Name (Print)____

(Last) (First)

ME 323 - Mechanics of Materials
Exam # 2
Date: November 6, 2019 Time: 8:00 – 10:00 PM

Instructions:

Circle your instructor's name and your class meeting time.

Gonzalez	Kokini	Zhao	Pribe
11:30-12:20PM	12:30-1:20PM	2:30-3:20PM	4:30-5:20PM

The only authorized exam calculator is the TI-30XIIS or the TI-30Xa.

Begin each problem in the space provided on the examination sheets.

Work on **ONE SIDE** of each sheet only, with only one problem on a sheet.

Please remember that for you to obtain maximum credit for a problem, you must present your solution clearly. Accordingly,

- coordinate systems must be clearly identified,
- free body diagrams must be shown,
- units must be stated,
- write down clarifying remarks,
- state your assumptions, etc.

If your solution cannot be followed, **it will be assumed that it is in error**.

When handing in the test, make sure that ALL SHEETS are in the correct sequential order.

Please review and sign the following statement:

Purdue Honor Pledge – "As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – We are Purdue."

Signature: ______________________________________

ME 323 Examination # 2 **Name __________________________________**
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<u>PROBLEM #1 (25 Points)</u>:

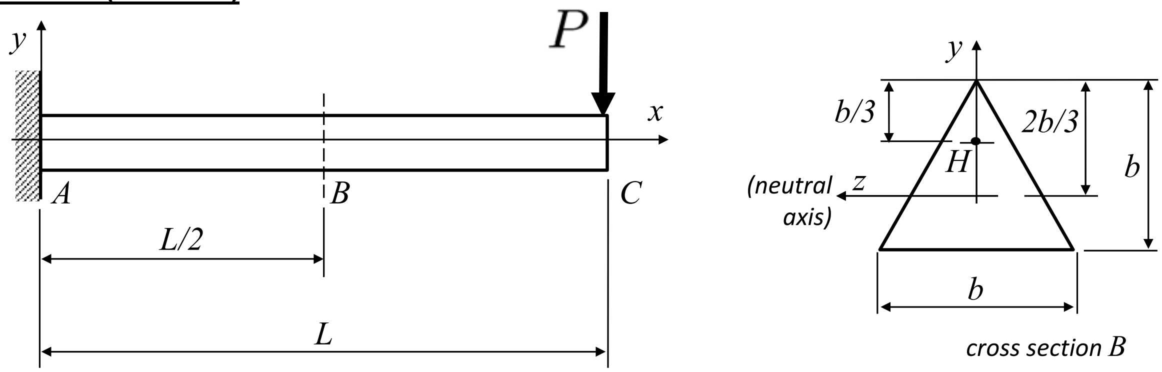


Figure 1

For the cantilever beam shown in the above figure:

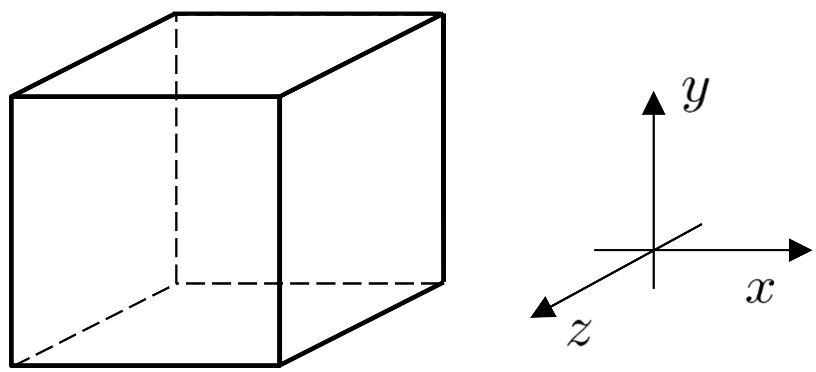
a) Determine the reactions at the wall.

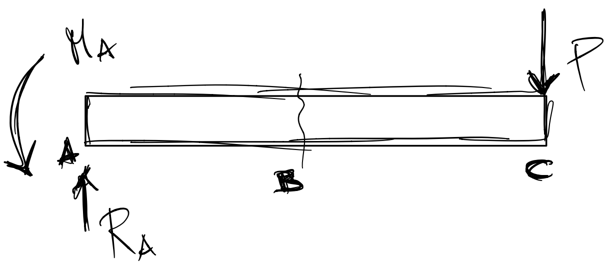
b) Determine the normal stress at point *H* of cross section *B*.

c) Determine the shear stress at point *H* of cross section *B*.

d) Show the state of stress at point *H* on the differential stress element shown below.

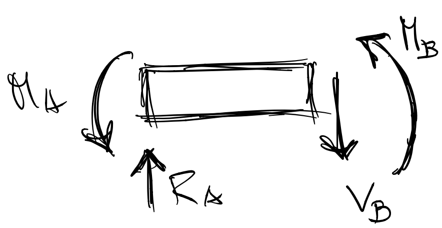
<u>Note</u>: $P = 25$ N, $L = 1$ m, $b = 0.03$ m, $I_{zz} = b^4/36$





$R_A = P = 25N$

$M_A = P \cdot L = 25Nm$

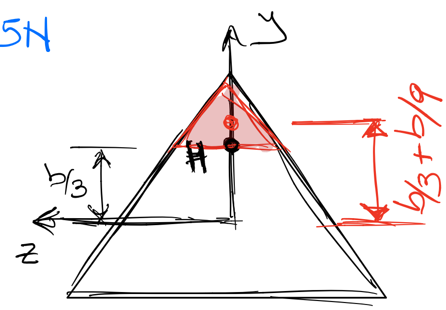


$$\left(\sum M\right)_B = 0 = M_A + M_B - R_A \cdot L/2$$

$$\Rightarrow M_B = PL/2 - PL = -PL/2$$

$$\sum F_y = 0 = R_A - V_B \Rightarrow V_B = P$$

$\Rightarrow$ @ B: $M_B = -PL/2 = -12.5\,Nm$ $\quad V_B = P = 25N$



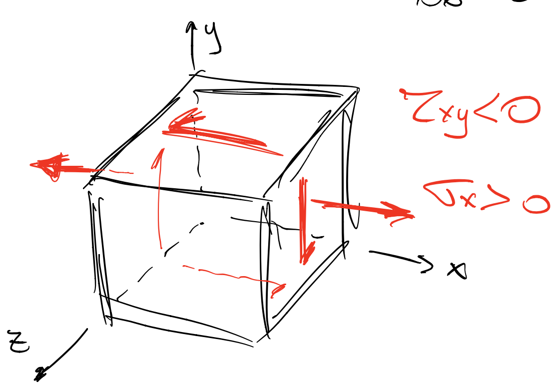
$$\sigma_x = -\frac{M_B\, y_H}{I_{zz}} = -\frac{(-PL/2)\, b/3}{b^4/36}$$

$$\sigma_x = \frac{6PL}{b^3} = 5.56\,MPa$$ Tension

$$\tau = \frac{V_B\, Q(y_H)}{I_{zz}\, t(y_H)}$$ with $t(y_H) = b/3$

$$Q(y_H) = A^* y^* = \frac{1}{2}\left(\frac{b}{3}\times\frac{b}{3}\right)\left(\frac{b}{3}+\frac{b}{9}\right) = \frac{b^2}{18}\times\frac{4b}{9} = \frac{2b^3}{81}$$

$$\Rightarrow \tau_{xy} = -\frac{P \cdot 2b^3/81}{b^4/36\;\, b/3} = -\frac{8P}{3b^2} = -74\,kPa$$



Name SOLUTION
(Print) **(Last)** **(First)**

PROBLEM #2 (25 points)

The beam BCD is fixed to the wall at B and supported by a roller at C. An external moment M_0 is applied at C. The beam has Young's modulus E and second moment of area I.

a) Draw a free-body diagram of the entire beam, and write down the equilibrium equations.

b) Use the second-order (or fourth-order) integration method to find the slope $v'(x)$ and deflection $v(x)$ of each segment of the beam. These can be left in terms of the unknown support reactions.

c) Write down the relevant boundary conditions and continuity conditions for the beam.

d) Use the boundary/continuity conditions to determine the reactions at B and C in terms of M_0 and L.

e) Determine the deflection at the free end D. Sketch the deflection curve over the length of the beam. The sketch does not need to be exact; show enough detail to indicate the boundary conditions.

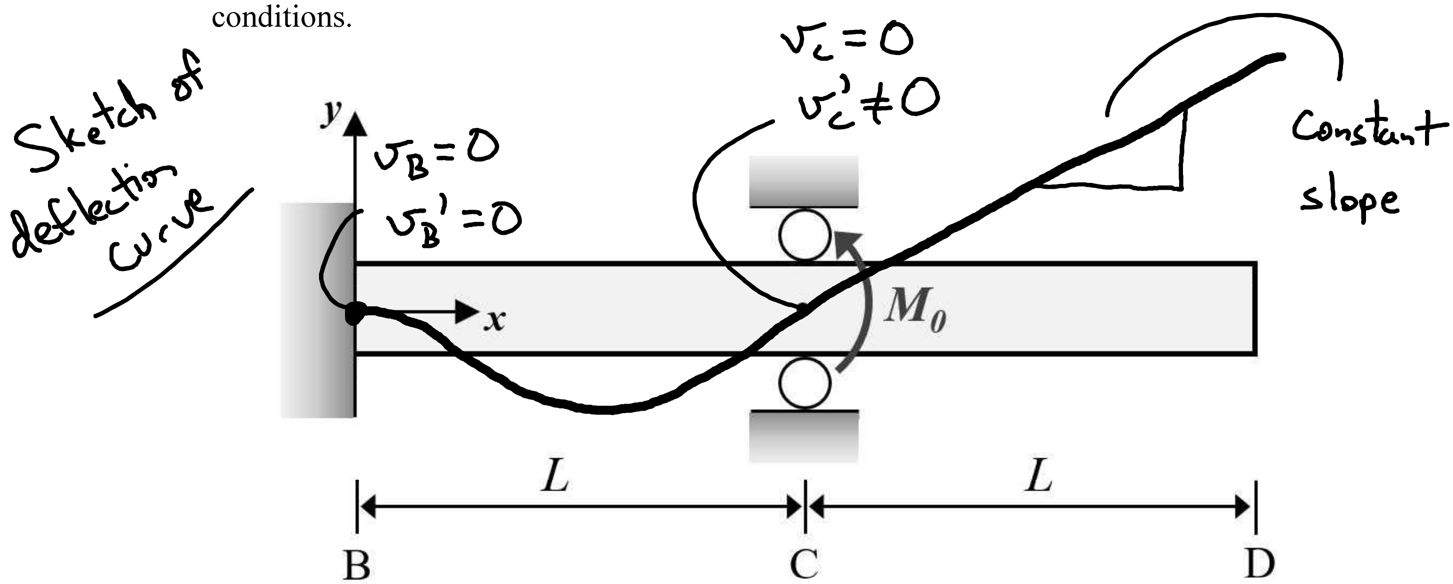
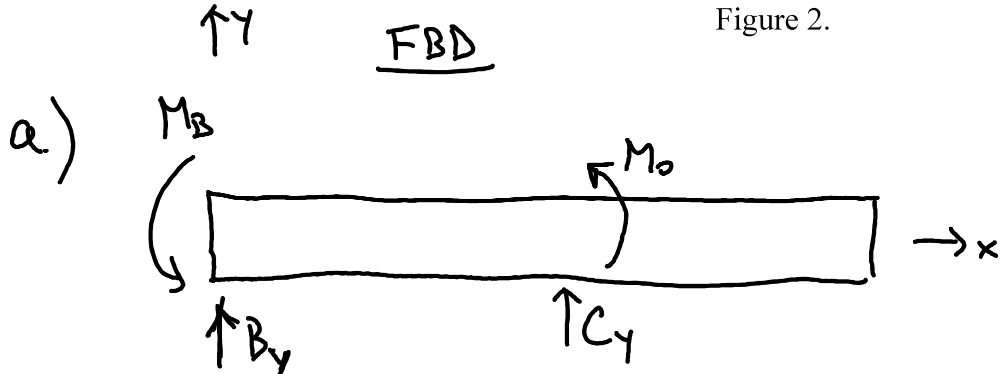


Figure 2.

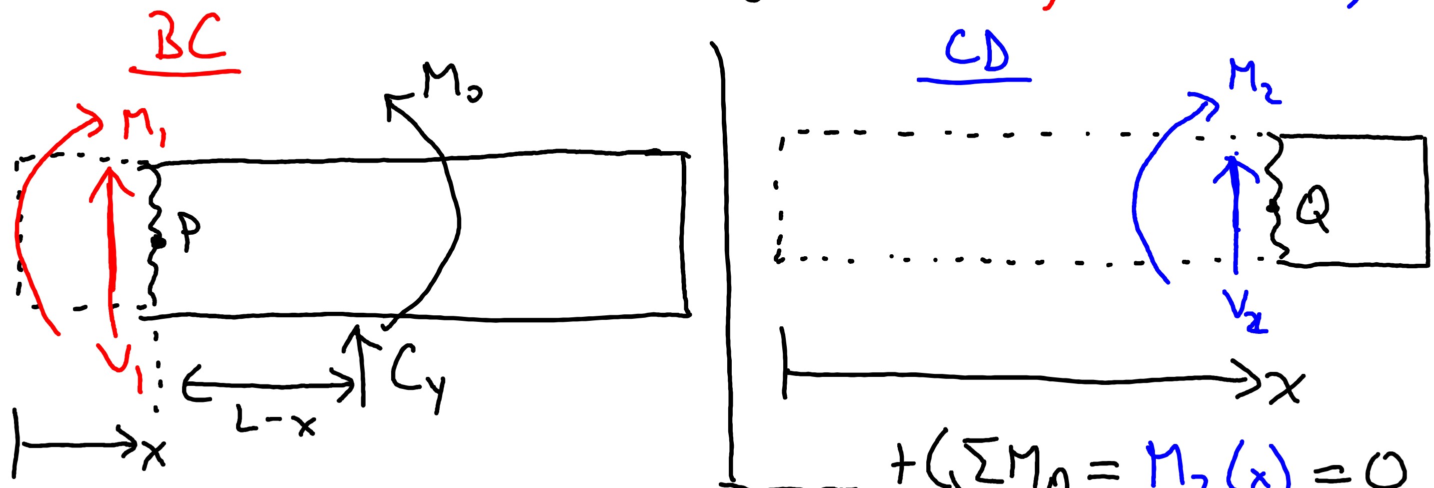


Equil.

$$+\circlearrowleft \Sigma M_B = M_B + M_0 + C_y L = 0 \quad (a)$$

$$+\uparrow \Sigma F_y = B_y + C_y = 0 \quad (b)$$

b.) Split the beam into 2 segments: BC (1) + CD (2)



$+\circlearrowleft \Sigma M_Q = M_2(x) = 0$

$\Rightarrow EI v_2''(x) = 0$

$$\boxed{\begin{aligned} EI v_2'(x) &= C_3 \\ EI v_2(x) &= C_3 x + C_4 \end{aligned}}$$

$+\circlearrowleft \Sigma M_P = -M_1(x) + M_o + C_y(L-x) = 0$

$\Rightarrow EI v_1''(x) = M_1(x) = -C_y x + C_y L + M_o$

$$\boxed{\begin{aligned} EI v_1'(x) &= -\tfrac{1}{2} C_y x^2 + C_y L x + M_o x + C_1 \\ EI v_1(x) &= -\tfrac{1}{6} C_y x^3 + \tfrac{1}{2} C_y L x^2 + \tfrac{1}{2} M_o x^2 + C_1 x + C_2 \end{aligned}}$$

NOTE: definite integrals, 4^{th}-order method, or calculating M_1 + M_2 using cuts that keep the left side of the beam is also acceptable

c.) Fixed at B

$\hookrightarrow$ $\boxed{\begin{aligned} v(0) &= v_1(0) = 0 \quad (c) \\ v'(0) &= v_1'(0) = 0 \quad (d) \end{aligned}}$

Continuity at C and roller support

$\hookrightarrow$ $\boxed{\begin{aligned} v_1(L) &= v_2(L) = 0 \quad (e) \\ v_1'(L) &= v_2'(L) \quad (f) \end{aligned}}$

d.) From eq. (c) + (d), $C_1 = C_2 = 0$

From eq. (e), $EIv_1(L) = 0 = \frac{-1}{6} C_y L^3 + \frac{1}{2} C_y L^3 + \frac{1}{2} M_0 L^2$

$$\Rightarrow \boxed{C_y = \frac{-3M_0}{2L}}$$

plug C_y into equilibrium (eq. (a) + (b)) $\longrightarrow$ $\boxed{B_y = \frac{3M_0}{2L} \quad M_B = \frac{1}{2} M_0}$

e.) Deflection at the free end D:

$v_D = v_2(2L) = \frac{1}{EI}\left(2C_3 L + C_4\right) \rightarrow$ Need C_3 + C_4

from eq. (f), $\cancel{EI} v_1'(L) = \cancel{EI} v_2'(L)$

$\Rightarrow \frac{-1}{2} C_y L^2 + C_y L^2 + M_0 L = C_3 \Rightarrow \underline{C_3 = \frac{-3M_0 L}{4} + M_0 L = \frac{M_0 L}{4}}$

from eq. (e), $EIv_2(L) = 0 = C_3 L + C_4$

$$\Rightarrow \underline{C_4 = \frac{-M_0 L^2}{4}}$$

$$\Rightarrow \boxed{v_D = v_2(2L) = \frac{1}{EI}\left(\frac{M_0 L^2}{2} - \frac{M_0 L^2}{4}\right) = \frac{M_0 L^2}{4EI}}$$

See above for sketch of deflection curve

Name SOLUTION
(Print) **(Last)** **(First)**

PROBLEM #3 (25 points)

A cantilevered beam BCDH is subjected to a vertical load P at the point C and an equal horizontal load P at the point D. The beam is made of a material with elastic modulus E, second moment of area I and cross-sectional area A. Assuming **the shear strain energy due to bending is negligible**, use Castigliano's theorem to determine:

a) the vertical (y-direction) deflection of point H
b) the horizontal (x-direction) deflection of point H

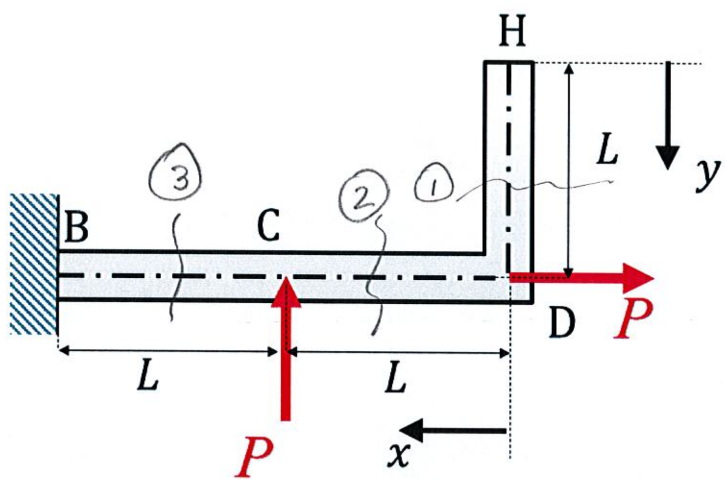
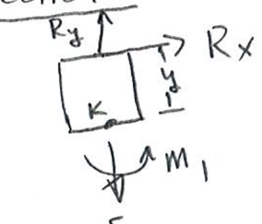


Figure 3

Shear strain energy is negligible

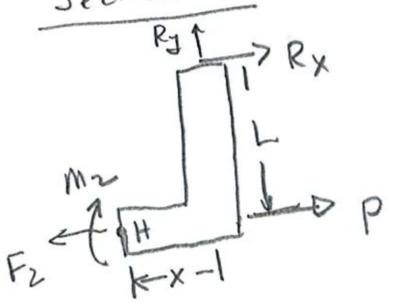
Section 1



$\sum M_K = M_1 - R_x y = 0$ $\boxed{M_1 = R_x y}$

$\sum F_y = R_y - F_1 = 0$ $\boxed{F_1 = R_y}$

Section 2



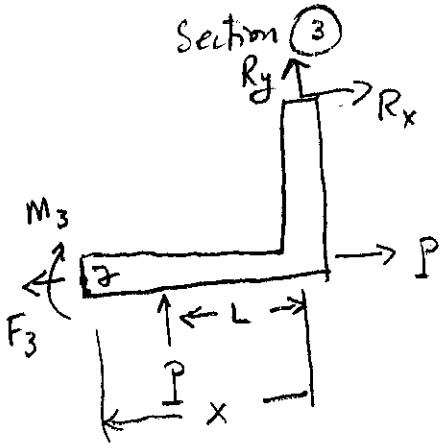
$\sum M_H = -M_2 - R_x L + R_y x = 0$

$\boxed{M_2 = R_y x - R_x L}$

$\sum F_x = -F_2 + R_x + P = 0$

$\boxed{F_2 = P + R_x}$

Name SOLUTION
(Print) (Last) (First)



$$\sum M_z = -M_3 + P(x-L) + R_y x - R_x L = 0$$

$$\boxed{M_3 = P(x-L) + R_y x - R_x L}$$

$$\sum F_x = P - F_3 + R_x = 0$$

$$\boxed{F_3 = P + R_x}$$

(a) δH_y

$$\delta H_y = \left.\frac{\partial U}{\partial R_y}\right|_{R_y=0} = \frac{1}{EI}\int_0^L M_1 \frac{\partial M_1}{\partial R_y}\,dy + \frac{1}{EA}\int_0^L F_1 \frac{\partial F_1}{\partial R_y}\,dy$$

$$+ \frac{1}{EI}\int_0^L M_2 \frac{\partial M_2}{\partial R_y}\,dx + \frac{1}{EA}\int_0^L F_2 \frac{\partial F_2}{\partial R_y}\,dx$$

$$+ \frac{1}{EI}\int_L^{2L} M_3 \frac{\partial M_3}{\partial R_y}\,dx + \frac{1}{EA}\int_L^{2L} F_3 \frac{\partial F_3}{\partial R_y}\,dx$$

$$\frac{\partial M_1}{\partial R_y} = 0 \quad \frac{\partial F_1}{\partial R_y} = 1 \quad \frac{\partial M_2}{\partial R_y} = x \quad \frac{\partial F_2}{\partial R_y} = 0 \quad \frac{\partial M_3}{\partial R_y} = x \quad \frac{\partial F_3}{\partial R_y} = 0$$

$$F_1 = 0 \qquad M_2 = 0$$

$$\delta H_y = \int_L^{2L} \left[P(x-L)\right] x\,dx = \frac{P}{EI}\left[\frac{x^3}{3} - L\frac{x^2}{2}\right]_L^{2L}$$

$$= \frac{P}{EI}\left[\frac{8L^3 - L^3}{3} - \frac{4L^3 - L^3}{2}\right] = \frac{7L^3}{3} - \frac{3L^3}{2}$$

$$\boxed{\delta H_y = \frac{5PL^3}{6EI}}$$

$$\delta H_y = \frac{\partial U}{\partial R_x} = \frac{1}{EI}\int_0^L m_1 \frac{\partial m_1}{\partial R_x}\,dy + \frac{1}{EA}\int_0^L F_1 \frac{\partial F_1}{\partial R_x}\,dy$$

$$+ \frac{1}{EI}\int_0^L m_2 \frac{\partial m_2}{\partial R_x}\,dx + \frac{1}{EA}\int_0^L F_2 \frac{\partial F_2}{\partial R_x}\,dx$$

$$+ \frac{1}{EI}\int_L^{2L} m_3 \frac{\partial m_3}{\partial R_x}\,dx + \frac{1}{EA}\int_L^{2L} F_3 \frac{\partial F_3}{\partial R_x}\,dx$$

$$m_1\Big|_{R_x=0}=0 \quad \frac{\partial m_1}{\partial R_x} = y \quad F_1\Big|_{R_y=0}=0 \quad \frac{\partial F_1}{\partial R_x}=0 \quad m_2\Big|_{R_x,R_y=0}=0 \quad \frac{\partial m_2}{\partial R_x}=-L \quad \frac{\partial m_3}{\partial R_x}=-L \quad \frac{\partial F_3}{\partial R_x}=1$$

$$\delta H_y = \frac{1}{EA}\int_0^L (P+\cancel{R_x}^{\,0})(1)\,dx + \frac{1}{EA}\int_L^{2L}(P+\cancel{R_x}^{\,0})(1)\,dx$$

$$+ \frac{1}{EI}\int_L^{2L}\left[P(x-L) + \cancel{R_y}^{\,0}x - \cancel{R_x}^{\,0}L\right](-L)\,dx$$

$$= \frac{PL}{EA} + \frac{P}{EA}(2L-L) + \underbrace{\frac{P}{EI}\int_L^{2L}(-xL+L^2)\,dx}_{\underbrace{\frac{PL}{2EI}\left[-x^2\Big|_L^{2L} + 2Lx\Big|_L^{2L}\right]}_{-\frac{PL^3}{2EI}}}$$

$$\boxed{\delta H_y = \frac{2PL}{EA} - \frac{PL^3}{2EI}}$$

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PROBLEM #4 (25 Points):

PART A – 4 points

Figure 4A shows a beam that is subjected to point load at multiple locations. The beam has a T-shaped cross section as shown in Figure 4B. Circle the correct answer for the following questions:

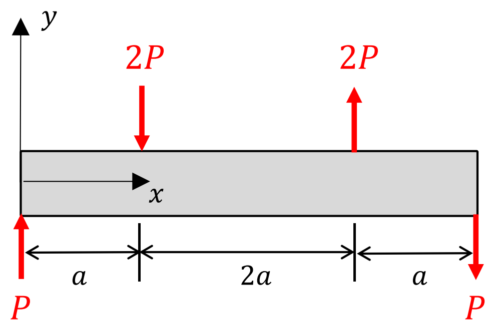


Figure 4A

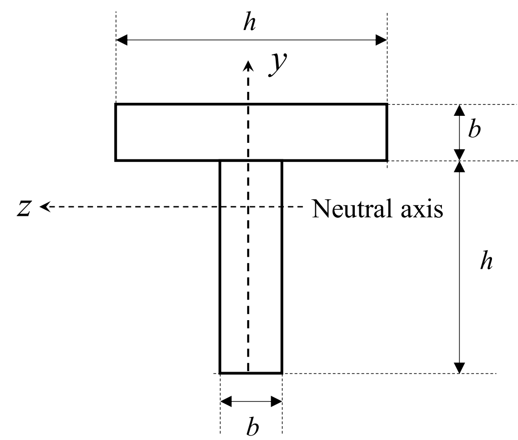
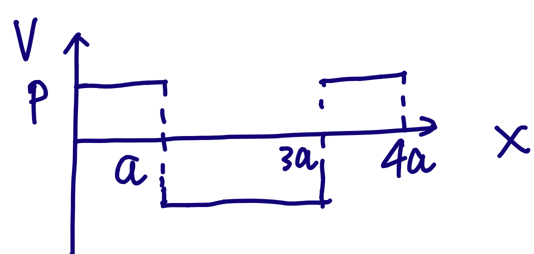


Figure 4B

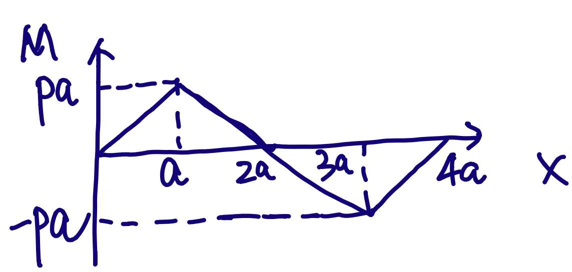
a) On which cross section the maximum tensile stress is attained?
(1) $x = 0$
(2) $x = a$
(3) $x = 2a$
(4) $x = 3a$
(5) $x = 4a$

b) On which cross section the maximum compressive stress is attained?
(1) $x = 0$
(2) $x = a$
(3) $x = 2a$
(4) $x = 3a$
(5) $x = 4a$

shear force diagram:



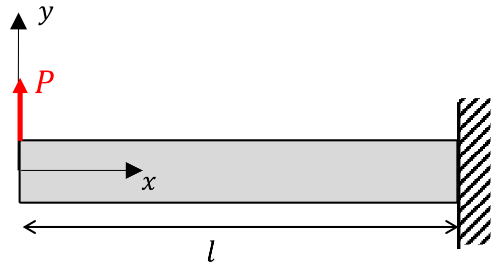
Internal moment diagram:



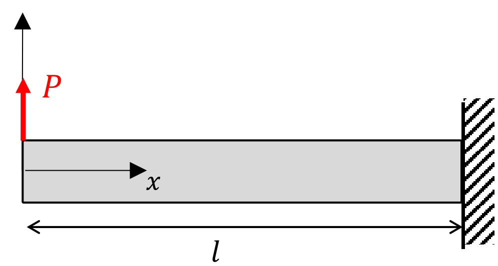
<u>PROBLEM #4 (cont.)</u>:

PART B – 9 points

Beam (i) and (ii) shown below are identical, except that beam (i) is made of steel, and beam (ii) is made of aluminum. Note that $E_{\text{steel}} > E_{\text{aluminum}}$.



Beam (i) – steel



Beam (ii) – aluminum

a) **TRUE** or **FALSE**: The two beams have the same second moment of area.

b) **TRUE** or **FALSE**: The two beams have the same magnitude of the maximum normal stress.

c) **TRUE** or **FALSE**: The two beams have the same magnitude of the maximum shear stress.

d) **TRUE** or **FALSE**: The two beams have the same magnitude of the maximum deflection.

e) Let v_{max} be the maximum deflection in beam (i). If the length of beam (i) increases from its original value l to a new value $2l$, and the same load is applied at the free end. The new value of the maximum deflection becomes v^*_{max}. Circle the correct answer:

(1) $v^*_{max} = v_{max}$.

(2) $v^*_{max} = 2v_{max}$.

(3) $v^*_{max} = 4v_{max}$.

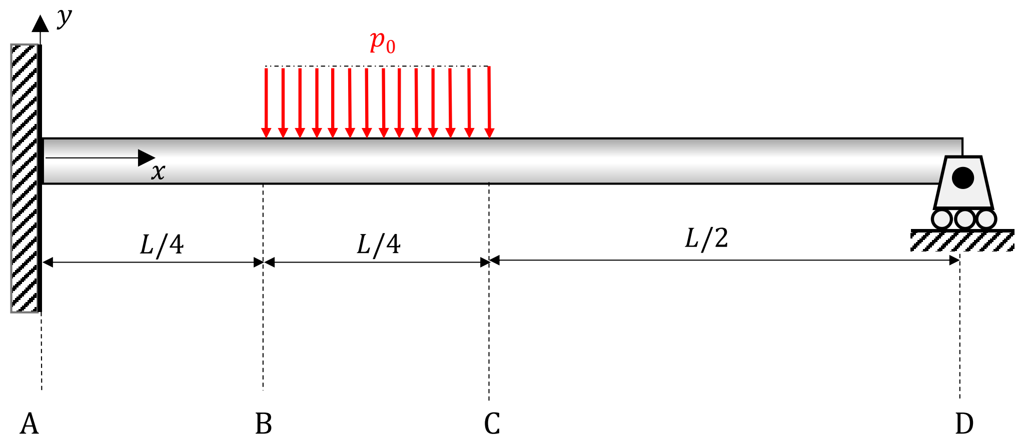
(4) $v^*_{max} = 8v_{max}$.

(5) $v^*_{max} = 16v_{max}$.

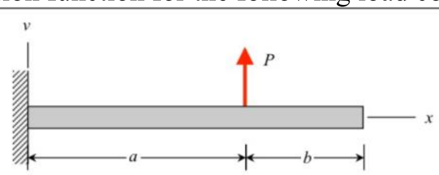
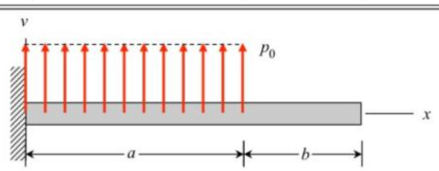
<u>PROBLEM #4 (cont.)</u>:

PART C – 6 points

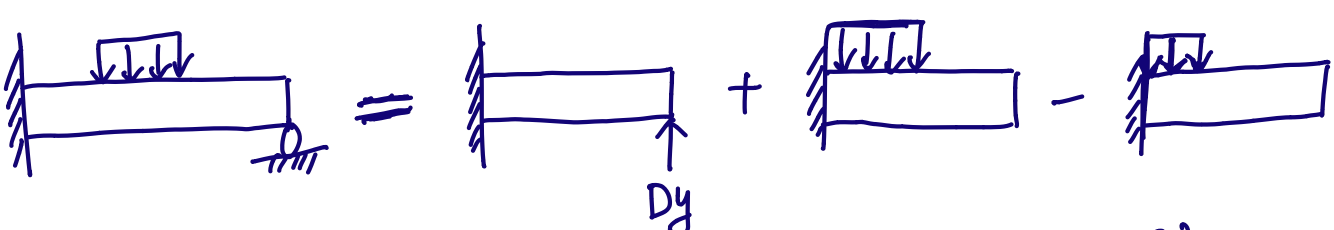
A cantilever beam shown below is supported by a roller at the end D and is subject to a distributed load of the magnitude p_0 through the section BC. Use the superposition method to determine the reaction force at the roller D. $L = 4\text{m}$, $p_0 = 4\text{kN/m}$.



The deflection function for the following load configuration is given:

	$v(x) = \frac{1}{6}\left[x^2(3a-x)\right]\frac{P}{EI}$; $0<x<a$ $= \frac{1}{6}\left[a^2(3x-a)\right]\frac{P}{EI}$; $a<x<L$
	$v(x) = \frac{x^2}{24}\left[6a^2-4ax+x^2\right]\frac{p_0}{EI}$; $0<x<a$ $= \frac{a^3}{24}\left[4x-a\right]\frac{p_0}{EI}$; $a<x<L$

superposition:



$v_{total} = v_1 + v_2 - v_3$

For the deflection at D:

$v_1 = \frac{1}{6} \times \left[4^2 \cdot (3\times 4-4)\right] \cdot \frac{D_y}{EI} = \frac{64}{3}\frac{D_y}{EI}$

$$v_2 = -\frac{2^3}{24}\left[4\times 4-2\right]\cdot\frac{4}{EI} = -\frac{56}{3}\frac{1}{EI}$$

$$v_3 = -\frac{1}{24}\left[4\times 4-1\right]\cdot\frac{4}{EI} = -\frac{5}{2}\cdot\frac{1}{EI}$$

$$v_{total}\Big|_D = v_1 + v_2 - v_3$$

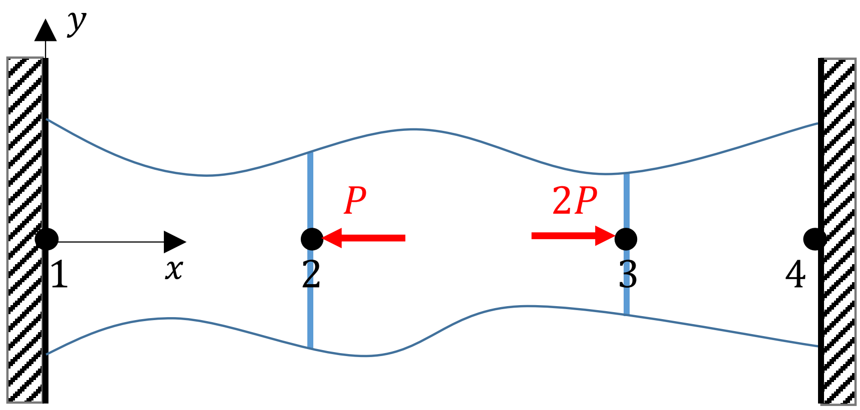
$$= \frac{64}{3}\frac{D_y}{EI} - \frac{56}{3}\frac{1}{EI} + \frac{5}{2}\cdot\frac{1}{EI} = 0$$

$$D_y = \frac{97}{128} \text{ KN}$$

PROBLEM #4 (cont.):

PART D – 6 points

The rod shown below has a variable cross section. A finite element model comprising of 3 elements and 4 nodes are shown on the figure. Two concentrated forces P and $2P$ are applied at the nodes 2 and 3, respectively. The concentrated force P = 1000 lb.



The finite element model has the global stiffness matrix as follows:

$k_1 = 1$

$k_2 + k_3 = 5$

$k_3 = 3, \quad k_2 = 2$

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -2 & 5 & -3 \\ 0 & 0 & -3 & 3 \end{bmatrix} \times 10^5 \ lb/in$$

a) Determine the remaining 13 matrix elements and fill the blank spaces.
b) Determine the displacement at nodes 2 and 3.
c) Determine the reaction force due to the wall at the node 4.

b): Enforce fixed-displacement B.C.

$$\begin{bmatrix} 3 & -2 \\ -2 & 5 \end{bmatrix} \times 10^5 \cdot \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} -P \\ 2P \end{Bmatrix} = \begin{Bmatrix} -1000 \\ 2000 \end{Bmatrix} lb$$

$3u_2 - 2u_3 = -1 \times 10^{-2}$ ①

$-2u_2 + 5u_3 = 2 \times 10^{-2}$ ②

① $\times 5$ + ② $\times 2$:

$$u_2 = -\frac{1}{11} \times 10^{-2} \text{ in}$$

① × 2 + ② × 3:

$$u_3 = \frac{4}{11} \times 10^{-2} \text{ in}$$

c):

$$F_3 = k_3 \cdot (u_4 - u_3)$$

$$= 3 \times 10^5 \cdot \left(-\frac{4}{11} \times 10^{-2}\right)$$

$$= -\frac{12}{11} \times 10^3 \text{ lb}$$

$$F_{wall} = F_3 = -\frac{12}{11} \times 10^3 \text{ lb}$$