(Last)
(First)

## ME 323 - Mechanics of Materials <br> Exam \# 2

Date: March 29, 2016 Time: 8:00 - 10:00 PM - Location: PHYS 114
Instructions:
Circle your lecturer's name and your class meeting time.

| Sadeghi | Gonzalez | Zhao |
| :--- | :--- | :--- |
| 8:30-9:20AM | $11: 30-12: 20 A M$ | $1: 30-2: 20$ PM |

Begin each problem in the space provided on the examination sheets. If additional space is required, use the yellow paper provided.

Work on one side of each sheet only, with only one problem on a sheet.
Please remember that for you to obtain maximum credit for a problem, you must present your solution clearly.
Accordingly,

- coordinate systems must be clearly identified,
- free body diagrams must be shown,
- units must be stated,
- write down clarifying remarks,
- state your assumptions, etc.

If your solution cannot be followed, it will be assumed that it is in error.
When handing in the test, make sure that ALL SHEETS are in the correct sequential order.
Remove the staple and restaple, if necessary.
Prob. 1 $\qquad$

Prob. 2 $\qquad$

Prob. 3 $\qquad$

Prob. 4 $\qquad$

Total $\qquad$

## Useful Equations

uniaxial tension, $\varepsilon_{\text {transverse }}=-\nu \varepsilon_{\text {longitudinal }}$
uniaxial tension, $\varepsilon_{y}=\varepsilon_{z}=-\nu \varepsilon_{x}$

$$
\begin{array}{ll}
\varepsilon_{x}=\frac{1}{E}\left[\sigma_{x}-v\left(\sigma_{y}+\sigma_{z}\right)\right]+\alpha \Delta T & \\
\varepsilon_{y}=\frac{1}{E}\left[\sigma_{y}-v\left(\sigma_{x}+\sigma_{z}\right)\right]+\alpha \Delta T & \\
\varepsilon_{z}=\frac{1}{E}\left[\sigma_{z}-v\left(\sigma_{x}+\sigma_{y}\right)\right]+\alpha \Delta T & e=\int_{0}^{L} \frac{F}{E A} d x+\int_{0}^{L} \alpha \Delta T d x \\
\gamma_{x y}=\frac{1}{G} \tau_{x y} \gamma_{x z}=\frac{1}{G} \tau_{x z} \gamma_{y z}=\frac{1}{G} \tau_{y z} & e=u \cos \theta+v \sin \theta \\
\varepsilon_{T}=\alpha \Delta T & e=\frac{F L}{E A}+L \alpha \Delta T \\
\varepsilon=\varepsilon_{\sigma}+\varepsilon_{T}=\frac{\sigma}{E}+\alpha \Delta T & \\
G=\frac{E}{2(1+v)} & \phi=\frac{T L}{G I_{p}} \\
\tau=\frac{T r}{I_{p}} & \\
\phi=\int_{0}^{L} \frac{T(x)}{G(x) I_{p}(x)} \mathrm{dx} &
\end{array}
$$

$I_{p}=\frac{\pi d^{4}}{32} \quad$ Polar moment of inertia for solid circular cross section
$I_{p}=\frac{\pi}{32}\left(d_{o}{ }^{4}-d_{i}{ }^{4}\right) \quad$ Polar moment of inertia for a circular hollow cross section
Factor of Safety $=\frac{\text { Yield Strength }}{\text { Allowable Stress }}$ or $\frac{\text { failure Stress }}{\text { Stress member }}$
$\frac{d V}{d x}=w(x) \quad \frac{d M}{d x}=V(x) \quad \Delta V=E I v^{\prime \prime} \quad \Delta M=-M_{0}$
$\sigma(x, y)=\frac{-E y}{\rho}=\frac{-M_{z z} y}{I_{z z}}$

$$
I_{z z}=\frac{b h^{3}}{12} \text { (rectangle), } I_{z z}=\frac{\pi r^{4}}{4} \text { (circle) }
$$

$\tau(x, y)=\frac{V Q}{I_{z z} t}=\frac{V A^{*} y^{*}}{I_{z z} t}$,

$$
\tau_{\max }=\frac{3 V}{2 A} \text { (rectangle), } \tau_{\max }=\frac{4 V}{3 A} \text { (circle) }
$$

$$
\text { Giga }=10^{9} \quad, \quad \text { Mega }=10^{6}
$$

Transformation of stress:

$$
\begin{gathered}
\sigma_{n}=\frac{\sigma_{x}+\sigma_{y}}{2}+\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right) \cos 2 \theta+\tau_{x y} \sin 2 \theta \quad \tau_{n t}=-\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right) \sin 2 \theta+\tau_{x y} \cos 2 \theta \\
\sigma_{1}=\sigma_{a v g}+R \quad \sigma_{2}=\sigma_{a v g}-R \quad \tan 2 \theta_{P}=\frac{2 \tau_{x y}}{\sigma_{x}-\sigma_{y}}, \sin 2 \theta_{P 1}=\frac{\tau_{x y}}{R}, \cos 2 \theta_{P 1}=\frac{\sigma_{x}-\sigma_{y}}{2 R} \\
\sigma_{a v g}=\frac{\sigma_{x}+\sigma_{y}}{2} \\
\sin 2 \theta_{P 2}=-\frac{\tau_{x y}}{R}, \cos 2 \theta_{P 2}=-\frac{\sigma_{x}-\sigma_{y}}{2 R} \\
\tau_{\text {max-inplane }}=\sqrt{\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right)^{2}+\tau_{x y}^{2}} \\
\frac{\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right)^{2}+\tau_{x y}^{2}}{\tan 2 \theta_{S}=-\frac{\sigma_{x}-\sigma_{y}}{2 \tau_{x y}}, \sin 2 \theta_{S 1}=-\frac{\sigma_{x}-\sigma_{y}}{2 R}, \cos 2 \theta_{S 1}=\frac{\tau_{x y}}{R}} \\
\sin 2 \theta_{S 2}=\frac{\sigma_{x}-\sigma_{y}}{2 R}, \cos 2 \theta_{S 2}=-\frac{\tau_{x y}}{R}
\end{gathered}
$$

ME 323 Examination \#2

March 29, 2016

## PROBLEM \#1 (28 points)

A beam with a uniform cross section is subjected to the loading shown below.

1. Draw the free body diagram of the beam on the next page.
2. Write the equilibrium equations by summing the forces and taking the moments at the left support ( $\mathrm{x}=0$ ).
3. Using discontinuity functions write the loading, shear force, bending moment, slope and deflection equations for the beam (i.e., do not determine or solve for the integration constants).
4. Write all boundary conditions for the beam.
5. Determine the reactions acting on the beam.

loading function $w(x)=$ $\qquad$

Shear function $V(x)=$

Moment function $M(x)=$ $\qquad$

Slope function $E I v^{\prime}(x)=$ $\qquad$

Deflection function $\operatorname{EIv}(x)=$
$\sum F_{y}=0=$
$\sum M_{A z z}=0=$

ME 323 Examination \#2

March 29, 2016

Name
(Print) (Last) (First)
Instructor $\qquad$

## PROBLEM \#2 (22 points)

The beam is subjected to the loading condition as shown below.

1. On the following blank page, draw the Free-Body Diagram of the beam and determine the reactions acting on the beam.
2. On the axes shown below, construct to scale plots of the internal shear force (V vs. x ), and the internal bending moment ( M vs. x ). Label all critical shear \& moment values and their respective units along the beam.


ME 323 Examination \#2

March 29, 2016

Name
(Print) (Last) (First)
Instructor
$\qquad$
$\qquad$

ME 323 Examination \#2

March 29, 2016

Name
(Print) (Last) (First)
Instructor $\qquad$

## PROBLEM \#3 (30 points)

The beam is subjected to the loading condition as shown below.

1. Determine the reactions forces acting on the beam.
2. Determine the maximum normal stress in the beam.
3. Determine the maximum shear stress in the beam.
4. Determine the normal and shear stresses at point A at $\mathrm{X}=\mathrm{L} / 4$ from the left end. Show the normal and shear stresses obtained for this point properly oriented on the differential element shown below.
5. Determine the principal stresses and absolute maximum shear stress for step 4.

Note: Point $A$ is located at $Y=1 \mathrm{in}$. and $Z=-0.5 \mathrm{in}$.


ME 323 Examination \#2

March 29, 2016

Name
(Print) (Last) (First)
Instructor
$\qquad$
$\qquad$

ME 323 Examination \#2

March 29, 2016

Name
(Print) (Last) (First)
Instructor $\qquad$

## PROBLEM 4 (20 Points):

PART A-6 points: The Mohr's circle shown in the figure corresponds to a three-dimensional state of stress with one principal stress equal to zero.


Circle the correct answer in the following two statements:

1) The normal stress acting on the planes of absolute maximum shear stress is $a$ :
(a) Compressive stress.
(b) Tensile stress.
2) The three principal stresses ( $\sigma_{1} \geq \sigma_{2} \geq \sigma_{3}$ ) act on planes with normal $p_{1}, p_{2}, p_{3}$ (also referred to as principal directions). For the $p_{2} p_{3}$ plane stress transformation, the normal stress acting on the planes of maximum shear stress is a:
(a) Compressive stress.
(b) Tensile stress.

PART B-6 points: A three-dimensional state of stress characterized by

$$
\tau_{x y}>0 \quad \sigma_{x}=\sigma_{y}=\sigma_{z}=\tau_{x z}=\tau_{y z}=0
$$

and $\sigma_{1} \geq \sigma_{2} \geq \sigma_{3}$. Circle the correct answer in the following statements:

| $\sigma_{1}=\tau_{x y} / 2$ | True | False |
| :---: | :---: | :--- |
| $\sigma_{2}=-\tau_{x y} / 2$ | True | False |
| $\sigma_{3}=-\tau_{x y}$ | True | False |
| $\sigma_{2}=0$ | True | False |
| $\sigma_{3}=0$ | True | False |
| $\tau_{\text {max }}^{\text {abs }}=\tau_{x y}$ | True | False |

## PART C - 8 points:



Beam (i) - Steel


Beam (ii) - Aluminum

Beams (i) and (ii) shown above are identical, except that beam (i) is made up of steel and beam (ii) is made up of aluminum. Note that $E_{\text {steel }} \geq E_{\text {aluminum }}$.
Let $|\sigma|_{\max ,(i)}$ and $|\sigma|_{\max ,(i i)}$ represent the maximum magnitude of flexural stress in beams (i) and (ii), respectively. Circle the correct relationship between these two stresses:
a) $|\sigma|_{\max ,(i)}>|\sigma|_{\max ,(i i)}$
b) $|\sigma|_{\max ,(i)}=|\sigma|_{\max ,(i i)}$
c) $|\sigma|_{\max ,(i)}<|\sigma|_{\max ,(i i)}$

Let $|\tau|_{\max ,(i)}$ and $|\tau|_{\max ,(i i)}$ represent the maximum magnitude of the xy-component of shear stress in beams (i) and (ii), respectively. Circle the correct relationship between these two stresses:
a) $|\tau|_{\max ,(i)}>|\tau|_{\max ,(i i)}$
b) $|\tau|_{\max ,(i)}=|\tau|_{\max ,(i i)}$
c) $|\tau|_{\max ,(i)}<|\tau|_{\max ,(i i)}$

Let $|\delta|_{\max ,(i)}$ and $|\delta|_{\max ,(i i)}$ represent the maximum magnitude of deflection in beams (i) and (ii), respectively. Circle the correct relationship between these two stresses:
a) $|\delta|_{\max ,(i)}>|\delta|_{\max ,(i i)}$
b) $|\delta|_{\max ,(i)}=|\delta|_{\max ,(i i)}$
c) $|\delta|_{\max ,(i)}<|\delta|_{\max ,(i i)}$

Let $\left|B_{y}\right|_{(i)}$ and $\left|B_{y}\right|_{(i i)}$ represent the vertical reaction at B in beams (i) and (ii), respectively. Circle the correct relationship between these two stresses:
a) $\left|B_{y}\right|_{(i)}>\left|B_{y}\right|_{(i i)}$
b) $\left|B_{y}\right|_{(i)}=\left|B_{y}\right|_{(i i)}$
c) $\left|B_{y}\right|_{(i)}<\left|B_{y}\right|_{(i i)}$

