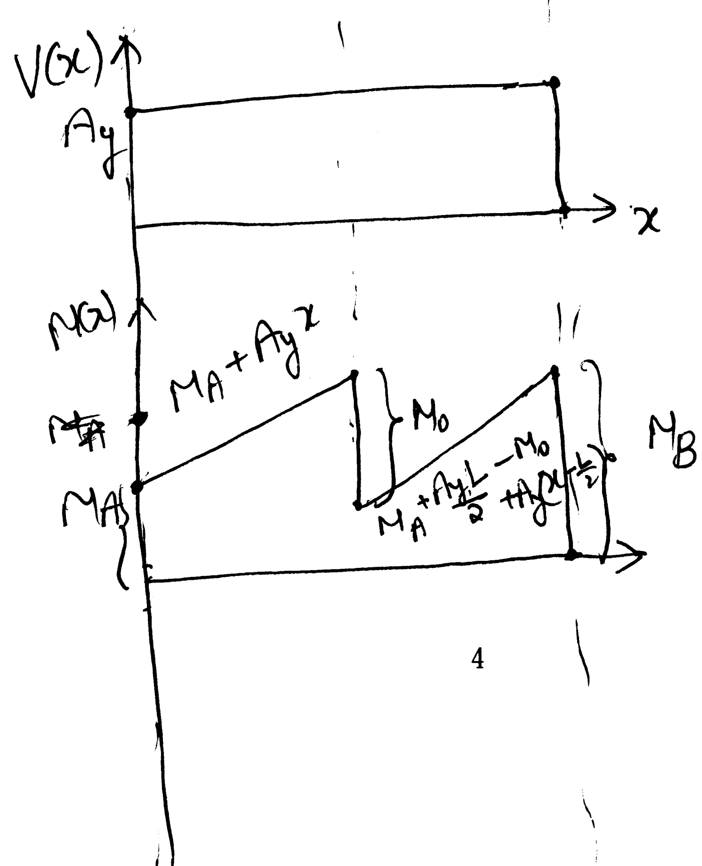


$A_x = B_x$ — ① ~~$= E_x$~~

$A_y + B_y = 0$ — ②

$A_y = -B_y$ ~~$= E_y$~~

$M_A - M_0 - M_B - B_y L = 0$ — ③



problem 1 — pg 2

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$$M(x) = M_A + A_y x \quad (0 \le x < L/2)$$

$$E\cancel{I}\cancel{\frac{d^2v}{d}} = M_A + \frac{A_y L}{2} - M_0 + A_y x \quad (L/2 < x < L)$$

$$EI\frac{d^2v}{dx^2} = M_A + A_y x$$

$$EI\frac{dv}{dx} = M_A x + A_y \frac{x^2}{2} + \cancel{c_1} \to 0$$

$$\left.\frac{dv}{dx}\right|_{x=0} = 0$$

$$EIv = M_A \frac{x^2}{2} + A_y \frac{x^3}{6}$$

Given ~~$v(x=0)$~~ $v(x=L/2) = 0$

$$\Rightarrow \frac{M_A L^2}{8} + \frac{A_y L^3}{48} = 0 \quad \text{— } 5$$

$$\frac{M_A}{1} + \frac{A_y L}{6} = 0$$

$$M_A = -A_y L/6$$

$$EI\frac{d^2v}{dx^2} = M_A + \cancel{\frac{A_y L}{2}} - M_0 + A_y x$$

$$EI\frac{dv}{dx} = (M_A - M_0)x + A_y\frac{x^2}{2} + d_1$$

$$\left.\left(\frac{dv}{dx}\right)\right|_{x=L} = 0$$

$$\Rightarrow (M_A - M_0)L + \frac{A_y L^2}{2} + d_1 = 0 \quad \text{— } 4$$

$$\Rightarrow d_1 = -(M_A - M_0)L - \frac{A_y L^2}{2}$$

Also continuity at $x = L/2$

$$\left.(dv/dx)\right|_{x=L/2^-} = \left.\left(\frac{dv}{dx}\right)\right|_{x=L/2^+}$$

$$\Rightarrow M_A\frac{L}{2} + \frac{A_y L^2}{8} = (M_A - M_0)\frac{L}{2} + \frac{A_y L^2}{8} + d_1$$

— 6

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Problem 1 pg 3

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$$M_A = -A_y L/6 \quad — \quad (5)$$

$$\cancel{M_A \frac{L}{2}} = \cancel{M_A \frac{L}{2}} - M_0 \frac{L}{2} - (M_A - M_0) L - \frac{A_y L^2}{2}$$

$$M_0 \frac{L}{2} - M_A L - \frac{A_y L^2}{2} = 0 \quad — \quad (6*)$$

$$M_0 \frac{L}{2} = \left(-A_y \frac{L}{6}\right) L + A_y \frac{L^2}{2}$$

$$M_0 \frac{L}{2} = \frac{A_y L^2}{2} \left(\frac{2}{3}\right)$$

$$\Rightarrow A_y \frac{L^2}{3} = M_0 \frac{L}{2} \Rightarrow \boxed{A_y = \frac{3}{2} \frac{M_0}{L}}$$

$$M_A = -\left(\frac{3}{2} \frac{M_0}{L}\right) \frac{L}{6} = -\frac{M_0}{4}$$

$$\boxed{M_A = -\frac{M_0}{4}}$$

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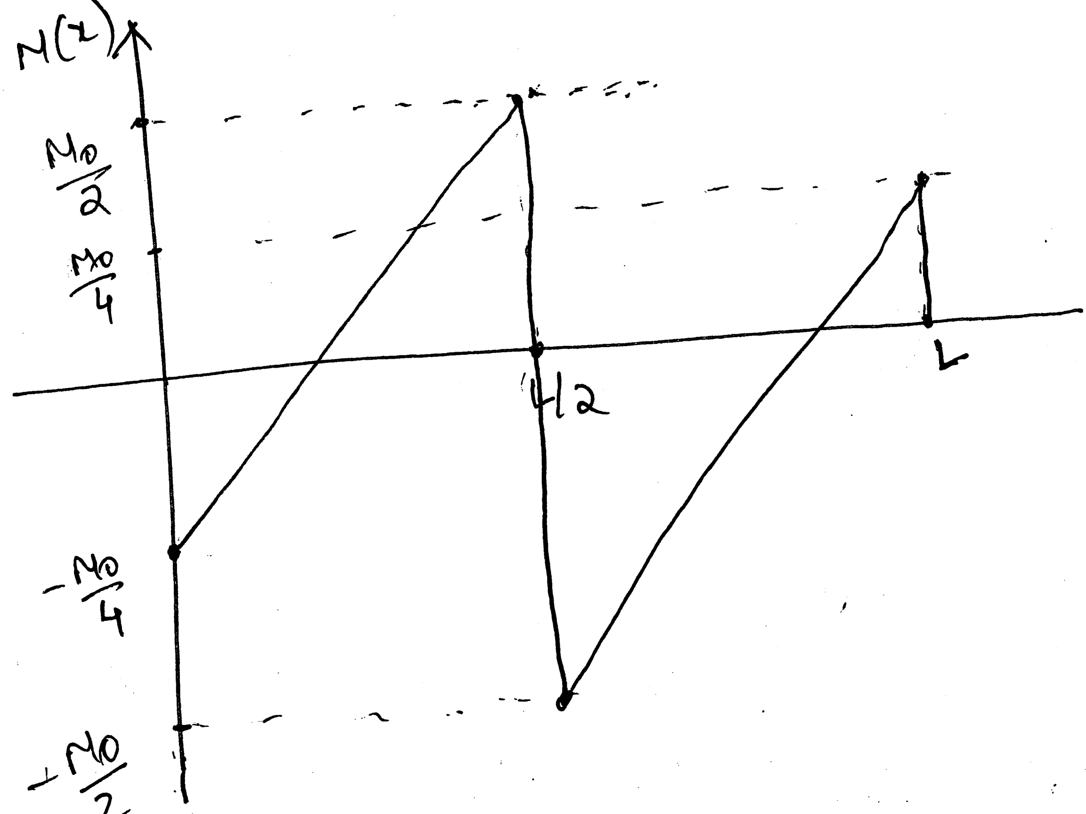
$M_A = -\frac{M_0}{4}$, $A_y = \frac{3}{2}\frac{M_0}{L}$, $\boxed{\text{problem 1, pg-3}}$



$-\frac{M_0}{4} + \frac{3}{2}\cdot\frac{M_0}{L}\cdot\frac{L}{2}$

$\frac{M_0}{2}$

$-M_0 + \frac{3}{2}\frac{M_0}{L}$

$-\frac{M_0}{4}$

$\frac{M_0}{4}$

$+\frac{3}{2}\frac{M_0}{L}L$

$M_B = -\frac{M_0}{4} - M_0$

$\frac{M_0}{4}$

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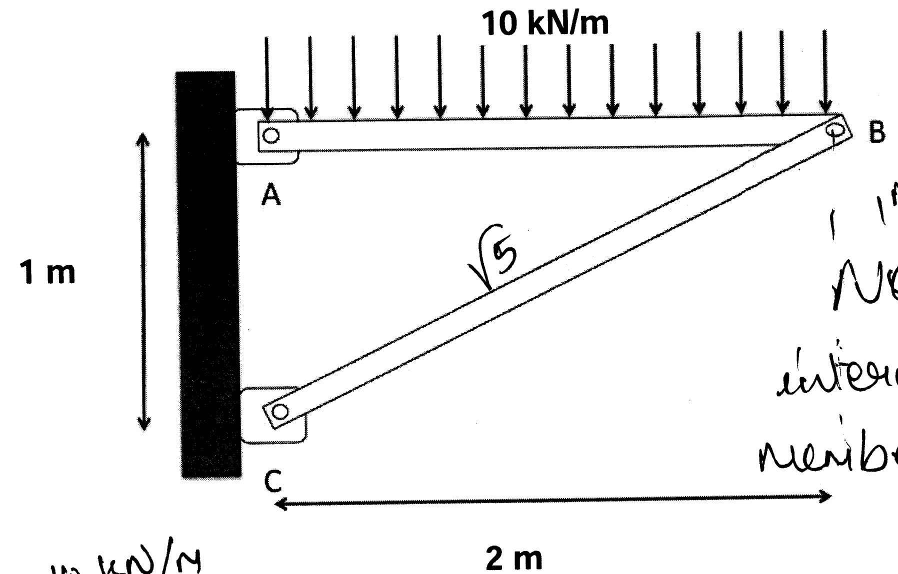
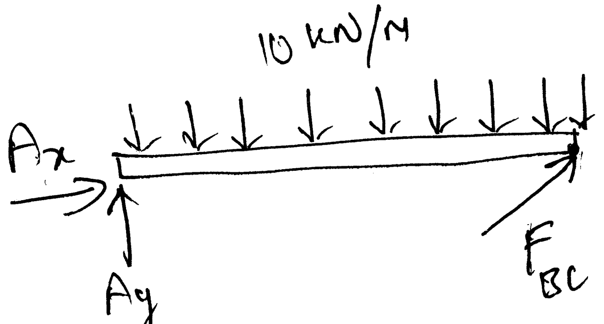
PROBLEM #2 (25 points)

A truss structure consists of two members AB and BC which are connected by pin joints. The **rigid** member AB is subjected to a distributed load of magnitude 10kN/m. The **deformable** member BC possesses a circular cross section of diameter **D**.

Determine the diameter of the truss BC in order to prevent buckling of the member BC with a factor of safety ***FS***=2.9. Consider that the modulus of the material used for the truss BC is ***E***=210GPa.



Need to find internal force of member BC.



$\Sigma F_y = 0$

$$A_y - (10)(2)\,\text{kN} + F_{BC}\frac{1}{\sqrt{5}} = 0$$

ΣM about $A = 0$

$$(10)(2)(1) - F_{BC}\left(\frac{1}{\sqrt{5}}\right)(2) = 0$$

$$F_{BC} = 10\sqrt{5}\ \text{kN}$$

Member BC → pin → pin supported

$$P_{cr} = \frac{\pi^2 E I}{(L_{eff})^2} \ ; \quad L_{eff} = L$$

problem 2 - pg 2

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$$P_{allow} = \frac{P_{cr}}{F.S} = \frac{\frac{(\pi^2)(210)(10^9)\frac{\pi}{64}d^4}{(\sqrt{5})^2}}{2.9}$$

$$= (7.016)(10^9)\,d^4$$

$\cancel{P}\; F_{BC} < P_{allow}$

$$(7.016)(10^9)\,d^4 > (10\sqrt{5})(10^3)$$

$$d^4 > \frac{(10\sqrt{5})}{(7.016)(10^6)} = (3.18)(10^{-6})$$

$d > 0.042\ M$

$d > \underline{\underline{4.2\ CM}}$

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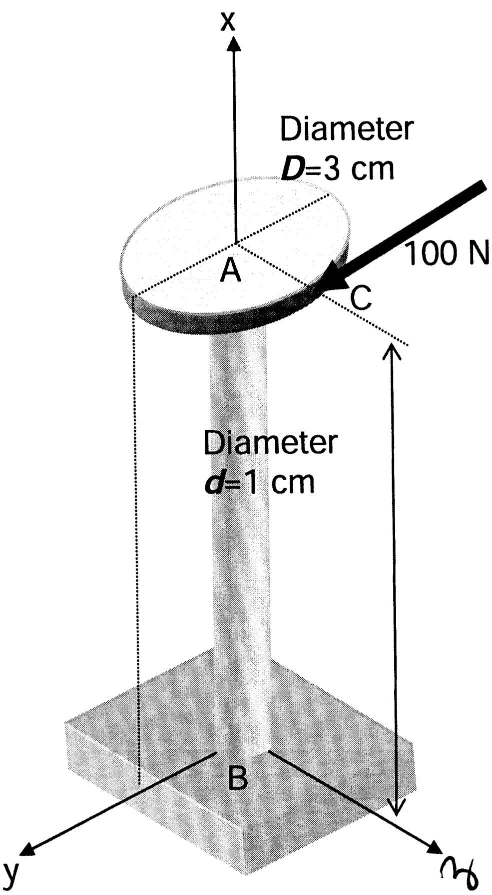
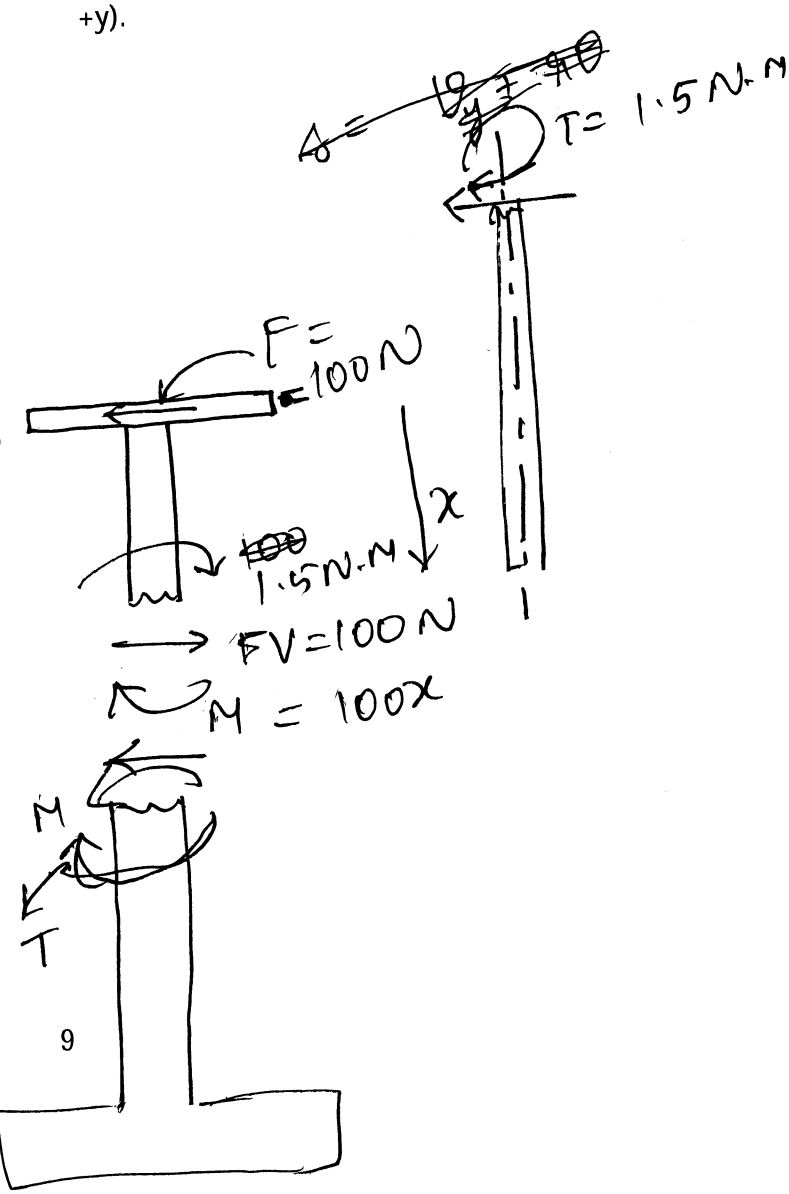
PROBLEM #3 (25 points)



A force is applied at point C to a (very thin) circular **rigid** plate with diameter ***D***=3cm. The plate is attached to an **elastic shaft** (AB) of diameter ***d***=1cm and length ***L***=10cm. All the strain energy is stored in the shaft (none in the plate since it is rigid). Also assume that strain energy stored due to transverse shear is negligible. The material is Aluminum (***E***= 73 GPa, ***G***=27 GPa)

Using the work-energy principle determine the deflection of point C in the direction of the applied force (i.e. along +y).

$(100)(0.015)$



T = 1.5 N.M
M = 100x
V = 100 N

$9.82(10^{-10})\ m^4 = I_p = \frac{\pi}{32} d^4 = \left(\frac{\pi}{32}\right)(0.01)^4$

3-2

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$$U_{torsion} = \frac{T^2 L}{2 G I_p} = \frac{(1.5)^2 \times 0.1}{(2)(27)(10^9)(9.82)(10^{-10})}$$

$$= 4.24\,(10^{-3}) \text{ joules}$$

$$= 0.00424$$

$$U_{bending} = \int_0^L \frac{(100x)^2}{(2)(73)(10^9) \times \frac{1}{2}(9.82)(10^{-10})}\,dx$$

$$= \int_0^{0.1} \frac{10^4}{(7.3)(9.82)} x^2\,dx$$

$$= \frac{10^4}{(7.3)(9.82)}\left(\frac{1}{3}\right)(0.1)^3$$

$$= \frac{10}{(7.3)(9.82)(3)} = 0.0465 \text{ joul.}$$

Total energy = 0.0507 joules

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PROBLEM #4.1 (4 + 4 points, no partial credit)

A thermal switch consists of a copper bar which under elevation of temperature closes a gap and closes an electrical circuit. The copper bar possesses a length ***L***, modulus ***E***, and coefficient of thermal expansion a. At room temperature T_0 the gap is ***d***.

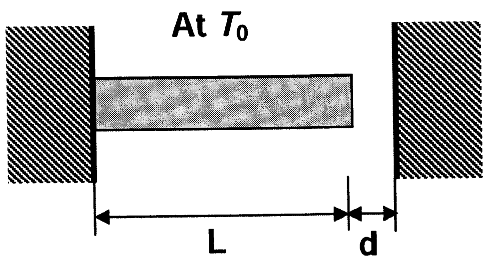
(1) The gap is closed at a temperature T_C calculated from equation **(circle one)**:

(a) $d = \alpha(T_C - T_0)$

(b) $d = L\alpha(T_C)$

(c) $d = L\alpha(T_C - T_0)$

(d) $d = E\alpha(T_C - T_0)$



(2) As the temperature is increased beyond T_C, the there exist stresses s in the bar equal to **(circle one)**:

(a) $\sigma = E\alpha(T - T_0)$

(b) $\sigma = -E\alpha(T - T_C)$

(c) $\sigma = -E\alpha(T - T_0)$

(d) $\sigma = EL\alpha(T - T_0)$

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PROBLEM #4.2 (4 + 4 points, no partial credit)

(1) A cylindrical pressure vessel with diameter D=2.5 m and wall thickness t=10 mm is filled with gas at an internal pressure of p=2400 kPa.

The absolute maximum shear stress in the vessel is **(circle one)**

$p = 2400$ kPa.

$\sigma_h = \frac{pr}{t}$

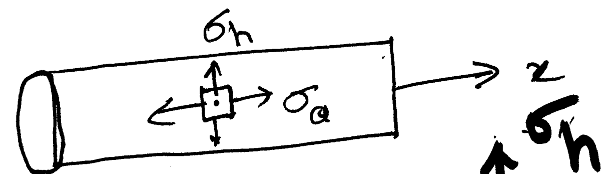
$\sigma_a = \frac{pr}{2t}$

(a) 300 MPa

(b) 150 MPa

(c) 75 MPa

(d) 37.5 MPa

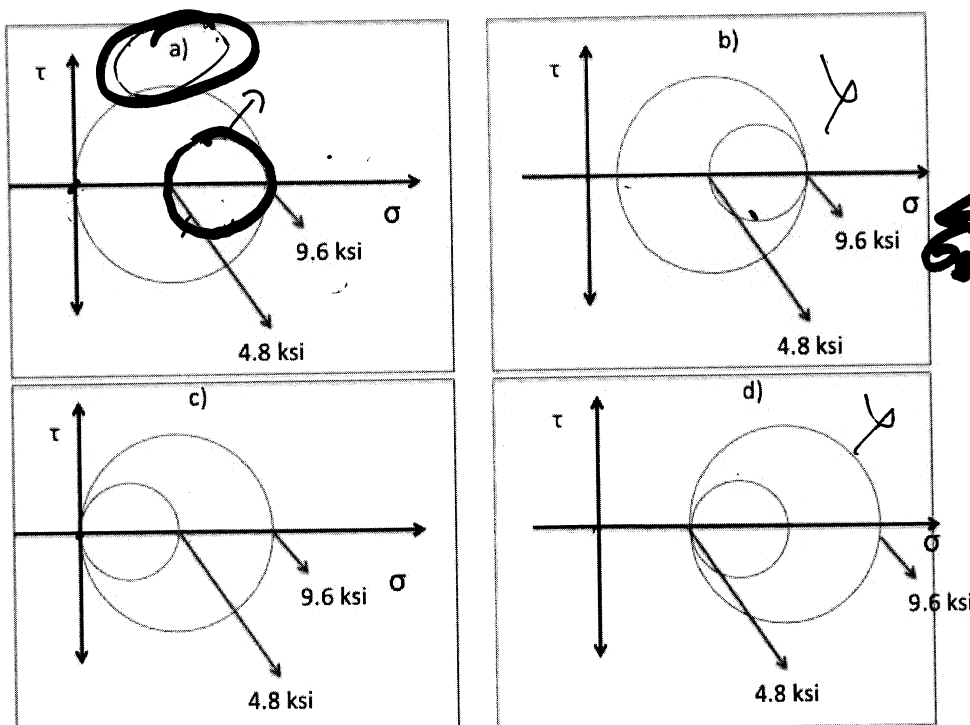


(2) A propane tank iun the shape of a cylindrical pressure vessel has a diameter D=12 in. and a wall thickness of t=1/8 in. The tank is pressurized to p=200 psi. Choose the Mohr's circle diagram that corresponds to the state of stress **(circle one)**.

$\frac{2400(10^3)1.25}{(10)(10^{-3})}$

$\sigma_h = 300$ MPa

$\sigma_a = 150$ MPa



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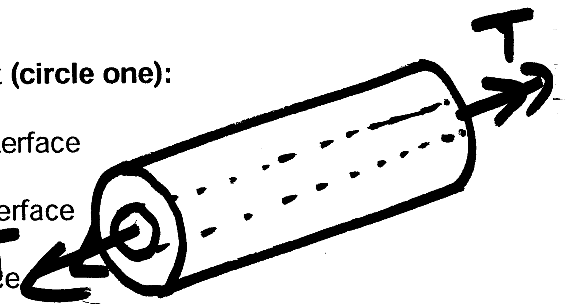
PROBLEM #4.3 (4 + 4 points, no partial credit)

A circular torsion bar consists of a core material (diameter D_{core}, shear modulus G_{core}) and a sleeve material (outer diameter D_{sleeve}, G_{sleeve}) bonded firmly together at the interface. An external torque T is applied to the composite shaft.



(1) In the analysis of this problem one needs to consider that **(circle one):**

(a) The stress distribution is possesses a jump at the interface

(b) The strain distribution is possesses a jump at the interface

(c) Both stress and strain possess a jump at the interface

(2) Which ones of the following statements are true regarding the maximum shear stress τ_{max} and its location in the bar? **(circle one)**

(a) The maximum shear stress τ_{max} always occurs in the sleeve at the outer surface of the sleeve.

(b) If the cross sectional area of the sleeve is greater than that of the core, then the maximum stress τ_{max} occurs on the outer surface of the sleeve; and if the cross sectional area of the core is greater than that of the sleeve then τ_{max} occurs on the outer surface of the core.

(c) If $G_{sleeve}D_{sleeve} > G_{core}D_{core}$ then τ_{max} occurs on the outer surface of the sleeve; and if $G_{sleeve}D_{sleeve} < G_{core}D_{core}$ then τ_{max} occurs on the outer surface of the core.

(d) If $G_{sleeve}J_{sleeve} > G_{core}J_{core}$ then τ_{max} occurs on the outer surface of the sleeve; and if $G_{sleeve}J_{sleeve} < G_{core}J_{core}$ then τ_{max} occurs on the outer surface of the core.

(e) The maximum shear stress τ_{max} always occurs at the outer surface of the core.

(f) None of the above.

$$\tau = G\rho \frac{d\phi}{dx} \rightarrow \quad \phi \rightarrow \text{twist}$$

$\left(\frac{d\phi}{dx}\right) \rightarrow$ same for core & sleeve

$$\left(\tau_{max}\right)_c = G_c \frac{d_c}{2} \frac{d\phi}{dx}$$

$$\left(\tau_{max}\right)_s = G_s \frac{d_s}{2} \left(\frac{d\phi}{dx}\right)$$

$$\frac{\left(\tau_{max}\right)_c}{\left(\tau_{max}\right)_s} = \frac{G_c d_c}{G_s d_s}$$