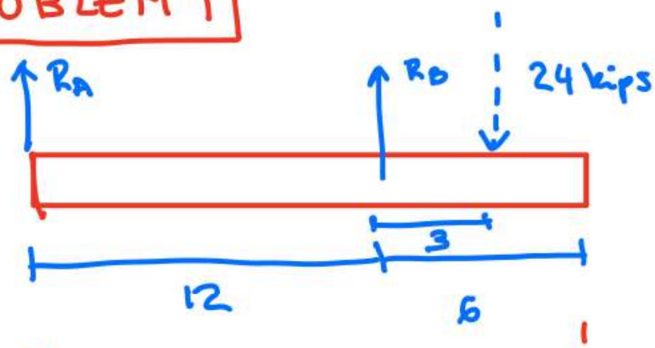
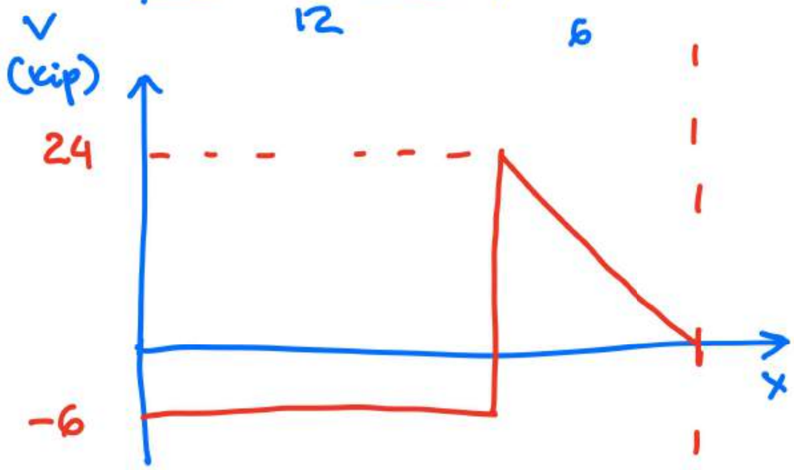
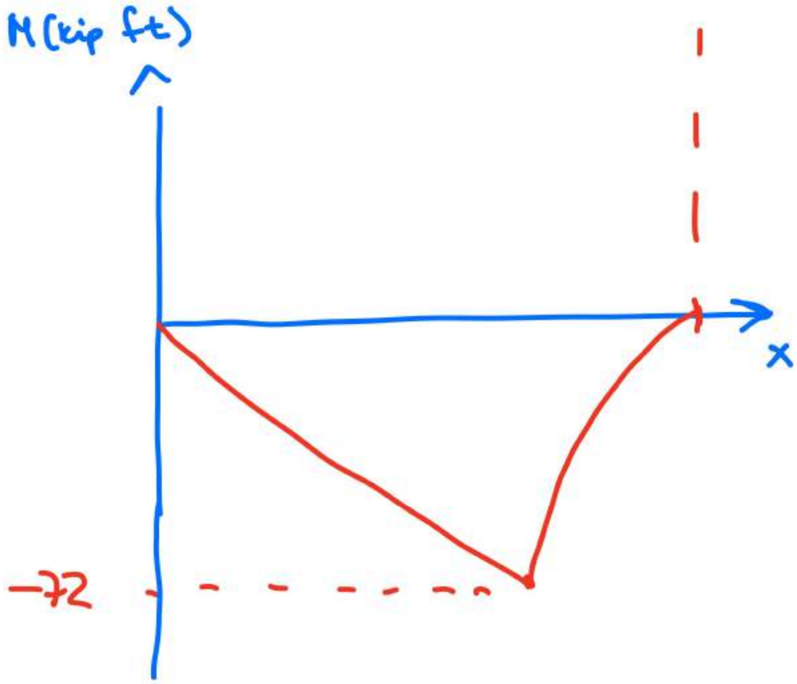
PROBLEM 1







$$\Sigma F_y = 0$$

$$R_A + R_B = 24$$

$$\Sigma M|_A = 0$$

$$R_B \cdot 12 - 24 \; 15 = 0$$

$$R_B = \frac{24 \cdot 15}{12} = 30$$

$$R_A = 24 - R_B$$

$$R_A = -6$$

$$\tau = \frac{V\,Q}{I t}$$

V_{max} at $x = 12$

$$V_{max} = 24 \text{ kip}$$

$$\sigma_x = \frac{-M y}{I}$$

$$|M_{max}| = 72 \text{ kip.ft}$$

σ_{MAX} at $x = 12$ $\quad y = \frac{h}{2}$

$$I = \frac{b\,h^3}{12} = \frac{0.25\,\text{ft} \cdot h^3}{12}$$

$$\left| \sigma\,(x=12,\ y=h/2) \right| = \sigma_{MAX} = \frac{72\ \text{kip}\cdot\cancel{ft}\ 12\cdot\cancel{h}}{0.25\ \cancel{ft}\ h^{\cancel{3}2}}\cdot\frac{\cancel{h}}{2}$$

$$\sigma_{MAX} = \frac{1728\ \text{kip}}{h^2}$$

$$\sigma_{MAX} < \sigma_{allow} = 21\ \text{ksi}$$

$$\frac{1728}{h^2} < 21 \quad \rightarrow \quad h > \sqrt{\frac{1728}{21}}$$

$$\boxed{h > 9.07\ \text{in}}$$

$$\tau^{max}\,(x=12,\ y=0) = \frac{V^{max}\cdot\cancel{3}\cdot\frac{h^2}{8}}{\frac{\cancel{3}\cdot h^3}{\cancel{12}\,4}\cdot\cancel{3}}$$

$$Q = \bar{y}\cdot A = \frac{h}{4}\cdot\frac{h}{2}\cdot 3$$

$$= \frac{24}{{}_2\cancel{8}h}\cdot\cancel{4} = \frac{12}{h}$$

$$\tau^{max} \leq 1\ \text{ksi}$$

$$\frac{12}{h} \leq 1 \quad \rightarrow \quad \boxed{h \geq 12\ \text{in}}$$

ME 323 – Mechanics of Materials
Midterm #2
March 30th, 2022

PURDUE UNIVERSITY | School of Mechanical Engineering

Name (Print) SOLUTION
(Last) **(First)**

PROBLEM # 2 (25 points)

The beam BCD is fixed to the wall at B and supported by a roller at C. A concentrated moment M_o is applied at D. The beam has Young's modulus $\boldsymbol{E}$ and second moment of area $\boldsymbol{I}$.

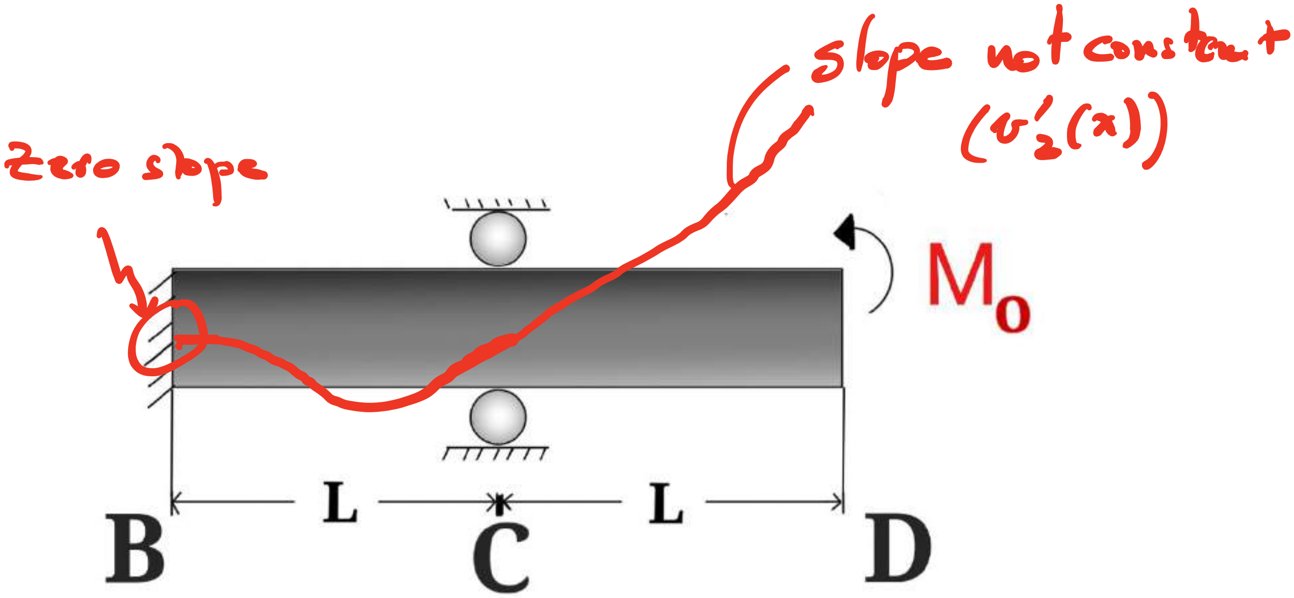
a) Draw a free-body diagram of the entire beam, and write down the equilibrium equations.

b) Use the second-order integration method to find the slope v'(x) and deflection v(x) of each segment of the beam. These can be left in terms of the unknown support reactions.

c) Write down the relevant boundary conditions and continuity conditions for the beam.

d) Use the boundary/continuity conditions to determine the reactions at B and C in terms of $\boldsymbol{M_o}$ ***and L.***

e) Determine the deflection of the free end D. Sketch the deflection curve over the length of the beam. The sketch does not need to be exact. Show enough detail to clearly indicate the boundary conditions.



- 3 solutions are provided.
- The FBD & equilibrium eqn's for the entire structure are the same
- The Boundary & Continuity conditions are the same for all 3 soln's
- The sketch of the deflection curve is the same.

ME 323 – Mechanics of Materials
Midterm #2
March 30th, 2022

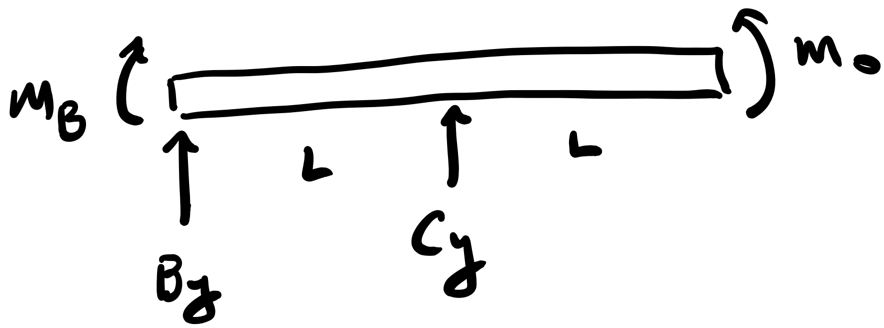
PURDUE UNIVERSITY | School of Mechanical Engineering

Name (Print) ______________________________

(Last) (First)

PROBLEM # 2 CONT.

FBD of entire beam:



$$\Sigma M_B = -M_B + C_y L + M_0 = 0$$

$$\boxed{M_B = M_0 + C_y L} \quad (1)$$

$$\Sigma F_y = B_y + C_y = 0 \qquad \boxed{B_y = -C_y} \quad (2)$$

B.C's:

$$v(0) = v_1(0) = 0$$

$$v'(0) = v_1'(0) = \theta_1(0) = 0$$

$$v(L) = v_1(L) = v_2(L) = 0$$

$$v_1'(L) = v_2'(L)$$

ME 323 – Mechanics of Materials
Midterm #2
March 30th, 2022

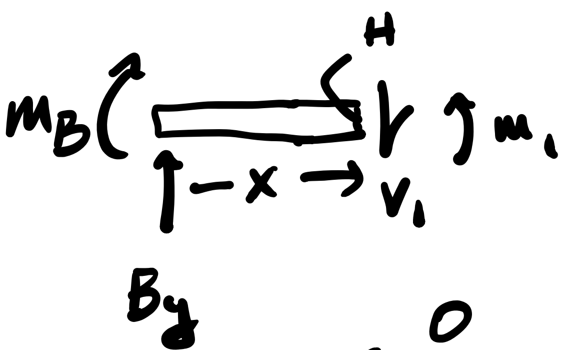
PURDUE UNIVERSITY | School of Mechanical Engineering

Name (Print) SOLUTION 1
(Last) (First)

PROBLEM # 2 CONT.

Solution 1 – Use definite integrals

Section BC $0 < x < L$



$$\Sigma M_H = M_1 - M_B - B_y x = 0$$

$$\boxed{M_1(x) = M_B + B_y x} = EI \frac{d\theta}{dx}$$

$$\theta_1(x) = \cancelto{0}{\theta_1(0)} + \frac{1}{EI}\int_0^x (M_B + B_y x)\,dx$$

$$= \frac{1}{EI}\left[M_B x + B_y \frac{x^2}{2}\right] = \frac{dv_1}{dx}$$

$$v_1(x) = \cancelto{0}{v_1(0)} + \frac{1}{EI}\int_0^x \left(M_B x + B_y \frac{x^2}{2}\right)dx$$

$$= \frac{1}{EI}\left[M_B \frac{x^2}{2} + B_y \frac{x^3}{6}\right]$$

$$\theta_1(L) = \frac{1}{EI}\left[M_B L + B_y \frac{L^2}{2}\right] = \theta_C \qquad (3)$$

$$v_1(L) = \cancel{\frac{1}{EI}}\left[M_B \frac{L^2}{2} + B_y \frac{L^3}{6}\right] = 0$$

ME 323 – Mechanics of Materials
Midterm #2
March 30th, 2022

School of Mechanical Engineering

Name (Print) ______________________________________

(Last) (First)

PROBLEM # 2 CONT.

$$M_B = -B_y \frac{L}{3}$$

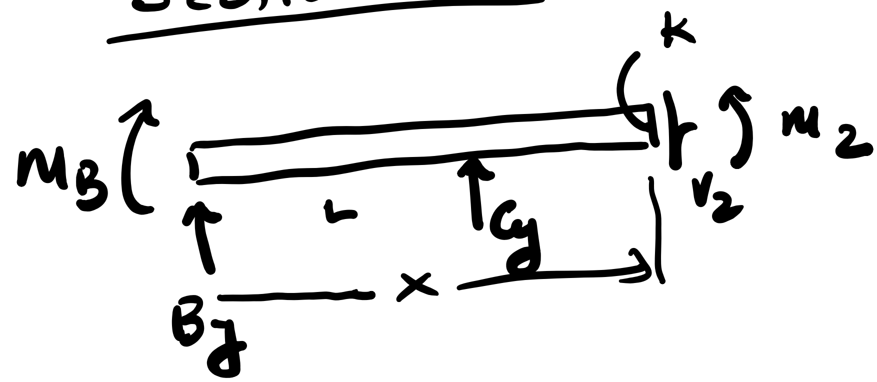
Use (1) & (2)

$$M_0 + C_y L = C_y \frac{L}{3} \qquad \boxed{C_y = -\frac{3M_0}{2L}}$$

$$B_y = -C_y \qquad \boxed{B_y = \frac{3M_0}{2L}}$$

$$M_B = M_0 - \frac{3M_0}{2L} L = \boxed{-\frac{M_0}{2} = M_B}$$

Section CD $\quad L < x < 2L$



$$\Sigma M_K = M_2 - M_B - B_y x - C_y (x - L) = 0$$

$$\boxed{M_2(x) = M_B + B_y x + C_y (x - L)} = EI \frac{d\theta_2}{dx}$$

ME 323 – Mechanics of Materials
Midterm #2
March 30th, 2022

PURDUE UNIVERSITY | School of Mechanical Engineering

Name (Print) ____

(Last) (First)

PROBLEM # 2 CONT.

$$\theta_2(x) = \underbrace{\theta_2(L)}_{\theta_C} + \frac{1}{EI}\int_L^x [M_B + B_y x + C_y x - C_y L)\,dx$$

$$\theta_2(x) = \theta_C + \frac{1}{EI}\Big[M_B(x-L) + \frac{B_y}{2}(x^2-L^2)$$

$$+ \frac{C_y}{2}(x^2-L^2) - C_y L(x-L)\Big] = \frac{dv_2}{dx}$$

$$v_2(x) = \cancel{v_2}^{\,0}(L) + \int_L^x \Big[\theta_C + \frac{1}{EI}\Big[M_B x - M_B L$$

$$+ B_y\frac{x^2}{2} - \frac{B_y L^2}{2} + C_y\frac{x^2}{2} - \frac{C_y L^2}{2} - C_y L x + C_y L^2\Big] dx$$

$$v_2(x) = \theta_C(x-L) + \frac{1}{EI}\Big[\frac{M_B}{2}(x^2-L^2) - M_B L(x-L)$$

$$+ \frac{B_y}{6}(x^3-L^3) - \frac{B_y L^2}{2}(x-L) + \frac{C_y}{6}(x^3-L^3)$$

$$- \frac{C_y L^2}{2}(x-L) - \frac{C_y L}{2}(x^2-L^2) + C_y L^2(x-L) \quad (4)$$

where θ_C is given by Equ (3).

ME 323 – Mechanics of Materials
Midterm #2
March 30th, 2022

PURDUE UNIVERSITY | School of Mechanical Engineering

Name (Print) ______________________________________

(Last) (First)

PROBLEM # 2 CONT.

In equ(4) replace $x = 2L$, θ_C (eqn 3),

M_B, B_y & C_y

$$\boxed{v_2(2L) = \frac{3}{4}\frac{M_0 L^2}{EI}}$$

ME 323 – Mechanics of Materials
Midterm #2
March 30th, 2022

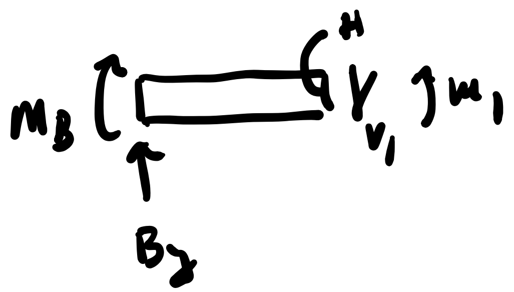
Name (Print) SOLUTION 2

(Last) (First)

PROBLEM # 2 CONT.

Solution 2 - Use indefinite integrals

Section BC $0 < x < L$



$$\Sigma M_H = M_1 - M_B - B_y x = 0$$

$$M_1(x) = M_B + B_y x = EI v_1''$$

$$EI v_1' = M_B x + B_y \frac{x^2}{2} + C_1$$

$$EI v_1 = M_B \frac{x^2}{2} + B_y \frac{x^3}{6} + C_1 x + C_2$$

B.C's at $x = 0$

$$v_1'(0) = C_1 = 0$$

$$v_1(0) = C_2 = 0$$

$$v_1'(L) = M_B L + B_y \frac{L^2}{2}$$

$$v_1(L) = M_B \frac{L^2}{2} + B_y \frac{L^3}{6} = 0$$

Write M_B & B_y in terms of C_y (Eqn's 1 & 2)

$$\boxed{C_y = -\frac{3M_0}{2L} = -B_y} \quad \boxed{M_B = -\frac{M_0}{2}}$$

ME 323 – Mechanics of Materials
Midterm #2
March 30th, 2022

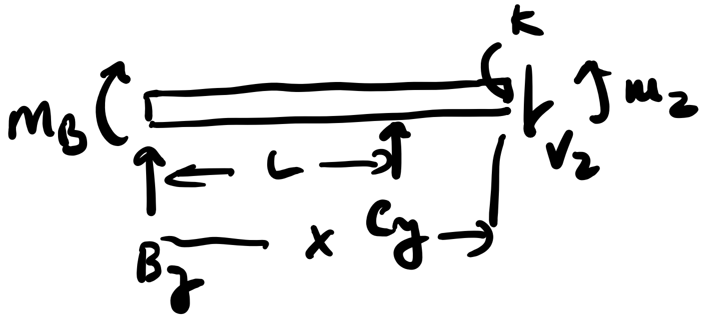
PURDUE UNIVERSITY | School of Mechanical Engineering

Name (Print) ______________________________________

(Last) (First)

PROBLEM # 2 CONT.

Section CD $\quad L < x < 2L$



$$\Sigma M_K = M_2 - M_B - B_y x - C_y (x-L) = 0$$

$$M_2(x) = M_B + B_y x + C_y x - C_y L = EI v_2''$$

$$EI v_2' = M_B x + B_y \frac{x^2}{2} + C_y \frac{x^2}{2} - C_y L x + C_3$$

$$EI v_2 = M_B \frac{x^2}{2} + B_y \frac{x^3}{6} + C_y \frac{x^3}{6} - C_y L \frac{x^2}{2} + C_3 x + C_4$$

BC's & continuity at $x = L$

$$v_2(L) = 0 \qquad v_2'(L) = v_1'(L)$$

$$v_2(L) = M_B \frac{L^2}{2} + B_y \frac{L^3}{6} + C_y \frac{L^3}{6} - C_y \frac{L^3}{2} + C_3 L + C_4 = 0$$

$$v_2'(L) = \cancel{M_B L} + \cancel{B_y \frac{L^2}{2}} + C_y \frac{L^2}{2} - C_y L^2 + C_3$$

$$= v_1'(L) = \cancel{M_B L} + \cancel{B_y \frac{L^2}{2}}$$

Solve: $-C_y \frac{L^2}{2} + C_3 = 0 \qquad C_3 = C_y \frac{L^2}{2}$

Replace C_y: $\boxed{C_3 = -\frac{3M_0 L}{4}}$

ME 323 – Mechanics of Materials
Midterm #2
March 30th, 2022

PURDUE UNIVERSITY School of Mechanical Engineering

Name (Print) ______________________________________

(Last) (First)

PROBLEM # 2 CONT.

Replace C_3, M_B, B_y & C_y in $v_2(L) = 0$

& solve for C_4

$$\boxed{C_4 = \frac{M_0 L^2}{4}}$$

Replace $x = 2L$ in $v_2(x)$

$$\boxed{v_2(2L) = \frac{3}{4}\frac{M_0 L^2}{EI}}$$

ME 323 – Mechanics of Materials
Midterm #2
March 30th, 2022

PURDUE UNIVERSITY | School of Mechanical Engineering

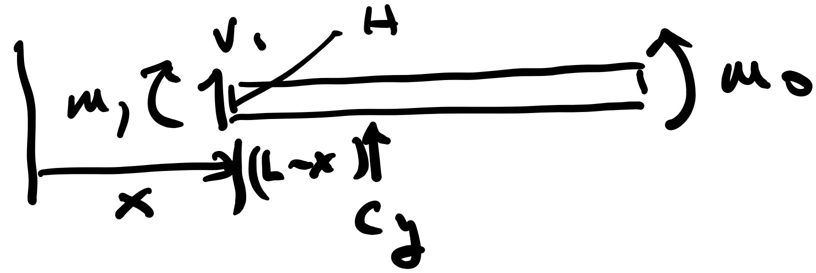
Name (Print) SOLUTION 3

(Last) (First)

PROBLEM # 2 CONT.

Solution 3 – Use the RHS of the beam

Section BC $\quad 0 < x < L$



$$\Sigma M_H = -M_1 + C_y(L-x) + M_0 = 0$$

$$M_1 = M_0 + C_y(L-x) = EIv_1''$$

$$EIv_1' = M_0 x + C_y L x - C_y\frac{x^2}{2} + C_1$$

$$EIv_1 = M_0\frac{x^2}{2} + C_y L\frac{x^2}{2} - C_y\frac{L^3}{6} + C_1 x + C_2$$

B.C's

$$v_1'(0) = C_1 = 0 \qquad v_1(0) = C_2 = 0$$

$$v_1'(L) = \frac{1}{EI}\left[M_0 L + C_y L^2 - C_y\frac{L^2}{2}\right]$$

$$v_1(L) = \frac{1}{\cancel{EI}}\left[M_0\frac{L^2}{2} + C_y\frac{L^3}{2} - C_y\frac{L^3}{6}\right] = 0$$

ME 323 – Mechanics of Materials
Midterm #2
March 30th, 2022

School of Mechanical Engineering

Name (Print) ______________________________
(Last) (First)

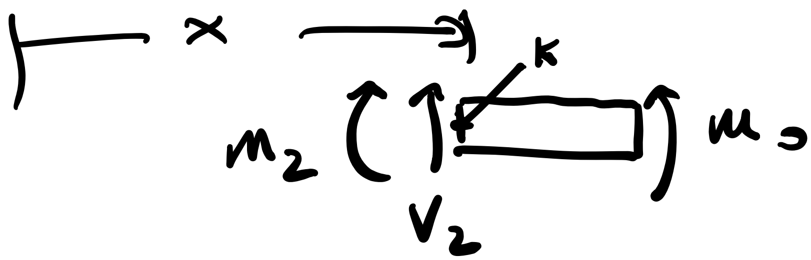
PROBLEM # 2 CONT.

Use eqn's (1) & (2) & solve

$$\boxed{C_y = -\frac{3}{2}\frac{M_0}{L} = -B_y}$$

$$\boxed{M_B = -\frac{M_0}{2}}$$

Section CD $L < x < 2L$



$$\Sigma M_K = M_0 - M_2 = 0$$

$$M_2(x) = M_0 = EI v_2''$$

$$EI v_2' = M_0 x + C_3$$

$$EI v_2 = M_0 \frac{x^2}{2} + C_3 x + C_4$$

Use $v_2(L) = v_1(L) = 0$

$$v_2'(L) = v_1'(L)$$

ME 323 – Mechanics of Materials
Midterm #2
March 30th, 2022

PURDUE UNIVERSITY | School of Mechanical Engineering

Name (Print) ____

(Last) (First)

PROBLEM # 2 CONT.

$$v_2'(L) = \cancel{m_0 L} + C_3 = v_1'(L) = \cancel{m_0 L} + C_1 L^2 - C_1 \frac{L^2}{2}$$

$$C_3 = C_1 \frac{L^2}{2} = \left(-\frac{3}{2}\frac{m_0}{\cancel{L}}\right)\left(\frac{L^{\cancel{2}}}{2}\right)$$

$$\boxed{C_3 = -\frac{3}{4} m_0 L}$$

$$v_2(L) = \frac{m_0 L^2}{2} + C_3 L + C_4 = 0$$

$$C_4 = -\left(-\frac{3}{4} m_0 L^2\right) - m_0 \frac{L^2}{2}$$

$$\boxed{C_4 = \frac{m_0 L^2}{4}}$$

$$EI v_2(2L) = \frac{m_0}{2}(4L^2) + \left(-\frac{3}{4} m_0 L\right)(2L) + \frac{m_0 L^2}{4}$$

$$\boxed{v_2(2L) = \frac{3 m_0 L^2}{EI}}$$

Exam 2: Q3

There are many different ways to do this question (6), so the grading was flexible, but the general procedure is:

1. Find two equilibrium equations.
2. Find M(x) in two sections.
3. Put M(x) in terms of only the redundant load.
4. Enforce BC using Castigliano's equation

$$\frac{\delta \mathrm{U}}{\delta A_y} = 0 \quad or \quad \frac{\delta \mathrm{U}}{\delta C_y} = 0 \quad \text{or} \quad \frac{\delta \mathrm{U}}{\delta M_A} = 0$$

5. Use the solved reaction to find other reactions.
6. Find the change in angle at C by using Castigliano's:

$$\theta = \left[\frac{\delta \mathrm{U}}{\delta M_C}\right]_{M_C=0}$$

Most common errors:

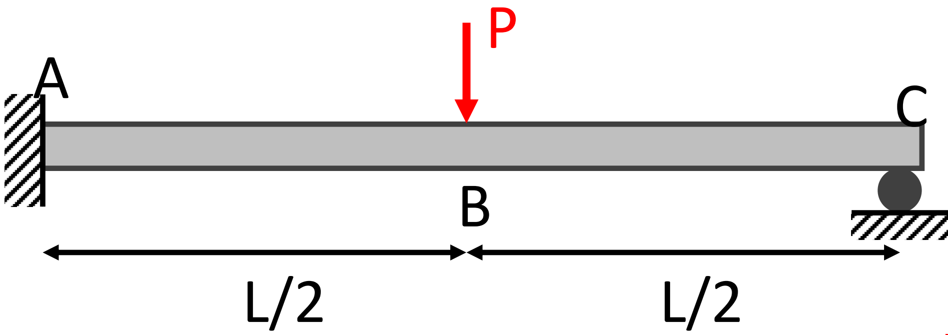
- Small math errors (just lose a few points)
- Taking the partial derivative without isolating for the redundant load.

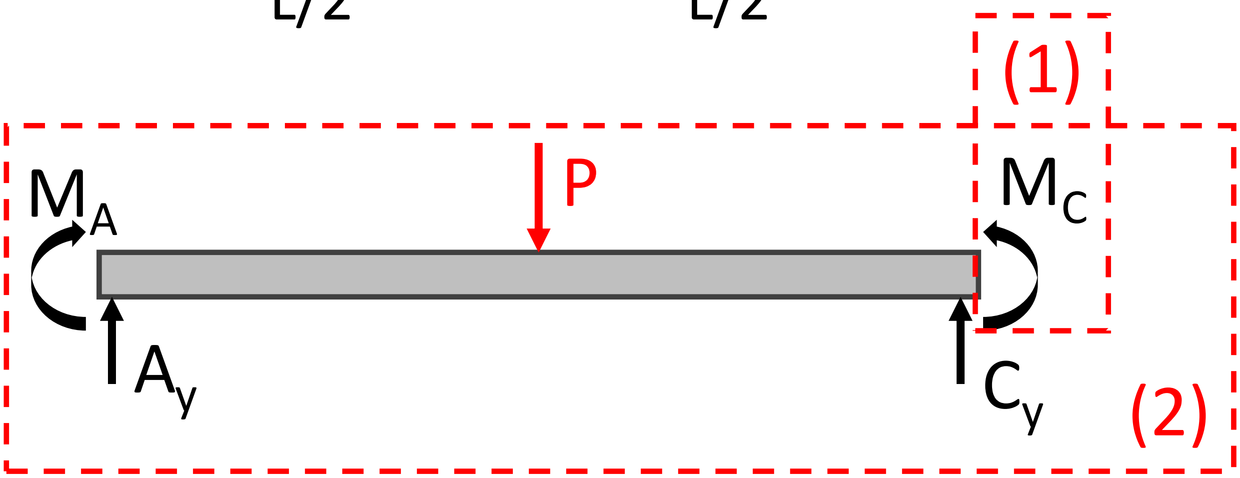
$$\frac{\delta}{\delta A_y}\left(A_y x + M_A\right) \neq x$$

$$\frac{\delta}{\delta A_y}\left(A_y x + M_A\right) = x + \frac{\delta M_A}{\delta A_y} = x - L$$

- Limits of integration on the integral of U.

Exam 2: Q3

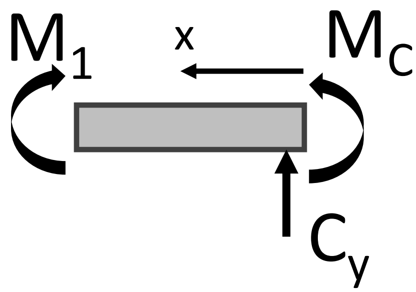




$$\left(\sum M\right)_A = -M_A + M_C - P\left(\frac{L}{2}\right) + C_y L = 0$$ (1) (1)

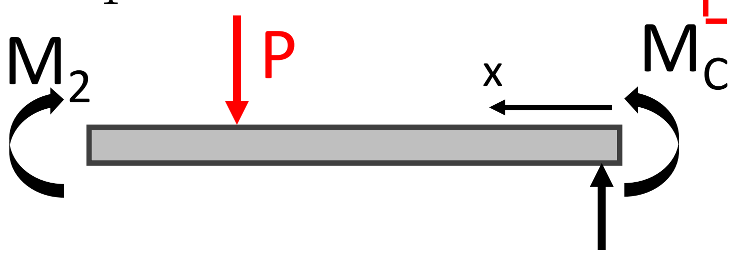
$$\sum F_y = A_y + C_y - P = 0$$ (1) (2)

(a) Example 1: Cy redundant, solve from the right (easiest method)



$$\left(\sum M\right)_1 = -M_1 + M_C + C_y x = 0$$

$$M_1 = M_C + C_y x \quad (2)$$



$$(\sum M)_2 = -M_2 + M_C + C_y x - P\left(x - \frac{L}{2}\right) = 0$$

$$M_2 = M_C + C_y x - P\left(x - \frac{L}{2}\right) \quad (2)$$

$$\frac{\delta M_1}{\delta C_y} = x \qquad \frac{\delta M_2}{\delta C_y} = x \qquad (2)$$

$$(2) \quad \left[\frac{\delta U}{\delta C_y}\right]_{M_C=0} = 0 = \left(\frac{1}{\mathrm{EI}}\right)\int_0^{L/2} M_1 \frac{\delta M_1}{\delta C_y} dx + \left(\frac{1}{\mathrm{EI}}\right)\int_{L/2}^{L} M_2 \frac{\delta M_2}{\delta C_y} dx$$

$$0 = \left(\frac{1}{\mathrm{EI}}\right)\int_0^{L/2} C_y x^2 dx + \left(\frac{1}{\mathrm{EI}}\right)\int_{L/2}^{L} C_y x^2 - P x^2 + P\left(\frac{L}{2}\right) x \, dx \quad (2)$$

Rearrange to make the integrals easier

$$0 = \left(\frac{1}{\mathrm{EI}}\right)\int_0^L C_y x^2 dx + \left(\frac{1}{\mathrm{EI}}\right)\int_{L/2}^L -Px^2 + P\left(\frac{L}{2}\right)x\,dx$$

Integrate and multiply by EI

$$0 = \left[\left(\frac{1}{3}\right)C_y x^3\right]_0^L + \left[-\left(\frac{1}{3}\right)Px^3 + P\left(\frac{L}{4}\right)x^2\right]_{L/2}^L$$

$$0 = \left(\frac{1}{3}\right)C_y L^3 - P\left(\frac{1}{3}\right)\left(\frac{7}{8}\right)L^3 + P\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)L^3$$

$$\left(\frac{1}{3}\right)C_y L^3 = P\left(\frac{14}{48}\right)L^3 - P\left(\frac{9}{48}\right)L^3 \longrightarrow \boxed{C_y = \frac{5}{16}P \quad (1)}$$

$$(2) \longrightarrow \boxed{A_y = \frac{11}{16}P \quad (1)}$$

$$(1) \longrightarrow \boxed{M_A = \frac{3}{16}PL \quad (1)}$$

(b)

$$\boxed{(2) \quad \left[\frac{\delta U}{\delta M_C}\right]_{M_C=0} = \theta} = \left(\frac{1}{\mathrm{EI}}\right)\int_0^{L/2} M_1\frac{\delta M_1}{\delta M_C}dx + \left(\frac{1}{\mathrm{EI}}\right)\int_{L/2}^L M_2\frac{\delta M_2}{\delta M_C}dx$$

$$\boxed{\frac{\delta M_1}{\delta M_C} = 1 \qquad \frac{\delta M_2}{\delta M_C} = 1 \qquad (2)}$$

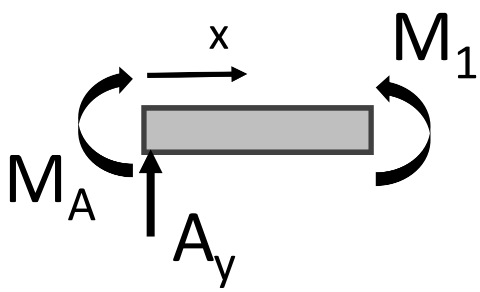
$$\boxed{\theta = \left(\frac{1}{\mathrm{EI}}\right)\int_0^{L/2} C_y x dx + \left(\frac{1}{\mathrm{EI}}\right)\int_{L/2}^L C_y x - Px + P\left(\frac{L}{2}\right)\,dx \qquad (2)}$$

$$\theta = \left(\frac{1}{EI}\right)\left[\left(\frac{1}{2}\right)C_y x^2\right]_0^L + \left(\frac{1}{EI}\right)\left[-\left(\frac{1}{2}\right)Px^2 + P\left(\frac{L}{2}\right)x\right]_{L/2}^L$$

$$\theta = \left(\frac{1}{\mathrm{EI}}\right)\left[\left(\frac{1}{2}\right)C_y L^2 - P\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)L^2 + P\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)L^2\right]$$

$$\theta = \left(\frac{1}{\mathrm{EI}}\right)\left[\left(\frac{5}{32}\right)PL^2 - \left(\frac{12}{32}\right)PL^2 + \left(\frac{8}{32}\right)PL^2\right] = \boxed{\left(\frac{1}{EI}\right)\left(\frac{1}{32}\right)PL^2 \quad (1)}$$

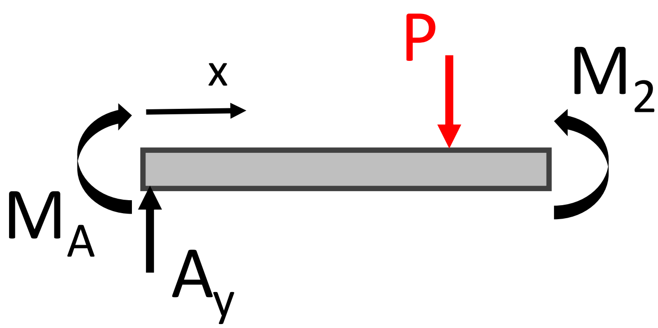
(a) Example 2: Ay redundant, solve from the left (reasonable method)



$$\left(\sum M\right)_1 = M_1 - M_A - A_y x = 0$$

$$\boxed{M_1 = M_A + A_y x \quad (2)}$$

(2) $\rightarrow M_A = M_C + P\left(\frac{L}{2}\right) - A_y L \rightarrow M_1 = M_C + P\left(\frac{L}{2}\right) + A_y(x - L)$



$$(\sum M)_2 = M_2 - M_A - A_y x + P\left(x - \frac{L}{2}\right) = 0$$

$$\boxed{(2) \quad M_2 = M_A + A_y x - P\left(x - \frac{L}{2}\right)} \qquad M_2 = M_C + A_y(x - L) - P(x - L)$$

$$\boxed{\frac{\delta M_1}{\delta A_y} = x - L \qquad \frac{\delta M_2}{\delta A_y} = x - L \qquad (2)}$$

$$\boxed{(2) \quad \left[\frac{\delta U}{\delta A_y}\right]_{M_C=0} = 0} = \left(\frac{1}{\mathrm{EI}}\right)\int_0^{L/2} M_1 \frac{\delta M_1}{\delta A_y} dx + \left(\frac{1}{\mathrm{EI}}\right)\int_{L/2}^{L} M_2 \frac{\delta M_2}{\delta A_y} dx$$

$$\boxed{\begin{aligned} 0 = &\left(\frac{1}{\mathrm{EI}}\right)\int_0^{L/2}\left[A_y(x - L) + P\left(\frac{L}{2}\right)\right](x - L)dx \\ &+ \left(\frac{1}{\mathrm{EI}}\right)\int_{L/2}^{L}\left[A_y(x - L) - P(x - L)\right](x - L)dx \qquad (2)\end{aligned}}$$

$$0=\left(\frac{1}{\mathrm{EI}}\right)\int_0^{L/2}\left[A_y(x^2-2xL+L^2)+P\left(\frac{L}{2}\right)x-P\left(\frac{L^2}{2}\right)\right]dx$$
$$+\left(\frac{1}{\mathrm{EI}}\right)\int_{L/2}^{L}\left[(x^2-2xL+L^2)(A_y-P)\right]dx$$

$$0=\left[A_y\left(\frac{1}{3}x^3-Lx^2+L^2x\right)+P\left(\frac{L}{2}\right)\left(\frac{x^2}{2}\right)-P\left(\frac{L^2}{2}\right)x\right]_0^{L/2}$$
$$+\left[\left(\frac{1}{3}x^3-Lx^2+L^2x\right)(A_y-P)\right]_{L/2}^{L}$$

$$0=A_y\left(\frac{7}{24}\right)+P\left(\frac{1}{16}\right)-P\left(\frac{1}{4}\right)+A_y\left(\frac{1}{24}\right)-P\left(\frac{1}{24}\right)$$

$$\left(\frac{1}{3}\right)A_y=P\left(\frac{14}{48}\right)-P\left(\frac{3}{48}\right)\longrightarrow \boxed{A_y=\frac{11}{16}P \quad (1)}$$

(2) $\longrightarrow \boxed{C_y=\frac{5}{16}P \quad (1)}$

(1) $\longrightarrow \boxed{M_A=\frac{3}{16}PL \quad (1)}$

(b)

$$\boxed{(2)\quad \left[\frac{\delta U}{\delta M_C}\right]_{M_C=0}=\theta}=\left(\frac{1}{\mathrm{EI}}\right)\int_0^{L/2}M_1\frac{\delta M_1}{\delta M_C}dx+\left(\frac{1}{\mathrm{EI}}\right)\int_{L/2}^{L}M_2\frac{\delta M_2}{\delta M_C}dx$$

$$\boxed{\frac{\delta M_1}{\delta M_C}=1 \qquad \frac{\delta M_2}{\delta M_C}=1 \qquad (2)}$$

$$\boxed{\begin{aligned}\theta&=\left(\frac{1}{\mathrm{EI}}\right)\int_0^{L/2}\left[A_y(x-L)+P\left(\frac{L}{2}\right)\right]dx \qquad (2)\\&+\left(\frac{1}{\mathrm{EI}}\right)\int_{L/2}^{L}\left[A_y(x-L)-P(x-L)\right]dx\end{aligned}}$$

$$\theta=\left(\frac{1}{\mathrm{EI}}\right)[-\left(\frac{1}{2}\right)A_yL^2+P\left(\frac{L}{2}\right)^2-P\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)L^2+P\left(\frac{1}{2}\right)L^2$$

$$\theta=\left(\frac{1}{\mathrm{EI}}\right)\left[-\left(\frac{11}{32}\right)PL^2+\left(\frac{12}{32}\right)PL^2\right]=\boxed{\left(\frac{1}{EI}\right)\left(\frac{1}{32}\right)PL^2 \quad (1)}$$

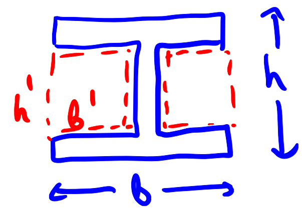
4.1

$M = 100\,kNm$

$\sigma = \dfrac{-My}{I_{zz}}$

$y_a = 160\,mm$; $y_b = 170\,mm$

I_{zz} :



$= \dfrac{1}{12} b h^3 - 2\left(\dfrac{1}{12} b' h'^3\right)$

$\therefore I_{zz,(a)} = \dfrac{1}{12}(200)(320)^3 - 2\left(\dfrac{1}{12} \times 90 \times 300^3\right)$

$= 141.13 \times 10^6\,mm^4$

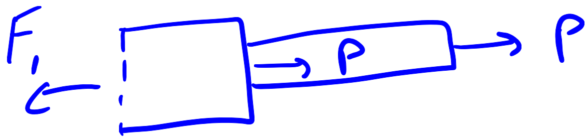
$I_{zz,(b)} = \dfrac{1}{12}(200)(340)^3 - 2\left(\dfrac{1}{12} \times 95 \times 300^3\right)$

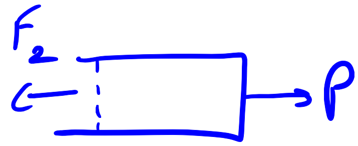
$= 227.57 \times 10^6\,mm^4$

$\dfrac{y_a}{I_{zz,(a)}} = 1.134 \times 10^{-6}\,mm^{-3} > \dfrac{y_b}{I_{zz,(b)}} = 0.747 \times 10^{-6}\,mm^{-3}$

$\therefore |\sigma_a| > |\sigma_b|$

4·2





$\therefore F_1 = 2P$ $\qquad$ $F_2 = P$

a) $$U_A = \frac{1}{2} F_1^2 \left(\frac{L}{AE}\right)_1 + \frac{1}{2} F_2^2 \left(\frac{L}{AE}\right)_2$$

$$= \frac{1}{2}\left(\frac{L}{AE}\right) 5P^2$$

b) $$U_B = \frac{1}{2}(2P)^2\left(\frac{L}{2AE}\right) + \frac{1}{2}(P)^2\left(\frac{L}{AE}\right)$$

$$= \frac{1}{2}\left(\frac{L}{AE}\right) 3P^2$$

c) $$U_c = \frac{1}{2}(2P)^2\left(\frac{L}{AE}\right) + \frac{1}{2}P^2\left(\frac{L}{2AE}\right)$$

$$= \frac{1}{2}\left(\frac{L}{AE}\right)\left(4.5P^2\right)$$

d) $$U_d = \frac{1}{2}(2P)^2\left(\frac{L}{2AE}\right) + \frac{1}{2}P^2\left(\frac{L}{2AE}\right)$$

$$= \frac{1}{2}\left(\frac{L}{2AE}\right)\left(2.5P^2\right)$$

$\therefore U_b < U_a$; $U_c < U_a$; $U_d < U_a$

4.3

a) $p(x) = EI\,u''''(x) = p_0 \cos\left(\frac{\pi x}{2L}\right)$

b) $\theta_A = 0$ (fixed end)

$\theta(x) = u'(x) \Rightarrow \theta_B = \theta(L) = u'(L)$

$\therefore \theta_B = \frac{p_0 L^3}{\pi^3 EI}(\pi^2 - 8)$

c) $V(x) = EI\,u'''(x)$

$R_A = V(0)$

$\therefore R_A = \frac{-2Lp_0}{\pi}$

$M(x) = EI\,u''(x)$

$M_A = M(0)$

$\Rightarrow M_A = \frac{2p_0 L^2}{\pi^2}(\pi - 2)$