Equation Sheet

Formulas

Hooke's law:

$\varepsilon = (L_f - L_i)/L_i$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu(\varepsilon_y + \varepsilon_z)\right]$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_y + \nu(\varepsilon_z + \varepsilon_x)\right]$$

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_z + \nu(\varepsilon_x + \varepsilon_y)\right]$$

Stress transformation and Mohr's circle:

$$\sigma_{x'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta_{xx'} + \tau_{xy}\sin 2\theta_{xx'}$$

$$\sigma_{y'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta_{xx'} - \tau_{xy}\sin 2\theta_{xx'}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta_{xx'} + \tau_{xy}\cos 2\theta_{xx'}$$

$$\sigma_{p1} = \sigma_{avg} + R \qquad \sigma_{p2} = \sigma_{avg} - R$$

$$\tau_{max} = R$$

$$\sigma_{avg} = \left(\frac{\sigma_x + \sigma_y}{2}\right)$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

Axial deformation, thermal expansion:

$$e = \frac{FL}{EA} + L\alpha\Delta T$$

$$F = \frac{EA}{L}(e - L\alpha\Delta T)$$

$$F = K(e - L\alpha\Delta T)$$

$$e = u\cos(\theta) + v\sin(\theta)$$

Torsion:

$$\tau = Gr\frac{\phi}{L}$$

$$\tau = \frac{Tr}{I_p}$$

$$\phi = \frac{TL}{GI_p}$$

$$K = \frac{GI_p}{L}$$

$$T = K\phi$$

$$I_{p\_Circular\_Cross\_Section} = \frac{\pi d^4}{32}$$

$$I_{p\_Hollow\_Circular\_Cross\_Section} = \frac{\pi(d_o^4 - d_i^4)}{32}$$

Stress due to bending moment:

$$\sigma(x,y) = \frac{-E(x)y}{\rho(x)} = \frac{-M(x)y}{I_{zz}}$$

$$I_{zz} = \frac{bh^3}{12} \text{ (rectangle)} \quad I_{zz} = \pi\frac{d^4}{64} \text{ (circle)}$$

Stress due to shear force:

$$\tau(x,y) = \frac{V(x)Q(y)}{I_z b} \quad Q(y) = \int_{A'} \eta\, dA = A'\bar{y}'$$

$$\left(\tau_{max}\right)_{N.A.} = \frac{3V}{2A} \text{(rectangle)} \quad \left(\tau_{max}\right)_{N.A.} = \frac{4V}{3A} \text{(circle)}$$

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Failure criteria, factor of safety:

von Mises Equivalent Stress:

$$\sigma_M = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_{p_1}-\sigma_{p_2}\right)^2+\left(\sigma_{p_2}-\sigma_{p_3}\right)^2+\left(\sigma_{p_3}-\sigma_{p_1}\right)^2}$$

von Mises Stress (Plane Stress):

$$\sigma_M = \sqrt{\sigma_{p_1}^2 - \sigma_{p_1}\,\sigma_{p_2} + \sigma_{p_2}^2}$$

$$FS = \frac{Failure\ Stress}{Allowable\ Stress}, \frac{Yield\ Strength}{State\ of\ Stress}$$

Buckling:

Critical buckling load for a pinned-pinned beam

$$P_{cr} = EI\frac{\pi^2}{L^2}$$

Critical buckling for fixed-fixed beam

$$P_{cr} = EI\frac{\pi^2}{(0.5L)^2}$$

Stress in pressure vessels:

$$\sigma_{spherical} = \frac{pr}{2t};\quad \sigma_h = \frac{pr}{t};\quad \sigma_a = \frac{pr}{2t}$$

2nd order equation to solve for deflection curve

$$EI\frac{d^2v(x)}{dx^2} = M(x)$$

Energy methods:

Strain energy: $U_{tot} = \int \frac{M^2}{2EI}dx + \int \frac{F^2}{2AE}dx + \int \frac{T^2}{2GI_p}dx + \int \frac{f_s V^2}{2AG}dx$ where f_s is the shape factor

- Castigliano's theorem for deflection Δ at a point in the direction of the force $\Delta = \partial U_{tot}/\partial P$ where P is the point force
- Castigliano's theorem for slope θ_C or angle of twist ϕ_C in the direction of an applied moment M or applied torque T is given by $\theta_C = \partial U_{tot}/\partial M$ or $\phi_C = \partial U_{tot}/\partial T$
- Work-energy theorem states that the deflection Δ at the point of application of a force P in the direction of applied force can be calculated by equating $\frac{1}{2}P\Delta = U_{tot}$
- Work-energy theorem states that the slope θ_C or angle of twist ϕ_C in the direction of an applied moment M or applied torque T or can be calculated by equating $\frac{1}{2}M\theta_C = U_{tot}$ or $\frac{1}{2}T\phi_C = U_{tot}$