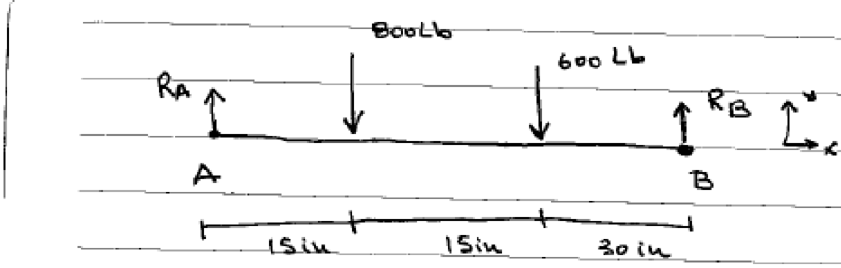
ME323 EXAM #2 Solution

Problem 1 Solution :



$\Sigma F_y = 0$

$R_A + R_B = +1400 \text{ Lb}$

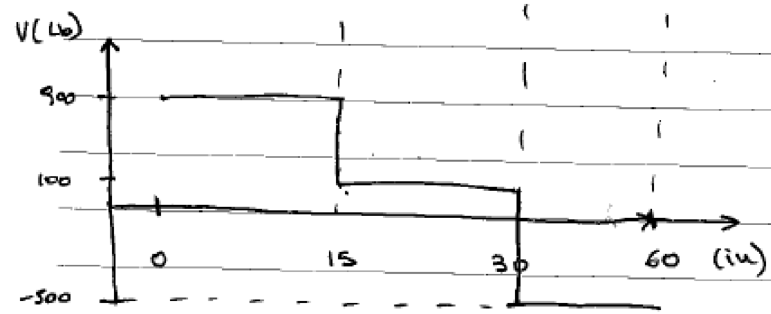
$\Sigma M_z = 0$

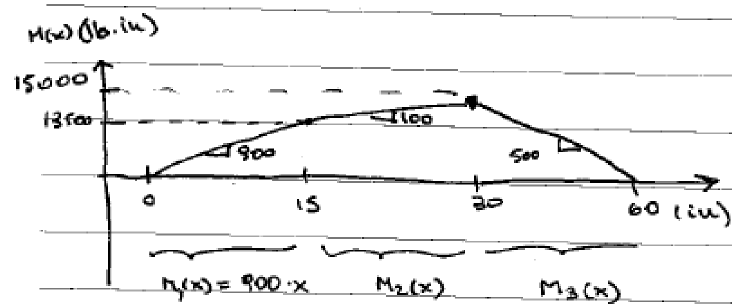
$800 \cdot 15 + 600 \cdot 30$

$= R_B \; 60$

$R_B = 500 \text{ Lb}$

$R_A = 900 \text{ Lb}$

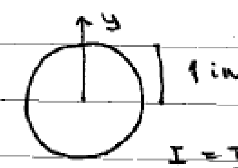




$M_1(x) = 900x$

$M_2(x) = 100x + 12000$

$M_3(x) = -500x + 30000$



$I = \dfrac{\pi r^4}{4}$

$M^{max} = 15{,}000 \text{ Lb.in} \quad \text{at } x = 30 \text{ in}$

$V^{max} = 900 \text{ Lb} \quad 0 \le x \le 15 \text{ in.}$

a) σ^{max} at $x = 30 \text{ in}$, $y = 1 \text{ in}$

$\sigma^{max} = \dfrac{M y}{I} = 19108.28 \text{ psi}$

b) τ^{max} at $0 \le x \le 15 \text{ in}$, $y = 0$

$\tau^{max} = \dfrac{4}{3} \dfrac{V}{A} = 382 \text{ psi}$

c) $\sigma_{allow} = 20.000 \text{ psi}$

$\sigma^{max} = \dfrac{M^{max} \cdot y}{\pi r^4/4} = \dfrac{M^{max} \cdot 4}{\pi r^3}$

$20.000 \text{ psi} = \dfrac{15{,}000 \cdot 4}{\pi r^3}$

$r = 0.98 \text{ in.}$

Problem 2 Solution:

a) $\sigma_x = 60\ MPa$, $\tau_{max,in-plane} = R = 50\ MPa$

$$\sigma_{p_1} = \sigma_{avg} \pm R = \frac{60+\sigma_y}{2} \pm 50 = 70\ MPa \quad \Rightarrow \quad \sigma_y = (70 \mp 50)\times 2 - 60 = \begin{cases} -20\ MPa \\ 180\ MPa \end{cases}$$

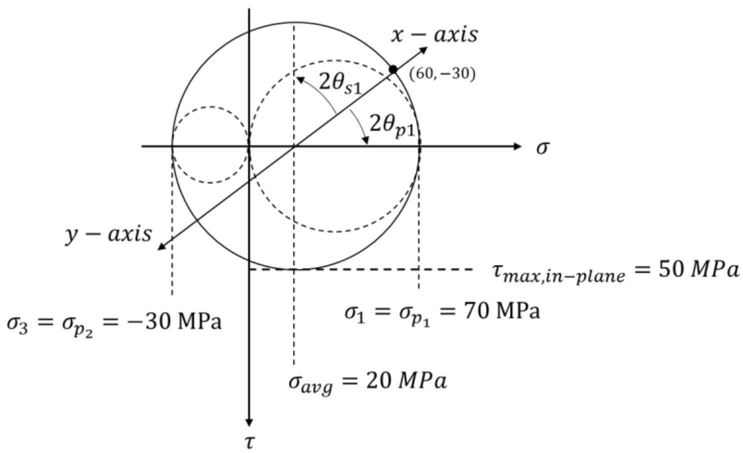
Since σ_y is compressive, then $\sigma_y = -20\ MPa$, $\sigma_{avg} = \frac{60-20}{2} = 20\ MPa$

The other in-plane principal stress $\sigma_{p_2} = \sigma_{avg} - R = 20 - 50 = -30\ MPa$

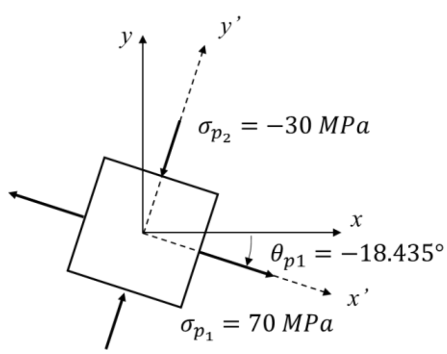
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \sqrt{\left(\frac{60+20}{2}\right)^2 + \left(\tau_{xy}\right)^2} = 50 \quad \Rightarrow \quad \tau_{xy} = \pm 30\ MPa$$

Since τ_{xy} is in the direction shown, then $\tau_{xy} = -30\ MPa$.

b)

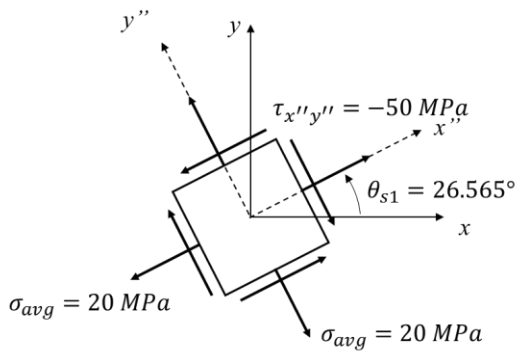


c)



$$2\theta_{p_1} = -\tan^{-1}\left(\frac{30}{60-20}\right) \Rightarrow 2\theta_{p_1} = -36.87° \Rightarrow \theta_{p_1} = -18.435°$$

d)



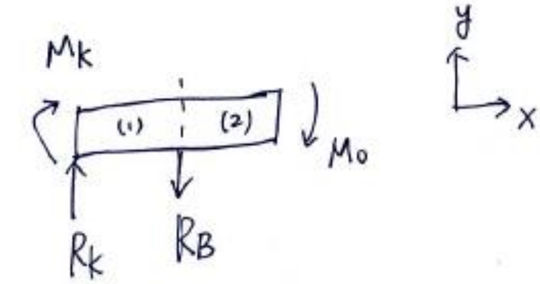
$$\theta_{s_1} = 45° + \theta_{p_1} = 26.565°$$

e) $\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2} = \frac{70+30}{2} = 50\ MPa$

Problem 3 Solution:

P3 Solution:

FBD for KC:

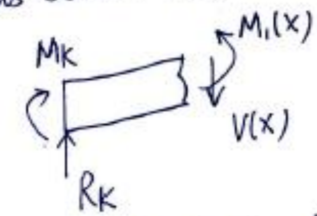


$\sum F_y = 0 \Rightarrow R_K = R_B$

$\sum M_K = 0 \Rightarrow M_K + R_B L + M_0 = 0, \quad M_K = -R_B L - M_0$

Mark KB as section (1), BC section (2)

For KB:



$M_1(x) = R_K x + M_K$

$= R_B(x-L) - M_0$

$EIv_1'' = M_1(x) = R_B(x-L) - M_0$

$EIv_1' = \frac{1}{2}R_B(x-L)^2 - M_0 x + C_1$

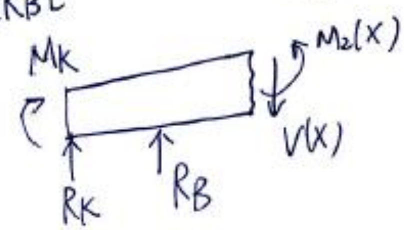
$EIv_1 = \frac{1}{6}R_B(x-L)^3 - \frac{1}{2}M_0 x^2 + C_1 x + C_2$

Boundary conditions:

$v_1'(0) = 0 \Downarrow C_1 = -\frac{1}{2}R_B L^2$

$v_1(0) = 0 \Downarrow C_2 = \frac{1}{6}R_B L^3$

For BC:



$M_2(x) = -M_0$

$$EI v_2'' = M_2(x) = -M_0$$

$$EI v_2' = -M_0 x + C_3$$

$$EI v_2 = -\frac{1}{2}M_0 x^2 + C_3 x + C_4$$

B.C.: $v_1'(L) = v_2'(L) \Rightarrow C_3 = C_1 = -\frac{1}{2}R_B L^2$

$v_1(L) = v_2(L) \Rightarrow C_4 = C_2 = \frac{1}{6}R_B L^3$

Compatibility condition:

Deflection at $x=L$ equals to the elongation of rod BH

$$\Rightarrow v(L) = \frac{-\frac{1}{2}M_0 L^2 - \frac{1}{3}R_B L^3}{EI} = \frac{R_B L}{EI} + \alpha\Delta T L$$

$$\Rightarrow R_B = -\frac{3}{8}\frac{M_0}{L} - \frac{3}{4}\alpha\Delta T EA$$

$$R_K = R_B$$

$$M_K = -R_B L - M_0 = -\frac{5}{8}M_0 + \frac{3}{4}\alpha\Delta T EAL$$

Deflection: $EI v_1(x) = \frac{1}{6}R_B(x-L)^3 - \frac{1}{2}M_0 x^2 - \frac{1}{2}R_B L^2 x + \frac{1}{6}R_B L^3$

$$EI v_2(x) = -\frac{1}{2}M_0 x^2 - \frac{1}{2}R_B L^2 x + \frac{1}{6}R_B L^3$$

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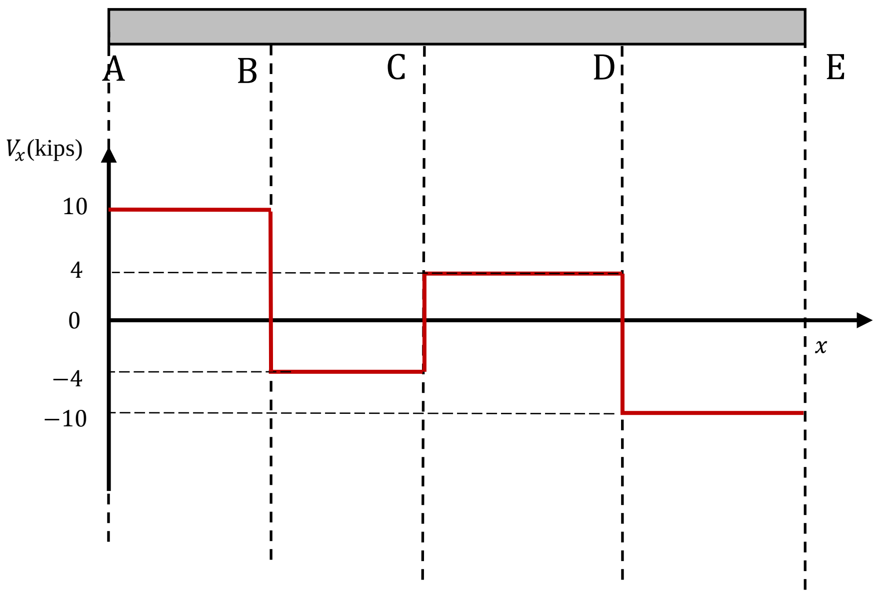
Problem 4 Solution:

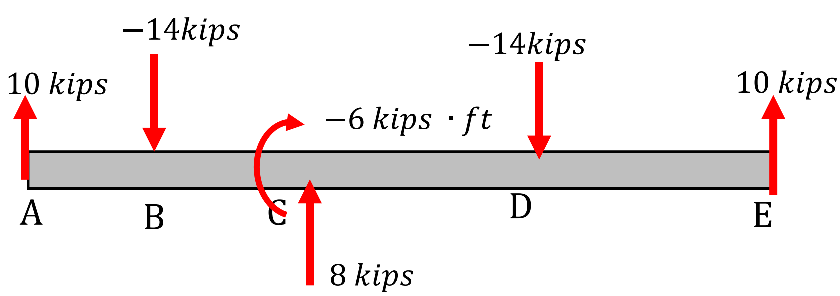
1.B

2.

a)



b)



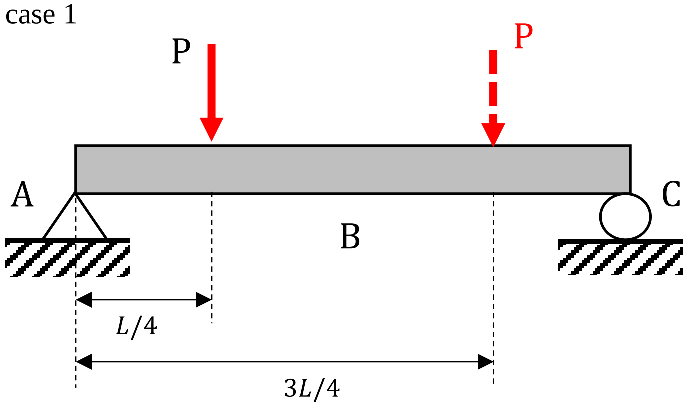
3.

D B

4.

Superposition Method:

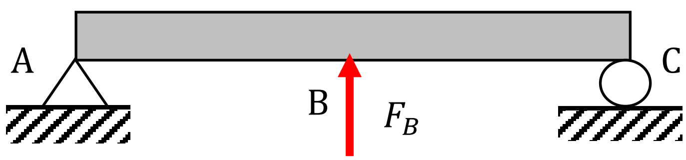
1) Loading case 1



Considering the symmetry, the Force P at $x = L/4$ (solid line) and P at $x = 3L/4$ (dashed line) will produce the same deflection for the roller B. Thus,

$$v_1\left(\frac{L}{2}\right) = \frac{-P \cdot \frac{L}{4} \cdot \frac{L}{2}}{6LEI}\left\{L^2 - \left(\frac{L}{4}\right)^2 - \left(\frac{L}{2}\right)^2\right\} = \frac{-PL^2}{48EI} \cdot \frac{11L^2}{16} \qquad \#(1)$$

2) Loading case 2



$$v_2\left(\frac{L}{2}\right) = \frac{F_B \cdot \frac{L}{2} \cdot \frac{L}{2}}{6LEI}\left\{L^2 - \left(\frac{L}{2}\right)^2 - \left(\frac{L}{2}\right)^2\right\} = \frac{F_B L^2}{24EI} \cdot \frac{L^2}{2} \qquad \#(2)$$

Compatibility condition:

$$v_1\left(\frac{L}{2}\right) + v_2\left(\frac{L}{2}\right) = 0 \quad \Rightarrow \quad F_B = \frac{11P}{16} \ (+)$$