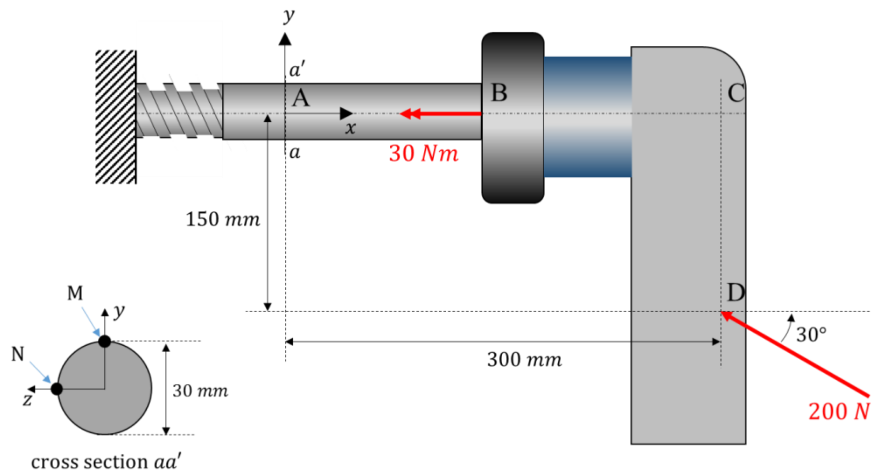
Problem 1 (25 points): A drill jammed in the wall is acted upon by a point load at D, and a torque at B, as shown in the figure below. For the given state of loading,

a) Determine the stress state at the point M on the cross section *aa*', and represent the stress state on an appropriate stress element.

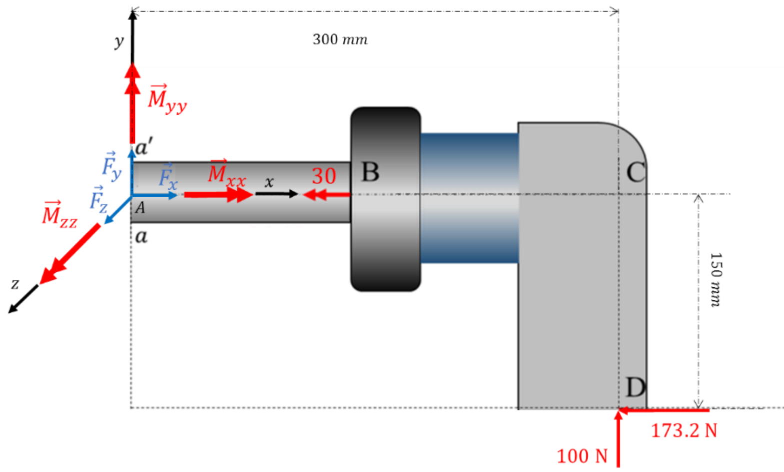
b) Determine the stress state at the point N on the cross section *aa*', and represent the stress state on an appropriate stress element.

c) Using a Mohr's circle, determine the absolute maximum shear stress $\tau_{max,abs}$ for the points M and N.



Problem Solution

Making a cut at aa' and drawing the FBD,



Force balance gives:

$$\Sigma\overrightarrow{F_A} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k} - 173.2\,\hat{\imath} + 100\,\hat{\jmath} = 0$$

$$\Rightarrow F_x = 173.2\,N \quad F_y = -100\,N \quad F_z = 0\,N$$

Moment balance gives:

$$\Sigma\overrightarrow{M_A} = M_{xx}\hat{\imath} + M_{yy}\hat{\jmath} + M_{zz}\hat{k} - 30\hat{\imath} + (0.30\,\hat{\imath} - 0.15\,\hat{\jmath})\text{x}\,(-173.2\,\hat{\imath} + 100\,\hat{\jmath}) = 0$$

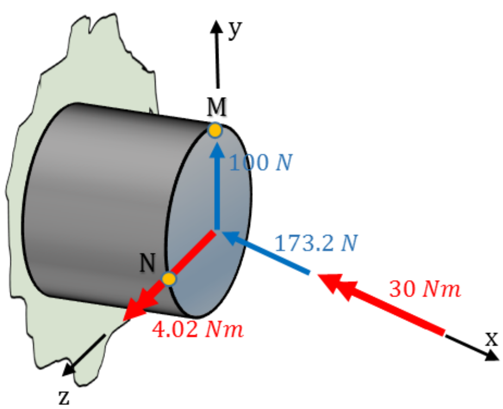
$$\Rightarrow M_{xx} = 30\,Nm \quad M_{yy} = 0\,Nm \quad M_{zz} = -4.02\,Nm$$

For the cross section,

$$A = \frac{\pi(0.030^2)}{4} = 0.00071\,m^2 \quad , I_{zz} = I_{yy} = \frac{\pi(0.030^4)}{64} = 3.975x10^{-8}\ m^4$$

$$I_p(\text{or } J) = \frac{\pi(0.030^4)}{32} = 7.95x10^{-8}\,m^4$$

State of stress at aa' represented on the positive face



State of Stress at M

$$\sigma_x = \frac{M_{zz}y}{I_{zz}} + \frac{F_x}{A} = \frac{(4.02)(0.015)}{3.975x10^{-8}} + \frac{173.2}{0.00071}$$

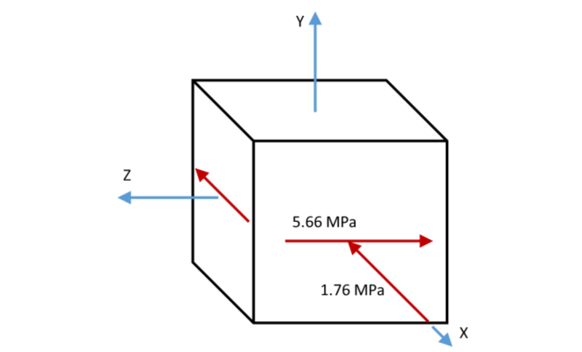
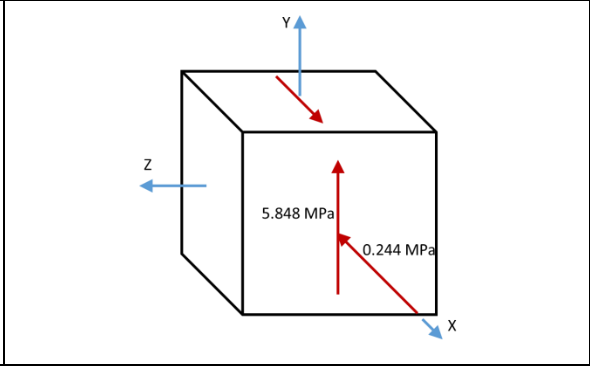
$$\sigma_x = 1.516\,MPa + 0.244\,MPa = 1.76\,MPa\,(C)$$

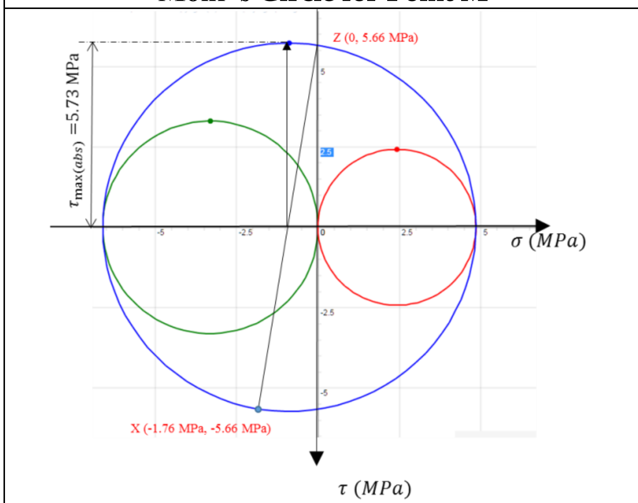
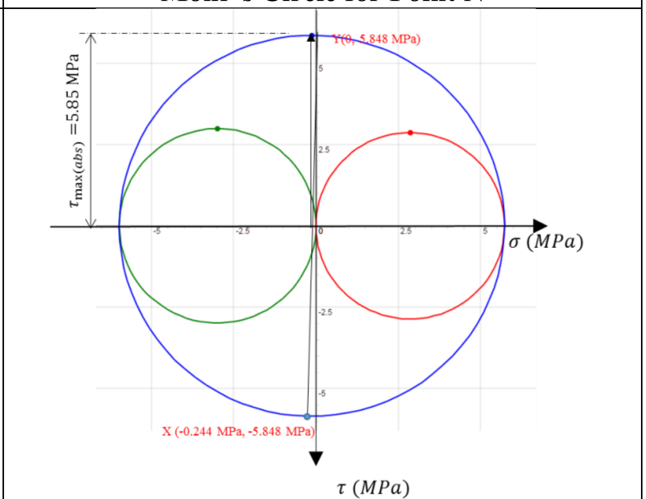
$$\tau_{xz} = \frac{M_{xx}r}{I_p} = \frac{(30)(0.015)}{7.95x10^{-8}} = 5.66x10^6\,MPa \;\rightarrow$$

State of Stress at N

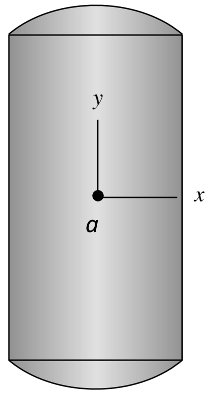
$$\sigma_x = \frac{F_x}{A} = -\frac{173.2}{0.00071} = \ 0.244\ MPa\ (C)$$

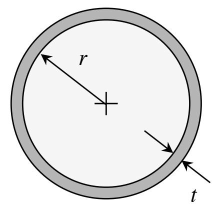
$$\tau_{xy} = \frac{M_{xx}r}{I_p} + \frac{F_zQ}{It} = \frac{(30)(0.015)}{7.95x10^{-8}} + \frac{4(100)}{3(0.00071)} = 5.66\ MPa + 0.188\ MPa = 5.848\ MPa\ \uparrow$$

<table>
<tr><th>Stress Element at M</th><th>Stress Element at N</th></tr>
<tr><td>
</td><td>
</td></tr>
</table>

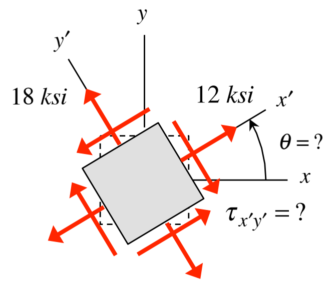
<table>
<tr><th>Mohr's Circle for Point M</th><th>Mohr's Circle for Point N</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$\tau_{max\ (abs)}$ = 5.73 MPa</td><td>$\tau_{max\ (abs)}$ = 5.85 MPa</td></tr>
</table>

Problem 2 (25 points) **SOLUTION**





tank cross section

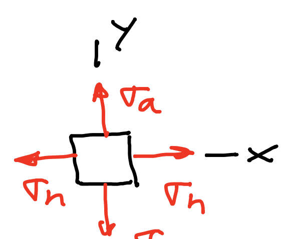


stress state at point a

The closed, thin-walled tank shown above has an inner radius of r and wall thickness t and contains a gas of pressure of p. The state of stress at point "a" on the tank is represented by stress components shown above for an unknown stress element rotation angle of θ. Note that $\tau_{x'y'}$ is in the direction shown; however, its magnitude is also unknown. For this problem, you are asked for the following (in no particular order):

- Determine the principal components of stress, σ_1 and σ_2.
- Determine the magnitude of $\tau_{x'y'}$.
- Draw the Mohr's circle for this state of stress. Show the location of the $x'-axis$ in your Mohr's circle. From this, determine the rotation angle θ.
- Determine the <u>*absolute maximum shear stress*</u> for this state of stress.

$$\sigma_{ave} = \frac{\sigma_{x'} + \sigma_{y'}}{2} = \frac{12+18}{2} = 15\text{ ksi}$$



Since $\left.\begin{array}{l}\sigma_1 = \sigma_h = 2\sigma_a \\ \sigma_2 = \sigma_a\end{array}\right\} \Rightarrow \sigma_1 = 2\sigma_2$

and: $\left.\begin{array}{l}\sigma_1 = \sigma_{ave} + R \\ \sigma_2 = \sigma_{ave} - R\end{array}\right\} \Rightarrow \sigma_{ave} + R = 2(\sigma_{ave} - R)$

↳ $R = \frac{1}{3}\sigma_{ave} = 5\text{ ksi}$

$\therefore \left\{\begin{array}{l}\sigma_1 = 15+5 = 20\text{ ksi} \\ \sigma_2 = 15-5 = 10\text{ ksi}\end{array}\right.$

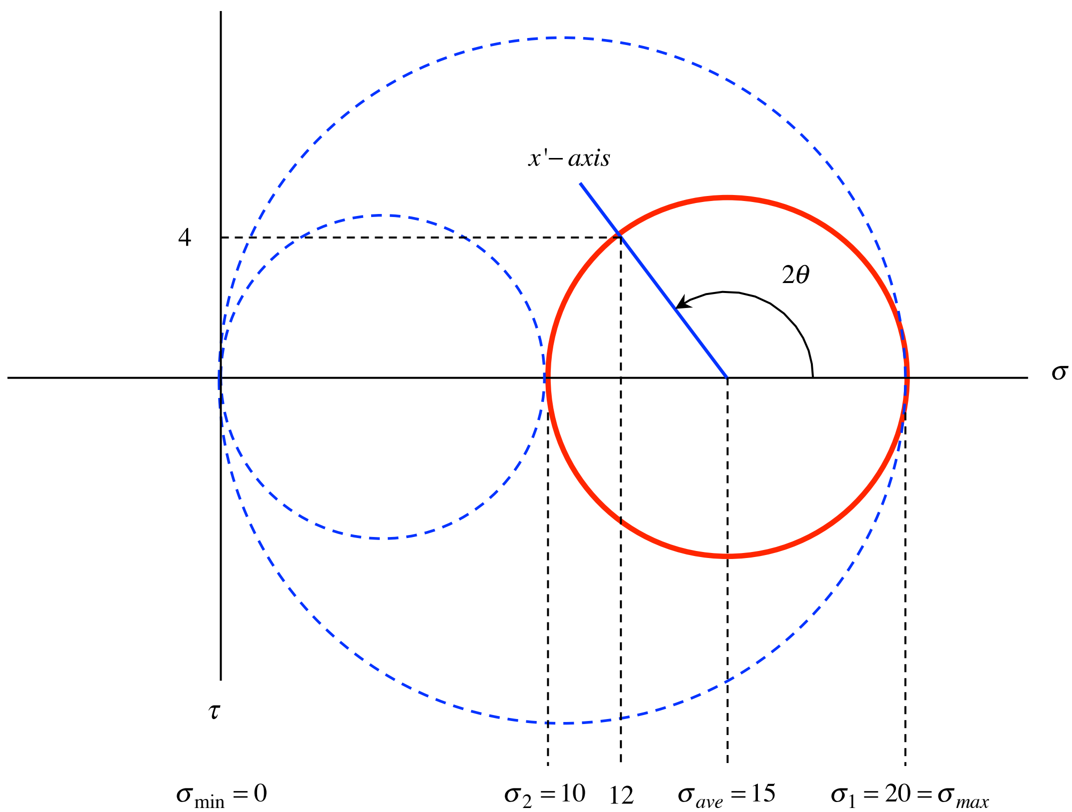
<u>Also</u>: $R^2 = \left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)^2 + \tau_{x'y'}^2$

↳ $\tau_{x'y'} = \sqrt{R^2 - \left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)^2} = \sqrt{5^2 - \left(\frac{12-18}{2}\right)^2} = 4\text{ ksi}$

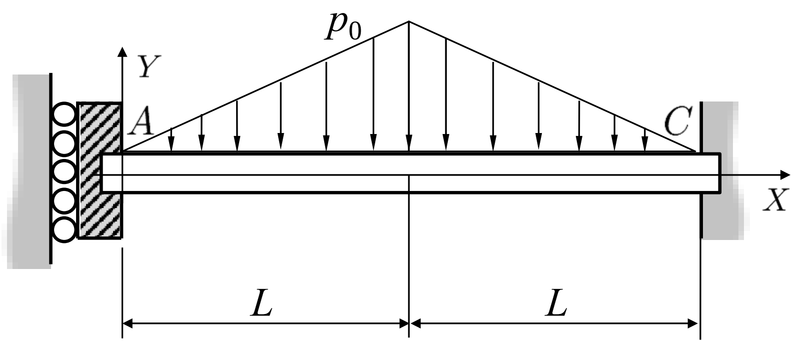
$$2\theta = 180^\circ - tan^{-1}\left(\frac{4}{15-12}\right) = 126.9^\circ \quad \Rightarrow \quad \theta = 63.4^\circ$$

Also from Mohr's circle:

$$\left|\tau\right|_{\max,abs} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{20}{2} = 10\ ksi$$



Problem 3 (25 Points):



The beam AC of constant flexural rigidity *EI* and length 2*L* shown in the figure is subjected to a triangular distributed load with a maximum intensity p_0 (load/length). The beam is fixed at *C* and supported by vertical rollers at *A*. Notice that <u>support *A* can have a vertical displacement</u> but it cannot rotate, therefore <u>the slope of the beam at *A* is zero</u>.

a) Draw the free body diagram for the beam and write down the equilibrium equations.

b) Justify whether the beam is statically indeterminate or not.

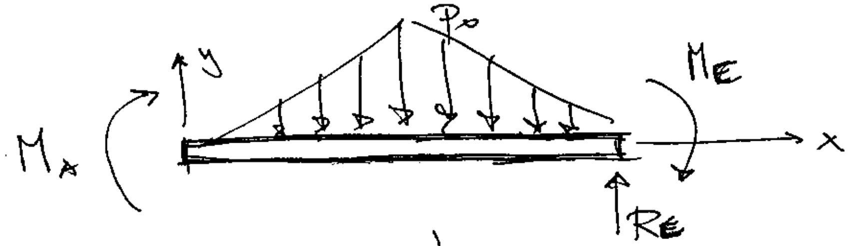
c) Write down the geometric boundary conditions at supports *A* and *C*.

d) Write down the load function *p*(*x*) using discontinuity functions.

e) Determine the deflection function for the beam using the discontinuity method. Express your answer in terms of *EI*, *L*, and p_0 and indicate which variables are unknown.

f) Following the suggested order, find:
   1. The reaction(s) at the support *A*;
   2. The deflection at *A*;
   3. The reaction(s) at the support *E*.

g) Sketch the deflection curve.



$v'_A = 0 \qquad v_E = 0 \qquad v'_E = 0$

Indeterminate

$$\oplus\uparrow\ \sum F = 0 = R_E - p_o L \Rightarrow \boxed{R_E = p_o L}$$

$$\oplus\curvearrowright\ \left(\sum M\right)_E = 0 = M_A + M_E - (p_o L)L$$

$$p(x) = M_A\langle x\rangle^{-2} + R_E\langle x-2L\rangle^{-1} + M_E\langle x-2L\rangle^{-2} + \frac{p_o}{L}\left[-\langle x\rangle^{1} + 2\langle x-L\rangle^{1}\right]$$

$$EI\, v^{IV} = p(x)$$

$$EI\, v''' = M_A\langle x\rangle^{-1} + R_E\langle x-2L\rangle^{0} + M_E\langle x-2L\rangle^{-1} + \frac{p_o}{L}\left[-\frac{1}{2}\langle x\rangle^{2} + \langle x-L\rangle^{2}\right]$$

$$EI\, v'' = M_A\langle x\rangle^{0} + R_E\langle x-2L\rangle^{1} + M_E\langle x-2L\rangle^{0} + \frac{p_o}{L}\left[-\frac{1}{6}\langle x\rangle^{3} + \frac{1}{3}\langle x-L\rangle^{3}\right]$$

$$EI\, v' = M_A\langle x\rangle^{1} + \frac{R_E}{2}\langle x-2L\rangle^{2} + M_E\langle x-2L\rangle^{1} + \frac{p_o}{L}\left[-\frac{1}{24}\langle x\rangle^{4} + \frac{1}{12}\langle x-L\rangle^{4}\right] + EI\, v'_A \quad (v'_A = 0)$$

$$EI\, v = \frac{M_A}{2}\langle x\rangle^{2} + \frac{R_E}{6}\langle x-2L\rangle^{3} + \frac{M_E}{2}\langle x-2L\rangle^{2} + \frac{p_o}{L}\left[-\frac{1}{120}\langle x\rangle^{5} + \frac{1}{60}(x-L)^{5}\right] + EI\, v_A$$

Unknowns $\{v_A, M_A, R_E, M_E\}$

$$\circledast\ v'_E = v'(2L) = M_A\, 2L + \frac{p_o}{L}\left[-\frac{1}{24}(2L)^4 + \frac{1}{12}L^4\right] = 0 \Rightarrow \boxed{M_A = \frac{7}{24}p_o L^2}$$

$$\circledast\ v_E = v(2L) = 0 = \frac{M_A}{2}(2L)^2 + \frac{p_o}{L}\left[-\frac{1}{120}(2L)^5 + \frac{1}{60}L^5\right] + EI\, v_A \Rightarrow \boxed{v_A = -\frac{1}{3}\frac{p_o L^4}{EI}}$$

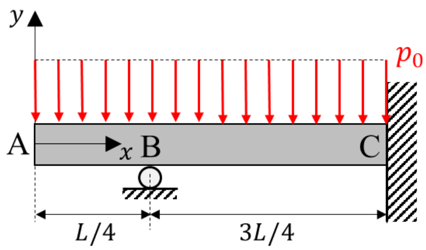
$$\circledast\ M_A + M_E - (p_o L)L = 0 \Rightarrow \boxed{M_E = \frac{17}{24}p_o L^2}$$

Problem 4.1 (3 points): The loading function and boundary condition for a beam is given below. Mark the figure below that this loading function describes.

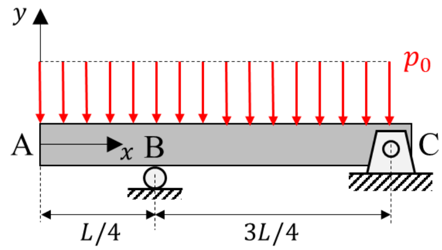
$$p(x) = -\frac{p_0}{L}\langle x\rangle^1 + B_y\left\langle x - \frac{L}{4}\right\rangle^{-1} + C_y\langle x - L\rangle^{-1} + M_C\langle x - L\rangle^{-2}$$

Boundary conditions: $x = \frac{L}{4}:\ v = 0 \qquad x = L:\ v = 0,\ v' = 0$

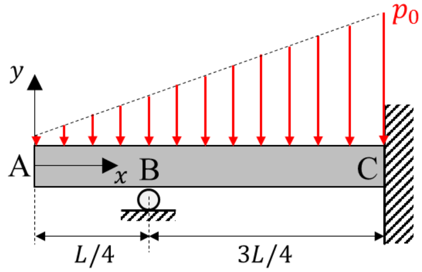
a)



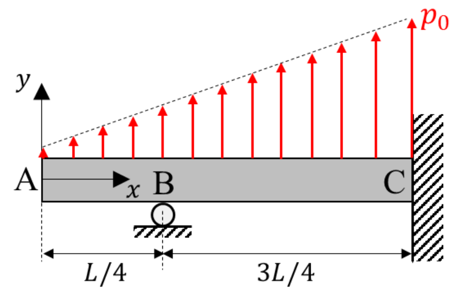
b)



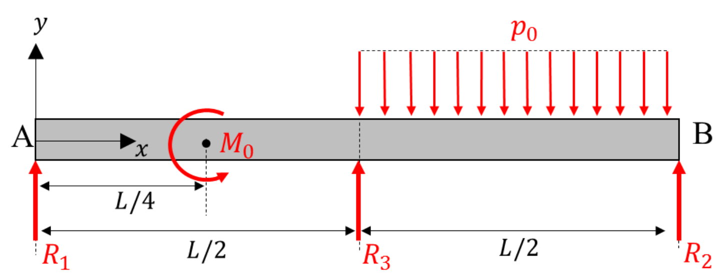
c)



d)



Problem 4.2 (3 points): Beam AB is subjected to the various loads as shown below. Which loading function describes the beam?



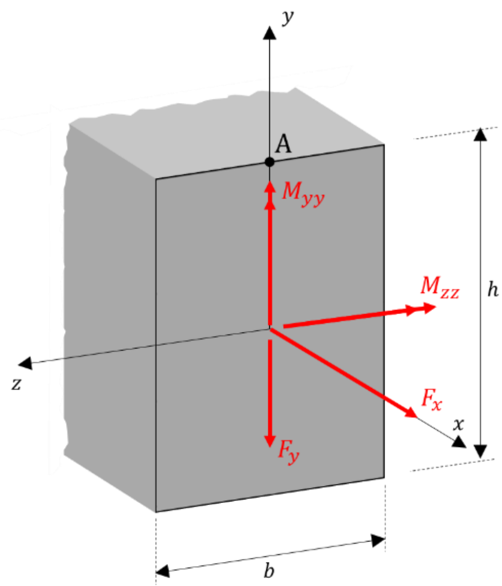
1) $p(x) = R_1\langle x\rangle^{-1} + M_0\left\langle x - \frac{L}{4}\right\rangle^{-2} + R_3\left\langle x - \frac{L}{2}\right\rangle^{-1} - p_0\left\langle x - \frac{L}{2}\right\rangle^{0} - R_2\langle x - L\rangle^{-1}$

2) $p(x) = R_1\langle x\rangle^{-1} - M_0\left\langle x - \frac{L}{4}\right\rangle^{-2} + R_3\left\langle x - \frac{L}{2}\right\rangle^{-1} - p_0\left\langle x - \frac{L}{2}\right\rangle^{0} + R_2\langle x - L\rangle^{-1}$

3) $p(x) = R_1\langle x\rangle^{0} - M_0\left\langle x - \frac{L}{4}\right\rangle^{-2} + R_3\left\langle x - \frac{L}{2}\right\rangle^{0} - p_0\left\langle x - \frac{L}{2}\right\rangle^{1} + R_2\langle x - L\rangle^{-1}$

4) $p(x) = R_1\langle x\rangle^{-1} - M_0\left\langle x - \frac{L}{4}\right\rangle^{-2} + R_3\left\langle x - \frac{L}{2}\right\rangle^{-1} - p_0\left\langle x - \frac{L}{2}\right\rangle^{1} - R_2\langle x - L\rangle^{-1}$

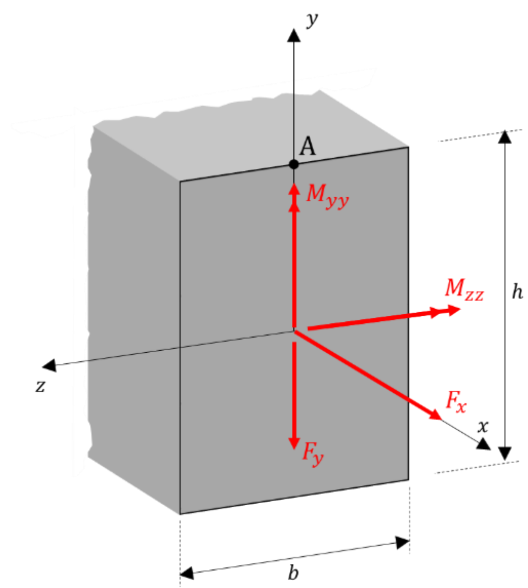
Problem 4.3 (10 points): A rectangular member is subject to a combination of loads. A cut is made somewhere along the beam and perpendicular to the x axis. At the cut surface, the loading condition is as shown below:



a) The normal stress at A is:

1.	$\sigma_x = \dfrac{M_{zz}\left(\frac{h}{2}\right)}{(bh^3/12)} + \dfrac{F_x}{bh}$	2.	$\sigma_x = \dfrac{M_{yy}\left(\frac{h}{2}\right)}{(bh^3/12)} + \dfrac{F_x}{bh}$
3.	$\sigma_x = -\dfrac{M_{zz}\left(\frac{b}{2}\right)}{(bh^3/12)} + \dfrac{F_x}{bh}$	4.	$\sigma_x = \dfrac{M_{yy}\left(\frac{b}{2}\right)}{(bh^3/12)} + \dfrac{F_x}{bh}$
5.	None of the above		

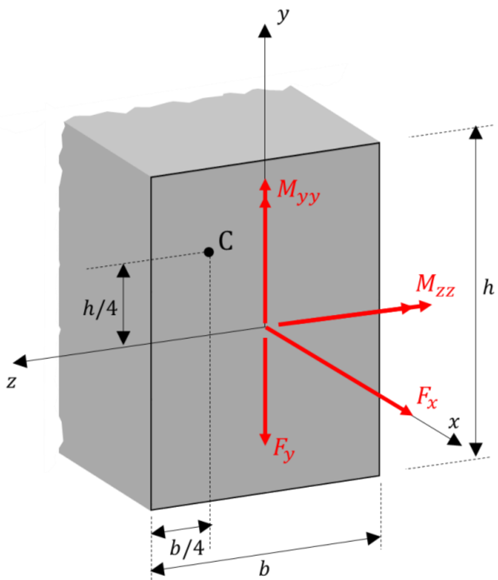
Problem 4.3 (continued):



b) The shear stress at A is:

1.	$\tau_{xy} = 0$	2.	$\tau_{xy} = \frac{F_y \left[\frac{bh}{2}\right]\left[\frac{b}{2}\right]}{(bh^3/12)t}$
3.	$\tau_{xy} = \frac{F_y}{bh}$	4.	$\tau_{xy} = \frac{M_{zz} \left[\frac{bh}{2}\right]\left[\frac{b}{2}\right]}{(bh^3/12)t}$
5.	None of the above		

Problem 4.3 (continued): A rectangular member is subject to a combination of loads. A cut is made somewhere along the beam and perpendicular to the x axis. At the cut surface, the loading condition is as shown below:

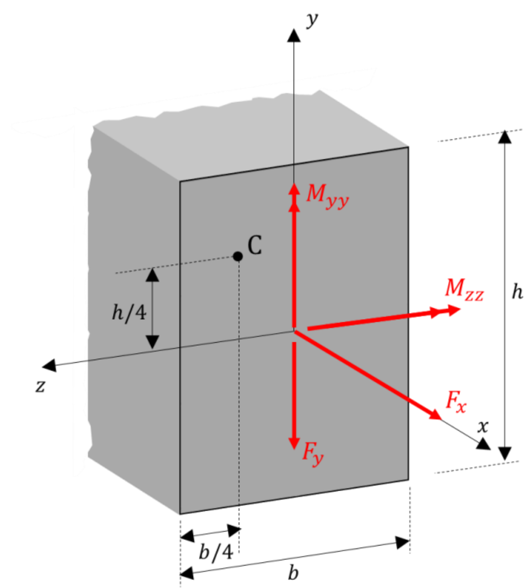


c) The normal stress at C is:

1. $\sigma_x = \dfrac{M_{zz}\left(\frac{h}{4}\right)}{(bh^3/12)} + + \dfrac{M_{yy}\left(\frac{b}{4}\right)}{bh^3/12} + \dfrac{F_x}{bh}$	2. $\sigma_x = -\dfrac{M_{zz}\left(\frac{h}{4}\right)}{(bh^3/12)} + \dfrac{M_{yy}\left(\frac{b}{4}\right)}{(bh^3/12)} + \dfrac{F_x}{bh}$
3. $\sigma_x = \dfrac{M_{zz}\left(\frac{h}{4}\right)}{(bh^3/12)} - \dfrac{M_{yy}\left(\frac{b}{4}\right)}{(bh^3/12)} + \dfrac{F_x}{bh}$	4. $\sigma_x = -\dfrac{M_{zz}\left(\frac{h}{4}\right)}{(bh^3/12)} - \dfrac{M_{yy}\left(\frac{b}{4}\right)}{(bh^3/12)} + \dfrac{F_x}{bh}$
5. None of the above	

$$\sigma_x = \frac{M_{zz}\left(\frac{h}{4}\right)}{(bh^3/12)} - \frac{M_{yy}\left(\frac{b}{4}\right)}{(hb^3/12)} + \frac{F_x}{bh}$$

Problem 4.3 (continued):



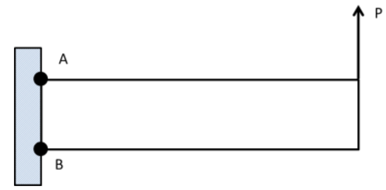
d) The shear stress at C is:

1.	$\tau_{xy} = -\dfrac{F_y\left[\frac{bh}{4}\right]\left[\frac{h}{4}+\frac{h}{8}\right]}{(bh^3/12)b}$	2.	$\tau_{xy} = \dfrac{F_y\left[\frac{bh}{4}\right]\left[\frac{b}{4}+\frac{b}{8}\right]}{(bh^3/12)b}$
3.	$\tau_{xy} = -\dfrac{F_y\left[\frac{bh}{4}\right]\left[\frac{h}{4}+\frac{h}{8}\right]}{(bh^3/12)h}$	4.	$\tau_{xy} = \dfrac{F_y\left[\frac{bh}{4}\right]\left[\frac{b}{4}+\frac{b}{8}\right]}{(bh^3/12)h}$
5.	None of the above		

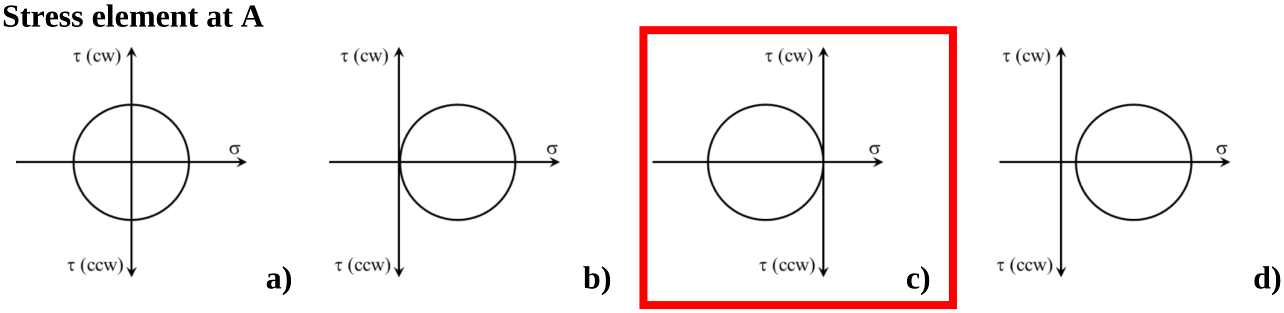
Problem 4.4 (3 points)

1. **Mohr' s circle is a graphical method to find**
   a) Bending stresses
   b) Principal stresses
   c) Torsional shear stresses
   d) None of the above
2. **Circle all of the loading conditions below that can lead to having the Mohr's circle being centered at the origin (more than one item can be circled):**
   a) uni-axial loading
   b) equal bi-axial loading
   c) pure torsion
   d) combined pure torsion and uni-axial loading
   e) combined flexural and shear stresses
3. **The abscissa (horizontal x-direction) of the Mohr's circle is a**
   (a) Shear stress
   (b) Normal stress
   (c) Normal as well as shear stress
   (d) None of the above
4. **The ordinate (vertical y-direction) of the Mohr's circle is a**
   (a) Shear stress
   (b) Normal stress
   (c) Normal as well as shear stress
   (d) None of the above
5. **In the Mohr's circle, the planes of maximum normal and shear stresses are:**
   (a) 45 degrees apart
   (b) 30 degrees apart
   (c) 90 degrees apart
   (d) None of the above
6. **In the Mohr's circle, the scales of ordinate and abscissa have to be:**
   (a) Ordinate scale has to be twice the abscissa
   (b) Abscissa scale has to be twice the ordinate
   (c) Abscissa scale has to be the same as the ordinate
   (d) None of the above

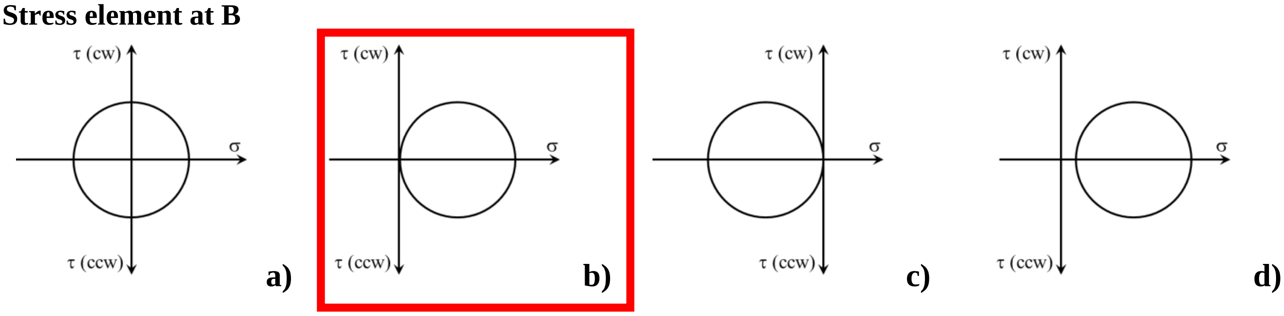
<u>Problem 4.5 (4 Points):</u> The cantilever beam is loaded as shown below. Which one of the Mohr's circle shown here represents the stress element at point A and B?



Stress element at A



Stress element at B



<u>Problem 4.6 (2 Points)</u>: The beam is loaded as shown below. Which one of the Mohr's circle shown here represents the stress element at point A?