

Name (Print) _____
(Last) (First)

**ME 323 - Mechanics of Materials
Exam # 2**

Date: April 10, 2019 Time: 8:00 – 10:00 PM - Location: FRNY G140

Instructions:

Circle your instructor's name and your class meeting time.

Gonzalez
11:30-12:20AM

Koslowski
1:30-2:20PM

Begin each problem in the space provided on the examination sheets.

Work on one side of each sheet only, with only one problem on a sheet.

Please remember that for you to obtain maximum credit for a problem, you must present your solution clearly.

Accordingly,

- coordinate systems must be clearly identified,
- free body diagrams must be shown,
- units must be stated,
- write down clarifying remarks,
- state your assumptions, etc.

If your solution cannot be followed, it will be assumed that it is in error.

When handing in the test, make sure that ALL SHEETS are in the correct sequential order.

Remove the staple and restaple, if necessary.

Prob. 1 _____

Prob. 2 _____

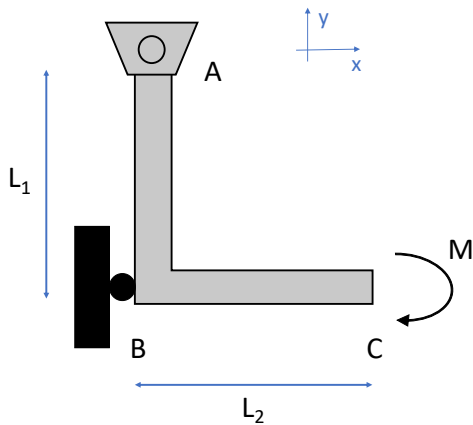
Prob. 3 _____

Prob. 4 _____

Total _____

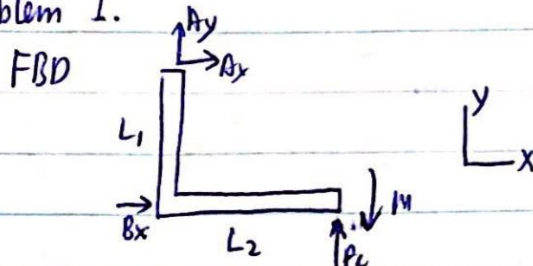
PROBLEM # 1 (25 points)

Using Castigliano's theorem determine the vertical displacement at **C** for the member shown in the figure. A moment M is applied at **C**. The cross section of the member is square and constant. Use A , I , and E constant along the member ABC . Ignore shear stress in the energy calculation.



Exam 2. ME323 Spring 2019

Problem 1.



Equilibrium: $\Sigma F_x = A_x + B_x = 0$

$\Sigma F_y = A_y + P_c = 0$

$\Sigma M_D = B_x L_1 + P_c L_2 - M = 0$

Solve: $A_x = \frac{P_c L_2 - M}{L_1}$, $A_y = -P_c$, $B_x = \frac{M - P_c L_2}{L_1}$



$U_{\text{axial}} = \frac{P_c^2 L_1}{2EA}$

$U_{\text{flex1}} = \int_0^{L_1} \frac{(P_c L_2 - M)^2 y^2}{2EI} dy$

$U_{\text{flex2}} = \int_0^{L_2} \frac{(P_c x - M)^2}{2EI} dx$

$U_{\text{total}} = U_{\text{axial}} + U_{\text{flex1}} + U_{\text{flex2}}$

$U_c = \frac{\partial U_{\text{total}}}{\partial P_c} \Big|_{P_c=0} = \left[\frac{P_c L_1}{EA} + \int_0^{L_1} \frac{P_c L_2 - M}{EI} x^2 dx + \int_0^{L_2} \frac{(P_c x - M)x}{EI} dx \right]$

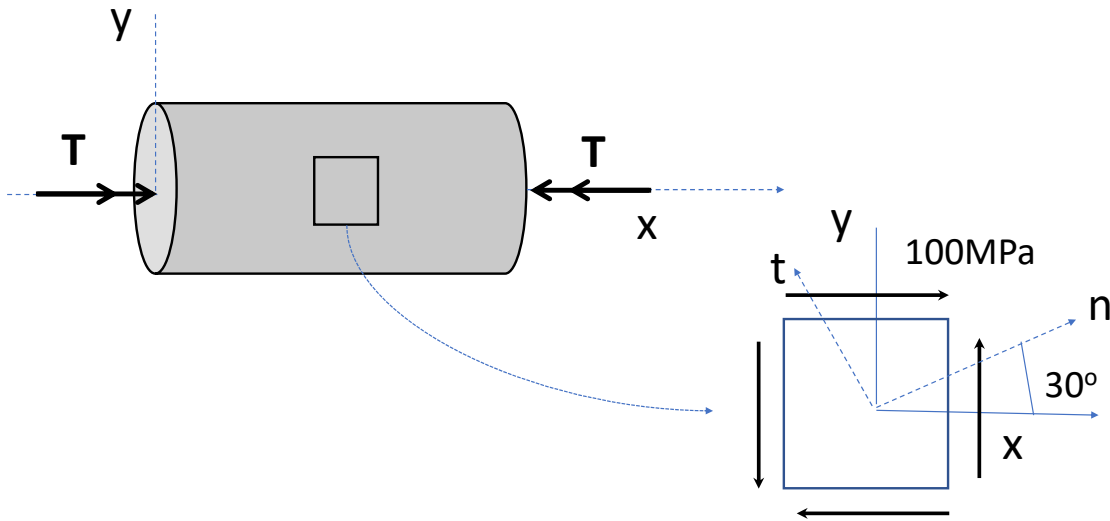
$= -\frac{M L_1 L_2}{3EI} - \frac{M L_2^2}{2EI} \quad (\uparrow)$

So that vertical displacement at C equals $\frac{M L_1 L_2}{3EI} + \frac{M L_2^2}{2EI}$,
points downward.

PROBLEM # 2 (25 points)

A solid circular aluminum rod has diameter $d=20$ mm and it is subjected to a torque T . An element in the surface of the rod has the state of stress shown ($\sigma_x = \sigma_y = 0, \tau_{xy} = 100MPa$) oriented with the x and y axis.

- 1) Calculate the torque T .
- 2) Construct a Mohr's circle for the state of stress in the element.
- 3) Using the Mohr's circle determine the principal stresses and show them on an oriented stress element.
- 4) Using the Mohr's circle determine the maximum shear stress and the normal stress and show them on an oriented stress element.
- 5) Using the Mohr's circle determine the state of stress in an element oriented 30° clockwise from the x axis, see figure. Show the stresses on an oriented stress element.



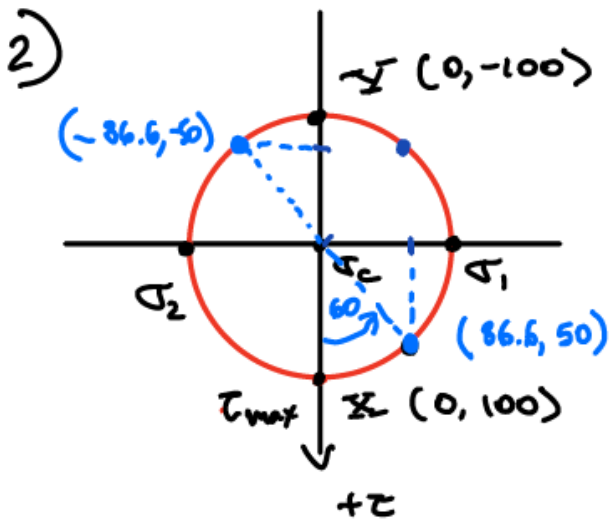
PROBLEM 2

$$1) \quad \tau = \frac{J T}{I_p}$$

$$\tau = \frac{2 J T}{\pi r^4} = 100 \text{ MPa}$$

$$T = \frac{100 \cancel{10^6} \pi \cancel{10^{-4}}}{2}$$

$$T = 157.1 \text{ N.m}$$



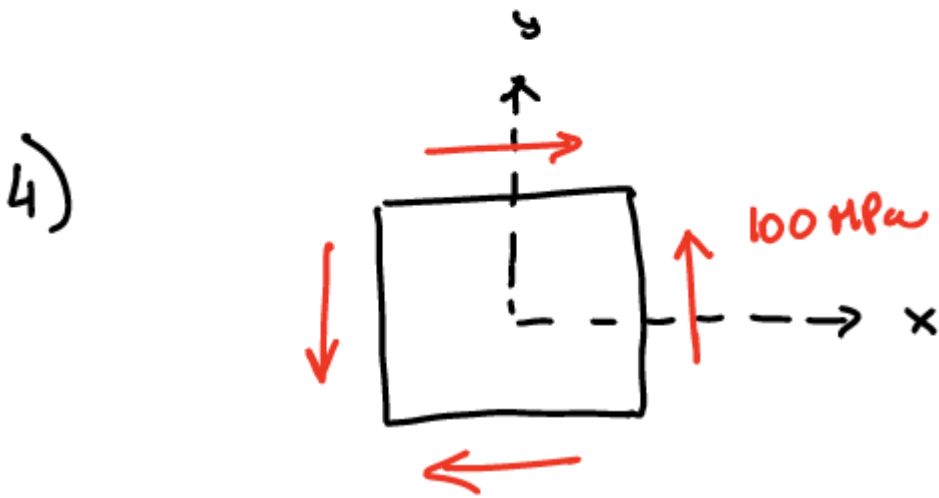
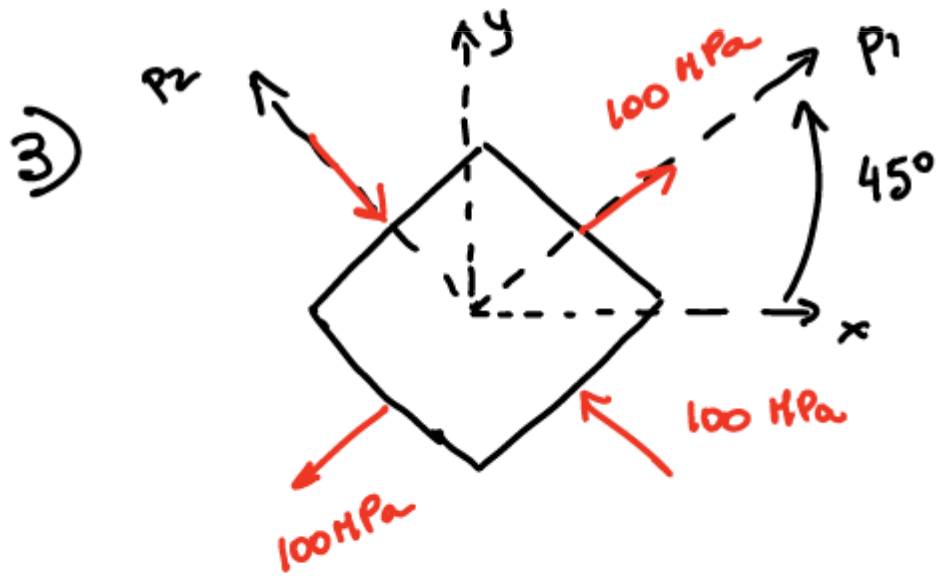
$$\sigma_c = 0$$

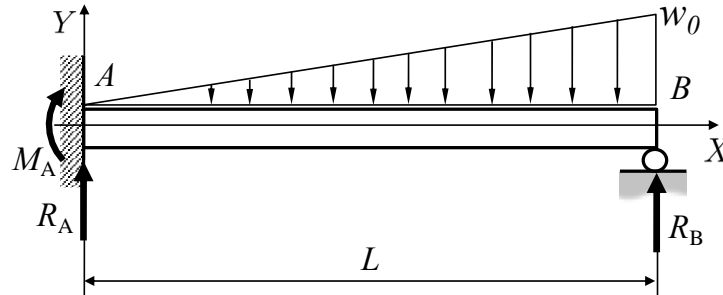
$$R = 100$$

$$\sigma_1 = 100 \text{ MPa}$$

$$\sigma_2 = 100 \text{ MPa}$$

$$\tau_{max} = 100 \text{ MPa}$$



PROBLEM # 3 (25 points)

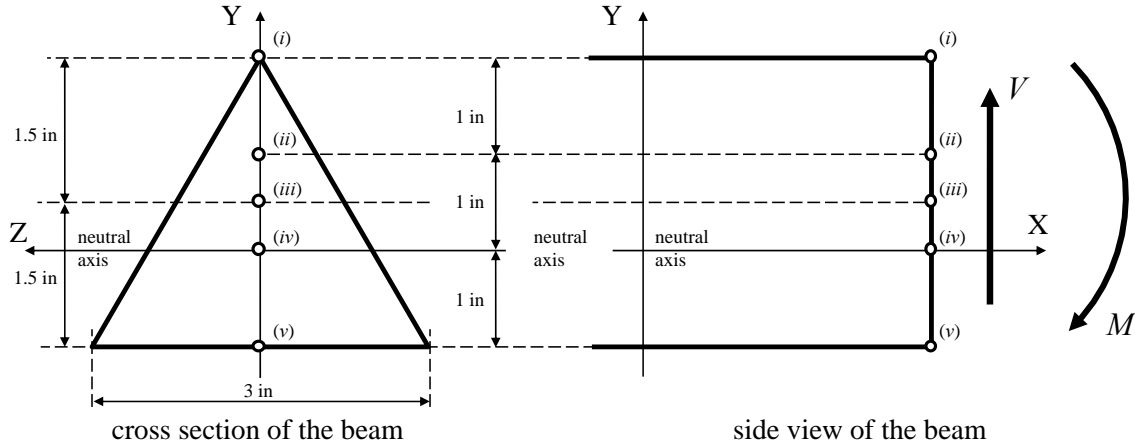
The uniform, linearly elastic beam shown in the figure supports a triangularly distributed load.

- 1) Write the equations of equilibrium and identify whether the structure is statically determinate or indeterminate.

Using the second-order method:

- 2) Determine the bending moment $M(x)$ of the beam (as a function of the reactions at A).
- 3) Determine the slope $v'(x)$ and deflection $v(x)$ of the beam.
- 4) Indicate the boundary conditions at supports A and B .
- 5) Solve for any constants of integration.
- 6) Write a system of three equations to determine the reactions at A and B , that is R_A , M_A , and R_B .
Note: do not solve for the reactions.
- 7) Sketch the deflection curve.

PROBLEM # 4A (points 15 points):



A shear force V and bending moment M act at a cross section of a triangular cross-sectioned beam. Consider the five points (i), (ii), (iii), (iv) and (v) on the beam cross section, as shown above. Match up the state of stress at each of these five points with the stress elements (a) through (o) shown below. If you choose ‘(o) NONE of the above’, provide a sketch of the correct state of stress for your answer.

The state of stress at point (i) is _____

The state of stress at point (ii) is _____

The state of stress at point (iii) is _____

The state of stress at point (iv) is _____

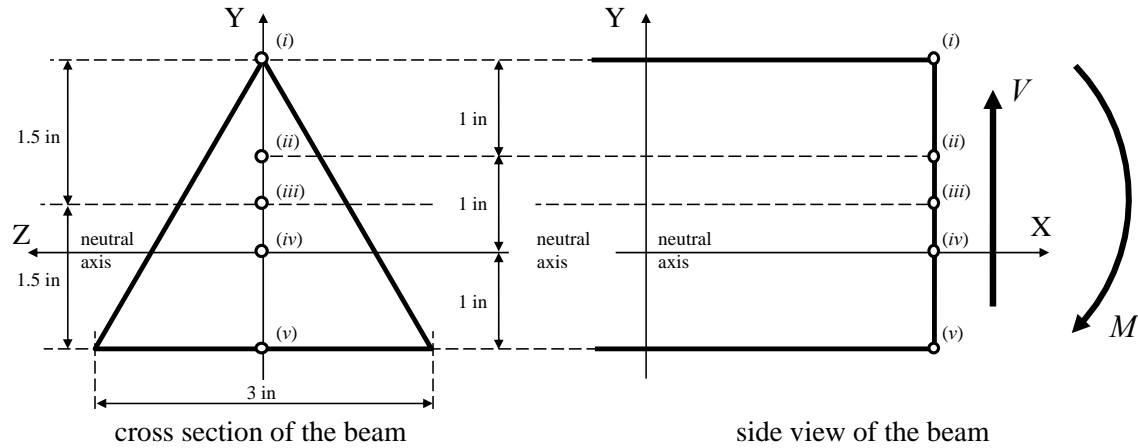
The state of stress at point (v) is _____

(a)	(b)	(c)	(d)	(e)
(f)	(g)	(h)	(i)	(j)
(k)	(l)	(m)	(n)	(o) NONE of the above

Problem 4A

- (i) A
- (ii) K
- (iii) K
- (iv) J
- (v) B

PROBLEM # 4B (points 10 points):

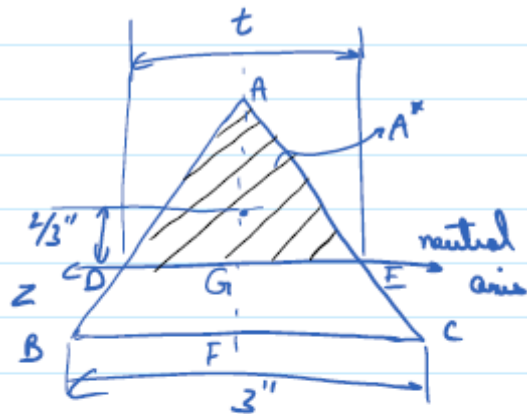


At the same triangular cross section along the beam, it is known that the shear force V is equal to 4 kips in the y -direction. Determine the shear stress at point (iv).

Given, $V = 4 \text{ kips}$

$\tau = ?$

$$\tau = \frac{VQ}{It} = \frac{VA\bar{y}^*}{It}$$



Finding t : $\frac{DG}{AG} = \frac{BF}{AF} \Rightarrow DG = \frac{0.5}{3} \times 2 = 1$

$$\Rightarrow t = 2 \cdot DG = 2''$$

$$I_z = \frac{1}{36} (BC)(AF)^3 = \frac{1}{36} \times 3 \times 3^3 = \frac{9}{4} \text{ in}^4$$

$$A^* (\text{shaded region}) = DG \times AG = 2 \text{ in}^2$$

$$y^* = \frac{2}{3}''$$

$$\begin{aligned} \therefore \tau_{\max} &= \frac{4000 \times 2 \times \frac{2}{3}}{\frac{9}{4} \times 2} = \frac{32000}{27} = 1185.18 \text{ psi} \\ &= \underline{\underline{1.185 \text{ ksi (in } + \text{ dir.)}}} \end{aligned}$$