**Name (Print)**__________________________________________________________

**(Last) (First)**

**ME 323 - Mechanics of Materials**
**Exam # 2**
**Date: July 23, 2014 Time: 11:00 – 12:00 PM (in class)**
**Location: ME 1130**
**Instructor: Nasir Bilal**

<u>Instructions:</u>

Begin each problem in the space provided on the examination sheets. If additional space is required, use the extra paper provided.

Work on one side of each sheet only, with only one problem on a sheet.

Please remember that for you to obtain maximum credit for a problem, you must present your solution clearly.

Accordingly,

- coordinate systems must be clearly identified,
- free body diagrams must be shown,
- units must be stated,
- write down clarifying remarks,
- state your assumptions, etc.

If your solution cannot be followed, it will be assumed that it is in error.

When handing in the test, make sure that ALL SHEETS are in the correct sequential order.

Remove the staple and restaple, if necessary.

Prob. 1 ______________________

Prob. 2 ______________________

Prob. 3 ______________________

Total ________________________

## Useful Equations

$$\sigma_{avge} = \frac{F_N}{A}, \ \tau_{avge} = \frac{V}{A}$$

$$FS = \frac{F_{fail}}{F_{allow,member}}, FS = \frac{\sigma_{fail}}{\sigma_{allow,member}}, \ FS = \frac{\tau_{fail}}{\tau_{allow,member}}$$

$$\varepsilon_{avge} = \frac{\Delta s' - \Delta s}{\Delta s} = \frac{\delta}{L_0}, \ \gamma = \lim_{\substack{\text{infinitesimally} \\ \text{small element}}} \left[\tfrac{\pi}{2} - \theta'\right]$$

$$\sigma = E\varepsilon, \ \nu = -\varepsilon_{lat}/\varepsilon_{long}, \ \tau = G\gamma, \ G = E/\left[2(1+\nu)\right]$$

$$u_B = u_A + \delta_{AB}, \ \delta_{AB} = \frac{P_{AB}L}{AE}, \ \delta_{AB} = \int_0^L \frac{P_{AB}(x)}{A(x)E(x)}dx, \ \delta_{thermal} = \alpha(\Delta T)L, \ \delta_{thermal} = \int_0^L \alpha(\Delta T)dx$$

$$\rho\phi = \gamma x, \ \gamma = \gamma_{\max}(\rho/c), \ \tau = \tau_{\max}(\rho/c), \ \tau = T\rho/J, \ \tau_{\max} = Tc/J$$

$$\theta_B = \theta_A + \phi_{AB}, \ \phi_{AB} = \frac{T_{AB}L}{GJ}, \ \phi_{AB} = \int_0^L \frac{T_{AB}(x)}{G(x)J(x)}dx, \ J = \int_A \rho^2 dA = \begin{cases} \tfrac{\pi}{2}c^4 \ \text{(solid)} \\ \tfrac{\pi}{2}c_{out}{}^4 - \tfrac{\pi}{2}c_{in}{}^4 \ \text{(hollow)} \end{cases}$$

$$\frac{dV}{dx} = w(x), \ \ \frac{dM}{dx} = V(x), \ \Delta V = P, \ \Delta M = -M_0$$

$$\sigma(x,y) = \frac{-Ey}{\rho} = \frac{-M_{zz}y}{I_{zz}}, \ I_{zz} = \frac{bh^3}{12} \ \text{(rectangle)}, \ I = \frac{\pi r^4}{4} \ \text{(circle)}$$

$$\tau = \frac{VQ}{I_{zz}t}, \ q = \frac{VQ}{I}, \ \tau_{\max} = \frac{3V}{2A} \ \text{(rectangle)}, \ \tau_{\max} = \frac{4V}{3A} \ \text{(circle)}$$

$$\sigma_{X'} = \left[\frac{\sigma_X + \sigma_Y}{2}\right] + \left[\frac{\sigma_X - \sigma_Y}{2}\right]\cos 2\theta + \left[\tau_{XY}\right]\sin 2\theta; \qquad \sigma_{Y'} = \left[\frac{\sigma_X + \sigma_Y}{2}\right] - \left[\frac{\sigma_X - \sigma_Y}{2}\right]\cos 2\theta - \left[\tau_{XY}\right]\sin 2\theta$$

$$\tau_{X'Y'} = -\left[\frac{\sigma_X - \sigma_Y}{2}\right]\sin 2\theta + \left[\tau_{XY}\right]\cos 2\theta; \quad \sigma_{1,2} = \left[\frac{\sigma_X + \sigma_Y}{2}\right] + \sqrt{\left[\frac{\sigma_X - \sigma_Y}{2}\right]^2 + \tau_{XY}{}^2}; \quad \tan 2\theta_p = \frac{\tau_{XY}}{\frac{\sigma_X - \sigma_Y}{2}}$$

$$e = \frac{FL}{EA} + L\alpha\Delta T$$

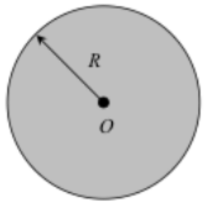
$$e = u\cos\theta + v\sin\theta$$

$$\sigma_1 = \sigma_{avg} + R, \ \sigma_2 = \sigma_{avg} - R, \ \ \tau_{\max} = R, \ \ \sigma_{avg} = \left(\frac{\sigma_x + \sigma_y}{2}\right), \ \ R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2{}_{xy}}$$

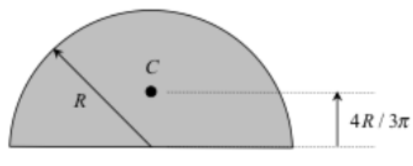
$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \quad \varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \quad \varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \ \ \gamma_{xz} = \frac{1}{G}\tau_{xz} \ \ \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$Factor\ of\ Safety = \frac{Yield\ Strength}{Allowable\ Stress}$$




$I_O = \frac{1}{4}\pi R^4$



**ME 323 Examination # 2** **Name** ___________________________________

**(Print) (Last) (First)**

**July 23, 2014**

**PROBLEM #1 (30 points)**

A load $P = 8\ kips$ is applied on the beam $ABC$ at point $B$ as shown in Figure *1(a)*. The cross section of the beam is shown in Figure *1(b)*.

a) Draw the shear force and bending moment diagram for the given beam.
b) Determine the maximum normal stress along the entire length of the beam.
c) Determine the maximum shear stress along the entire length of the beam.

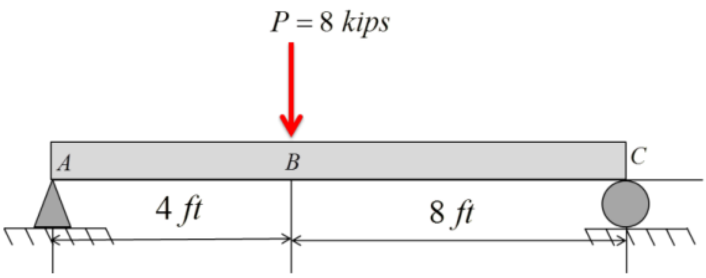



Figure 1(a)

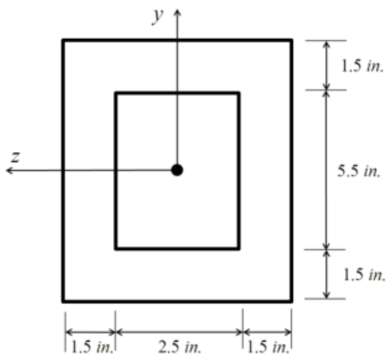
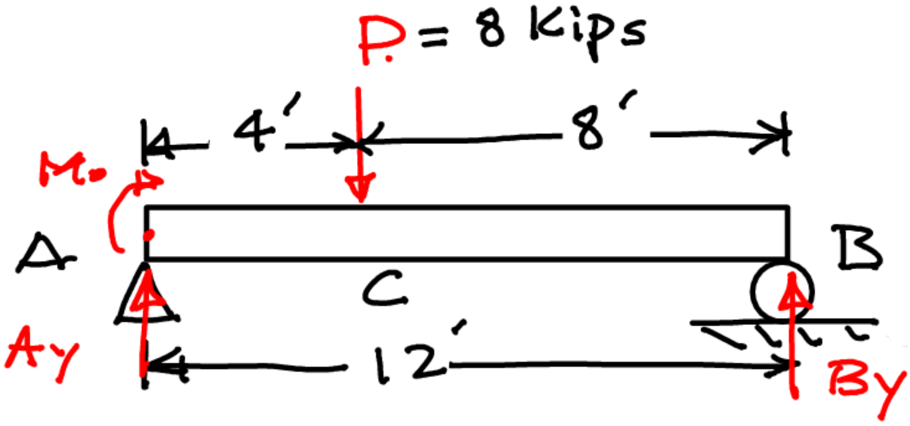
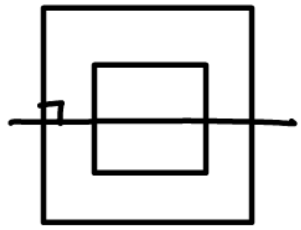



Figure 1(b)



$\circlearrowleft + \sum M_B = 0$

$$-A_y(12) + 8(8) = 0$$

$$\boxed{A_y = \frac{64}{12} = 5.33 \text{ kips}}$$

$+\uparrow \sum F_y = 0$

$$A_y + B_y - 8 = 0$$

$$B_y = 8 - A_y = 8 - 5.33$$

$$\boxed{B_y = 2.67} \text{ kips}$$

The beam has two section.

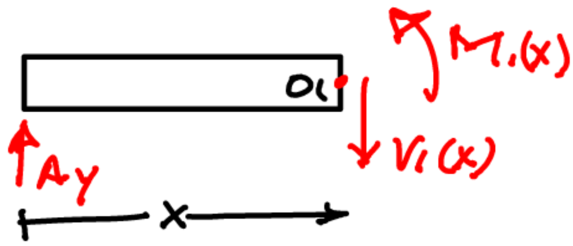
section AC. $0 \leq x \leq 4$

$\uparrow \sum F_y = 0$

$\Rightarrow$

$$A_y - V_1(x) = 0$$

$$\boxed{V_1(x) = 5.33 \text{ kips}}$$




$\circlearrowleft + \sum M_{o1} = 0$

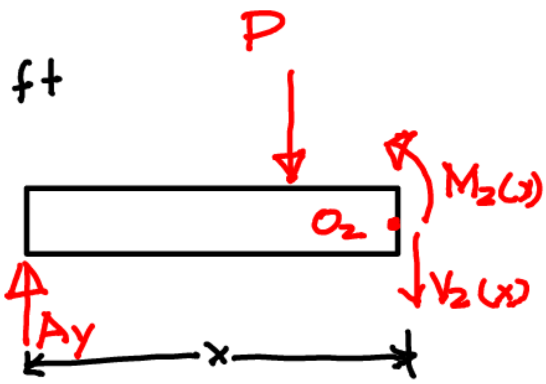
$$-A_y x + M_1(x) = 0$$

$$\Rightarrow M_1(x) = A_y x$$

$$\boxed{M_1(x) = 5.33x} \text{ kip-ft}$$

Section CB. $4 \leq x \leq 12$

$+\uparrow \sum F_y = 0 \quad A_y - P - V_2(x) = 0$



$$\Rightarrow V_2(x) = A_y - P = 5.33 - 8$$

$$\boxed{V_2(x) = 2.67 \text{ kips}}$$

$$\circlearrowleft + \sum M_{o2}(x) = 0$$

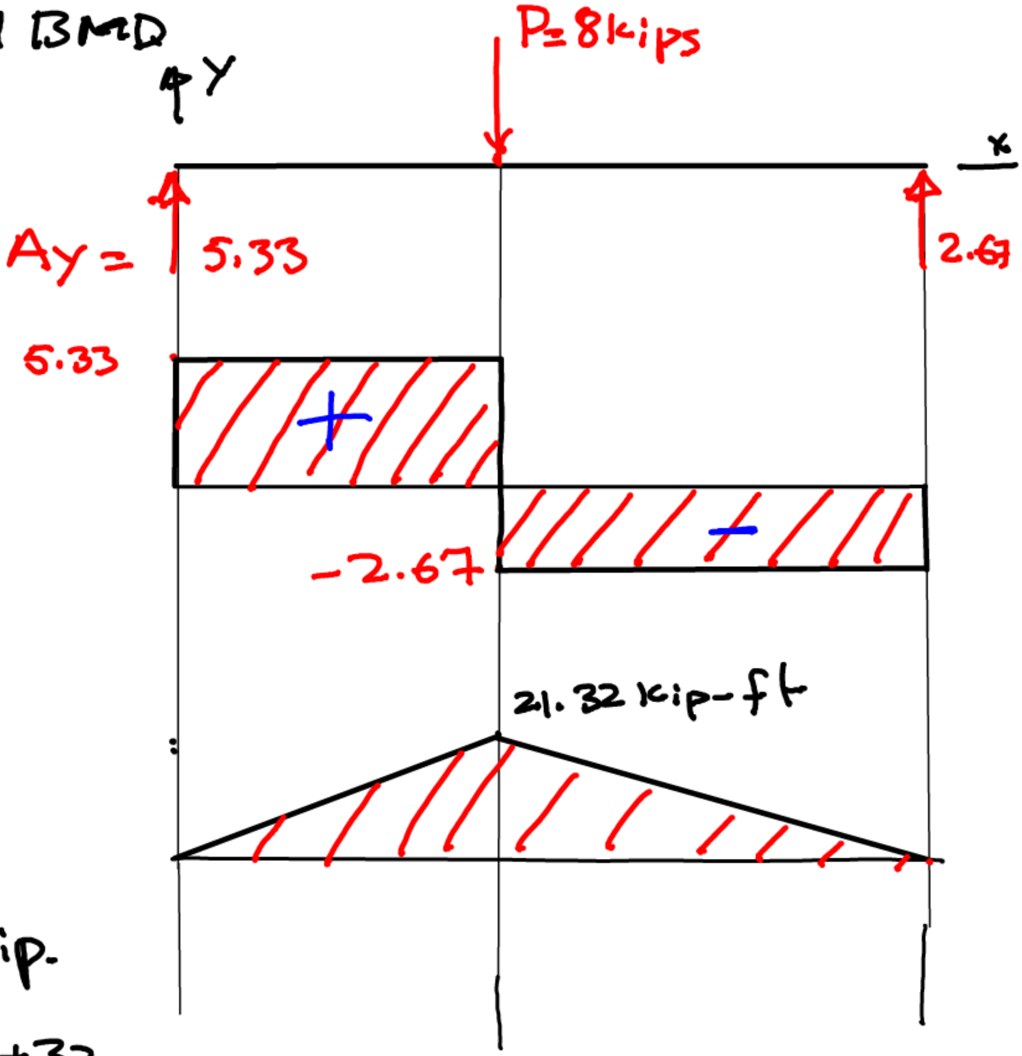
$$-A_y(x) + P(x-4) + M_2(x) = 0$$

$$M_2(x) = +5.33x - 8(x-4)$$

$$= 5.33x - 8x + 32.$$

$$\boxed{M_2(x) = -2.67x + 32}$$

Draw the SF and BMD




$M_1(x) = 5.33x$

@ $x = 0$   0

@ $x = 4$

$M_1(x) = 21.32$ kip.

$M_2(x) = -2.67x + 32$

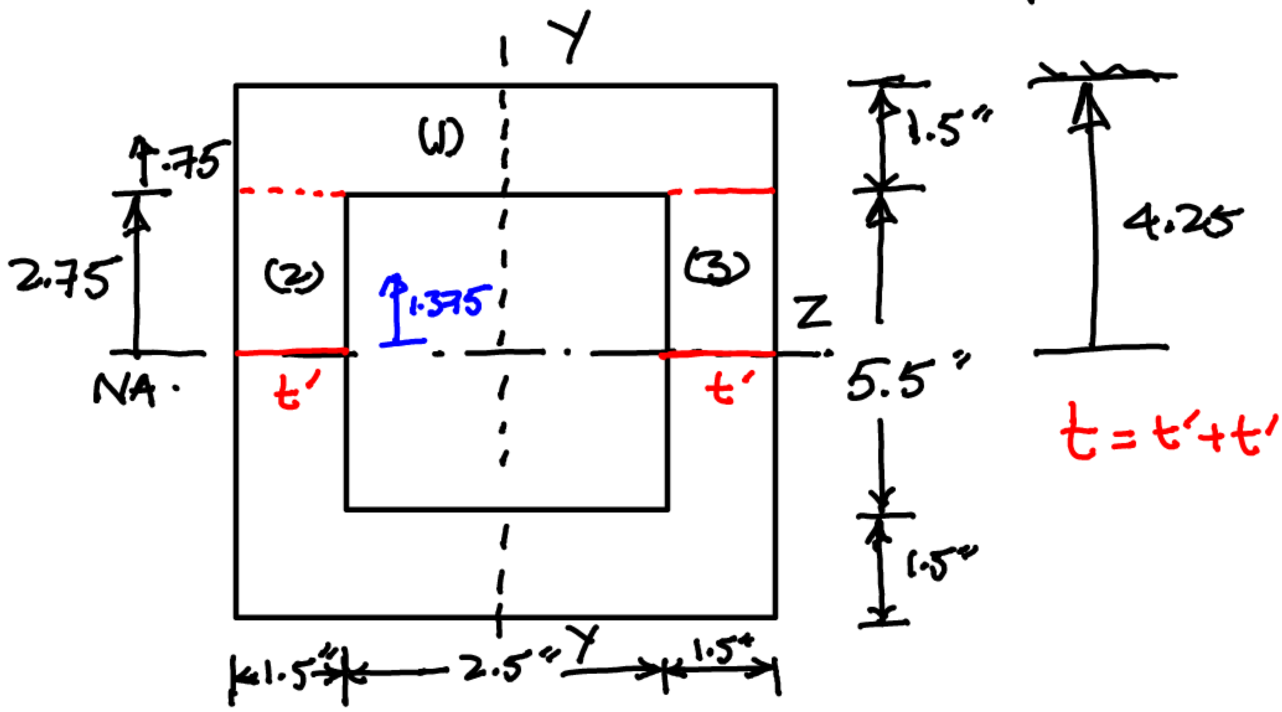
@ $x=4$; $= 21.32$ kip-ft

@ $x = 12 =$   0

$\therefore$

| |
|---|
| Max shear force = 5.33 kips |
| Max bending moment = 21.32 kip-ft |

Now look at the two cross-sections of the beam.




Since the cross-section is symmetric, the neutral axis is at the middle.

$$I_{zz} = \left(\frac{1}{12} bh^3\right)_{Out} - \left(\frac{1}{12} bh^3\right)_{Inner}$$

$$= \frac{1}{12}\left(5.5(8.5)^3 - \frac{1}{12}(2.5)(5.5)^3\right]$$

$$\boxed{I_{zz} = 246.813 \text{ in}^4}$$

The max bending stress.

$$\sigma_x = -\frac{MY}{I} = \frac{(-21.32\times10^3\times 12)\times 4.25}{246.8125}$$

$$\sigma_x = \pm 4405.45 \quad \text{psi}$$

$$\boxed{\sigma_x = \pm 4405 \quad \text{Ksi}}$$

To find max shear stress we have

$$\boxed{\tau = \frac{VQ}{It}}$$

max shear is 5.33 kips.

$I = 358.74 \text{ in}^4$

& $Q = A'\bar{y}' = A_1'\bar{y}_1' + A_2'\bar{y}_2' + A_3'\bar{y}_3'$

$= (5.5 \times 1.5)(3.5) + 2(1.5 \times 2.75)(1.375)$

$Q = 40.21 \text{ in}^3$

$t = (1.5 \times 2) = 3''$

$V = 5.33 \text{ kips} = 5.33 \times 10^3$

$\tau = \dfrac{(5.33 \times 10^3)(40.21)}{246.8125\,(3)}$

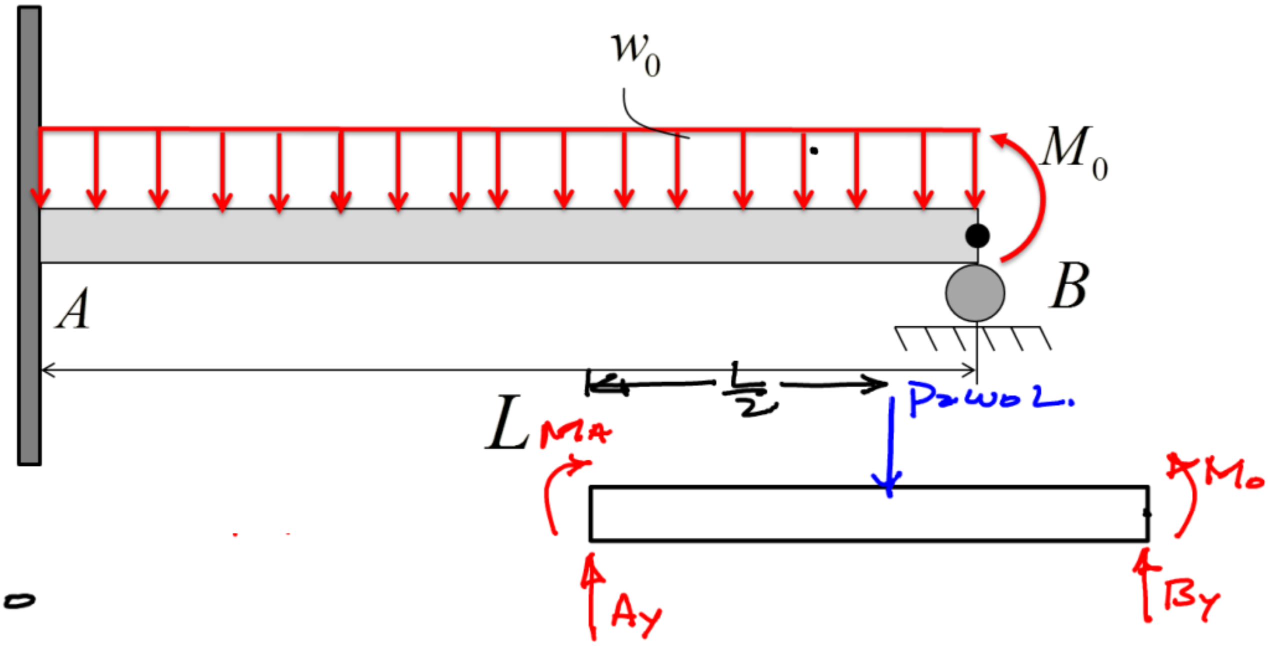
$\boxed{\tau = 289.5 \text{ psi}}$

**ME 323 Examination # 2** Name ______________________________
(Print) (Last) (First)

**July 23, 2014**

**PROBLEM #2 (35 points)**

Beam $AB$ carries a uniformly distributed load of intensity $\omega_0$ as shown in the Figure. A concentrated moment $M_0$ is applied at end B. Determine the support reaction at end B.




$\circlearrowleft + \sum M_B = 0$

$$-A_y(L) - M_A + w_0L\left(\frac{L}{2}\right) + M_0 = 0$$

$$\boxed{A_y L + M_A = \frac{w_0 L^2}{2} + M_0} \longrightarrow (1)$$

$\uparrow \sum F_y = 0$

$$A_y + B_y - w_0 L = 0$$

$$\boxed{A_y + B_y = w_0 L} \longrightarrow (2)$$

$$p(x) = -w_0$$

$$\boxed{EI\, v'''' = -w_0} \longrightarrow (3)$$

integrate 4 times and apply B.C.

$$EI\, v''' = -w_0 x + C_1.$$

$$@\, x = 0 \quad v''' = V = A_y$$

**ME 323 Examination # 2** Name ______________________________
(Print) (Last) (First)

$$\boxed{EI\,v''' = -w_0 x + A_y} \longrightarrow (4)$$

integrate again.

$$EI\,v'' = -\frac{w_0 x^2}{2} + A_y x + C_2.$$

@ $x=0$, $v'' = M = M_B$.

$$\therefore \boxed{EI\,v'' = -\frac{w_0 x^2}{2} + A_y x + M_A} \longrightarrow (5)$$

integrate.

$$EI\,v'(x) = \frac{-w_0 x^3}{6} + \frac{A_y x^2}{2} + M_A x + C_3.$$

@ $x = 0$, $v'(0) = 0$

$$EI\,v(0) = 0 = 0 + 0 + 0 + C_3.$$

$$\Rightarrow \boxed{C_3 = 0}$$

$$\boxed{EI\,v'(x) = \frac{-w_0 x^3}{6} + \frac{A_y}{2} x^2 + M_A x} \longrightarrow (6)$$

$$EI\,v(x) = \frac{-w_0 x^4}{24} + \frac{A_y x^3}{6} + \frac{M_A x^2}{2} + C_4$$

@ $x=0$ $v(0)=0 \Rightarrow C_4 = 0$

$$\therefore \boxed{EI\,v(x) = -\frac{w_0 x^4}{24} + \frac{A_y x^3}{6} + \frac{M_A x^2}{2}} \longrightarrow (7)$$

apply the BC. @ $x = L$, $v(L) = 0$

$$0 = \frac{-w_0 L^4}{24} + \frac{A_y L^3}{6} + \frac{M_A L^2}{2} = 0$$

$$\Rightarrow \boxed{M_A = \frac{w_0 L^2}{12} - \frac{A_y L}{3}}$$

sub $M_A$ in (1)

$$\boxed{A_y L + M_A = \frac{w_0 L^2}{2} + M_0} \longrightarrow (1)$$

$$A_y L + \frac{w_0 L^2}{12} - \frac{A_y L}{3} = \frac{w_0 L^2}{2} + M_0$$

$$\Rightarrow A_y \frac{2}{3} L = \frac{w_0 L^2}{2} - \frac{w_0 L^2}{12} + M_0$$

$$\boxed{A_y = \frac{5}{8} w_0 L + \frac{3}{2} \frac{M_0}{L}.}$$

Sub in 2

$$A_y + B_y = w_0 L.$$

$$B_y = w_0 L - \frac{5}{8} w_0 L - \frac{3}{2} \frac{M_0}{L}.$$

$$\boxed{B_y = \frac{3}{8} w_0 L - \frac{3}{2} \frac{M_0}{L}.}$$

**ME 323 Examination # 2** **Name** ___________________________________

**(Print) (Last) (First)**

**July 23, 2014**

**PROBLEM #3 (35 points)**

The state of stress at a point is shown in Figure *3(a)*. The figure *3(b)* shows an inclined plane *aa* cut through the same point in the material but at an orientation angle $\theta$.

a) Determine the value of the angle $\theta$, between zero and 90° such that no normal stress acts on plane *aa*.
b) Sketch a stress element having plane *aa* as one of its sides and show all stresses acting on the element.

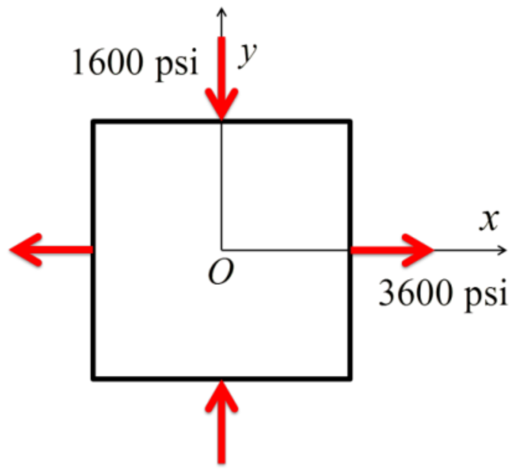



Figure 3(a)

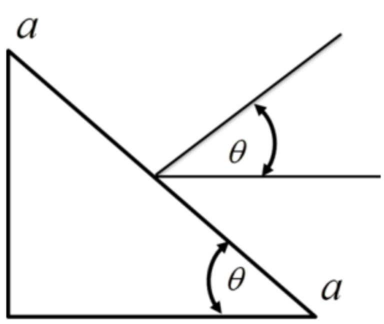



Figure 3(b)

| 19 July, 2014 | ME323 Exam-II |
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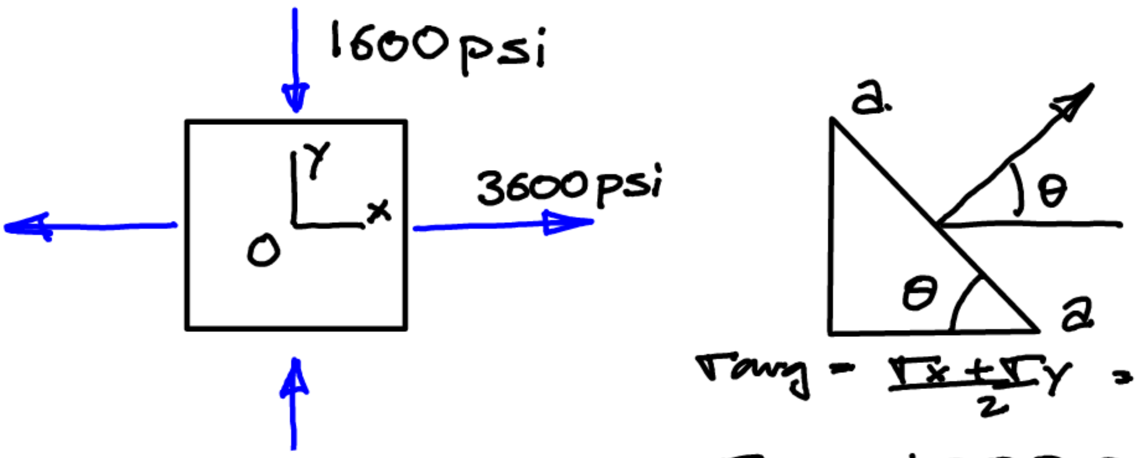



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{3600-1600}{2}$$

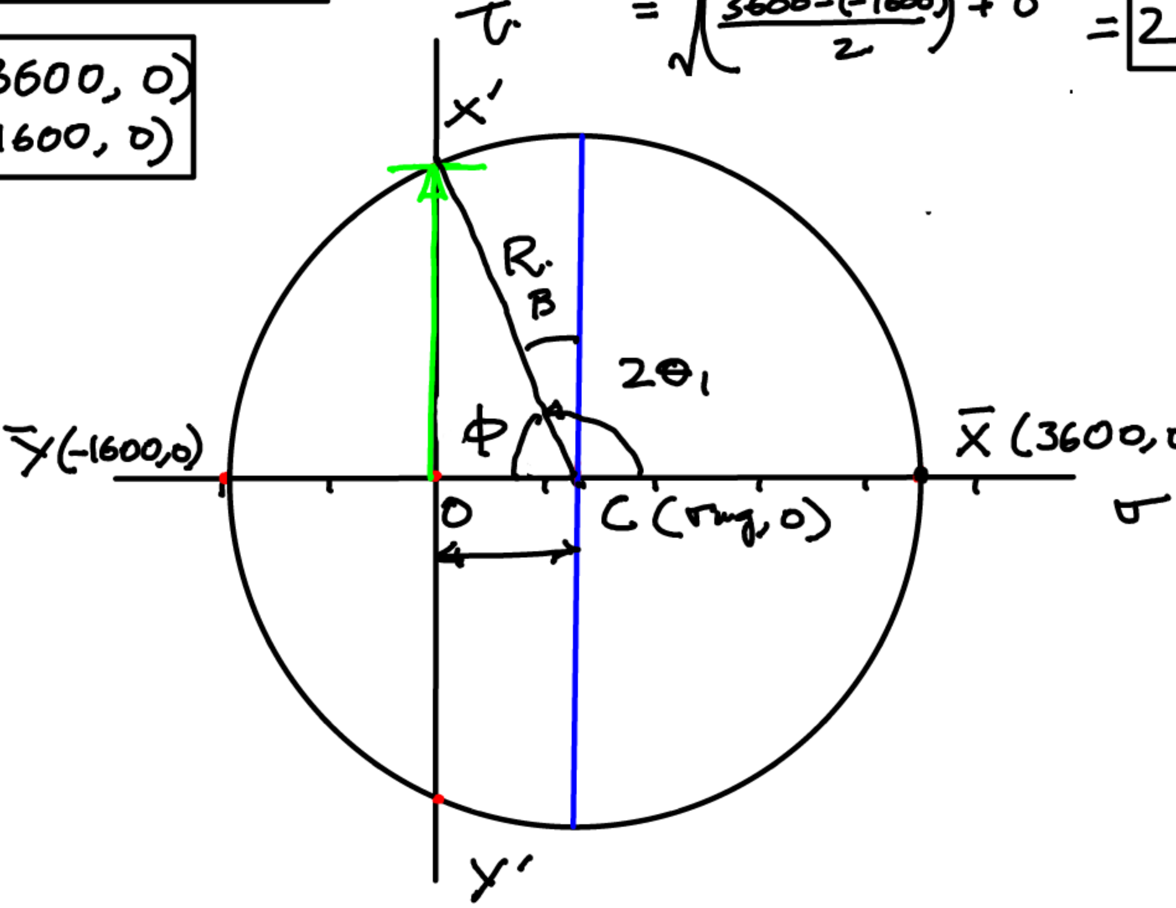
$$\sigma_{avg} = 1000 \text{ psi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{3600-(-1600)}{2}\right)^2 + 0^2} = \boxed{2600 = R}$$

$$\boxed{\begin{array}{l} \sigma_x = 3600 \text{ psi} \\ \sigma_y = -1600 \text{ psi} \\ \tau_{xy} = 0 \end{array}}$$

$$\boxed{\begin{array}{l} \bar{X} = (3600, 0) \\ \bar{Y} = (-1600, 0) \end{array}}$$




The value of shear stress is $\overline{OX'}$.

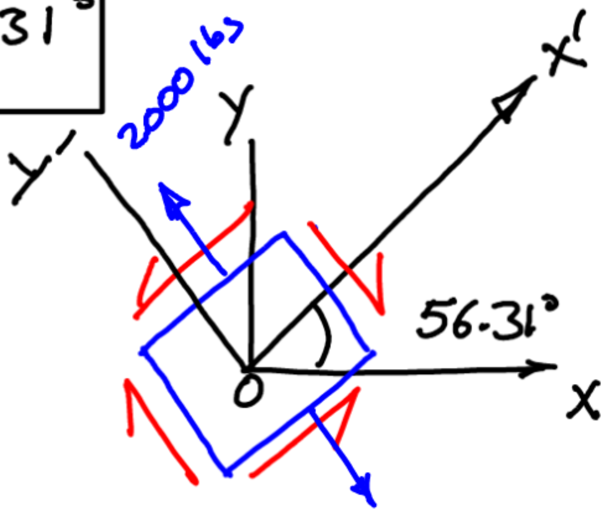
$$\cos(\phi_1) = \frac{OC}{R.} = \frac{1000}{2600}$$

$$\phi_1 = \cos^{-1}\left(\frac{1000}{2600}\right) \Rightarrow \phi_1 = 67.38°$$

$$\sigma x' = R \sin(\Phi_1)$$
$$= 2600 \sin(67.38)^\circ$$
$$= 2399.99$$
$$\boxed{\sigma x' = 2400 \text{ psi}}$$

The plane with no normal stress is located at.

$$180 - 67.38^\circ$$
$$2\theta = 112.62^\circ$$
$$\Rightarrow \boxed{\theta = 56.31^\circ}$$




Check:

Use the given angle in stress transformation equation to find the value of stress @ $\theta = 56.31^\circ$ as shown above.

$$\sigma n = \left(\frac{\sigma x + \sigma y}{2}\right) + \left(\frac{\sigma x - \sigma y}{2}\right) \cos 2\theta + \tau xy \sin 2\theta.$$
$$= \left(\frac{3600 - 1600}{2}\right) + \left(\frac{3600 - (-1600)}{2}\right) \cos(2.56.31)^\circ + 0$$
$$\boxed{\sigma n = -0.0056574.}$$
$$\tau nt = -\left(\frac{\sigma x - \sigma y}{2}\right) \sin 2\theta + \tau xy \cos 2\theta.$$
$$\tau nt = -\left(\frac{3600 - (-1600)}{2}\right) \sin(2.56.31) + 0$$
$$\boxed{\tau nt = -2399.99 \simeq -2400 \text{ psi}}$$