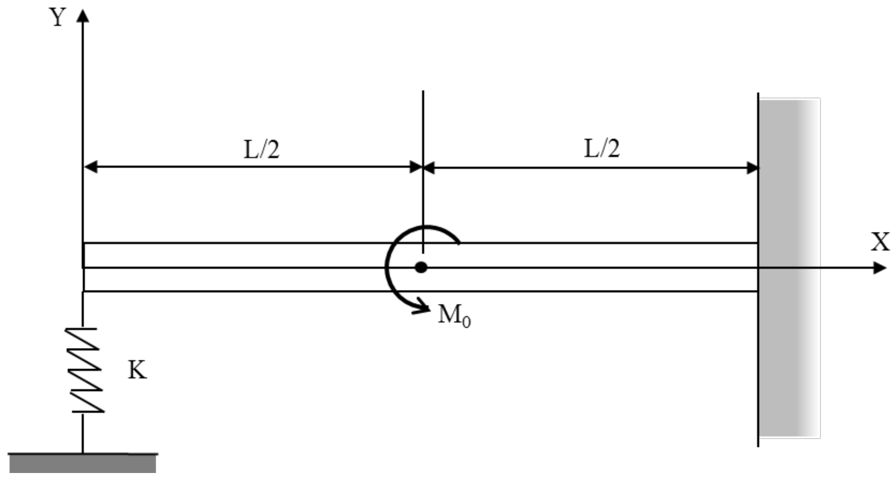
ME 323 Examination #2 | **Name ______________________________**
(Print) (Last) (First)

November 7, 2013 | **Instructor ___________________________**

PROBLEM #1 (15 points)

The cantilever beam shown below is subject to a couple at mid-span and supported by a spring at the left end. Using superposition method (tables shown on page 3), determine the force in the spring; leave your answer in terms of E, I, M_o, L, and K. Spring constant K is known.



Deflection of the left end due to the force applied by the spring

$$\delta_{fs_{left\,end}} = \frac{F_s L^3}{3EI}$$

Deflection of the left end due to couple applied at mid-span

$$\delta_{M_{o\,left\,end}} = -\frac{M_o \frac{L}{2}}{2EI}\left(2L - \frac{L}{2}\right) = -\frac{3M_o L^2}{8EI}$$

Deflection of the left end at the spring

$$\delta_{left\,end} = -\frac{F_s}{K}$$

Therefore,

$$-\frac{F_s}{K} = \frac{F_s L^3}{3EI} - \frac{3M_o L^2}{8EI}$$

Solving for F_s, we obtain,

$$F_s = \frac{\frac{3M_o L^2}{8EI}}{\frac{L^3}{3EI} + \frac{1}{K}}$$

Method II – using singularity function

$$p(x) = F_s < x >^{-1} - M_o < x - \frac{L}{2} >^{-2}$$

$$V(x) = F_s < x >^{0} - M_o < x - \frac{L}{2} >^{-1}$$

$$M(x) = F_s < x >^{1} - M_o < x - \frac{L}{2} >^{0}$$

$$EIv^{'}(x) = \frac{F_s}{2} < x >^{2} - M_o < x - \frac{L}{2} >^{1} + C_1$$

$$EIv(x) = \frac{F_s}{6} < x >^{3} - \frac{M_o}{2} < x - \frac{L}{2} >^{2} + C_1 x + C_2$$

$$v^{'}(x = L) = 0 \rightarrow \frac{F_s}{2}(L)^2 - M_o(\frac{L}{2}) + C_1 = 0 \rightarrow C_1 = -\frac{F_s L^2}{2} + \frac{M_o L}{2}$$

$$v(x = L) = 0 \rightarrow \frac{F_s L^3}{6} - \frac{M_o}{2}\frac{L}{4}^{2} + C_1 L + C_2 = 0$$

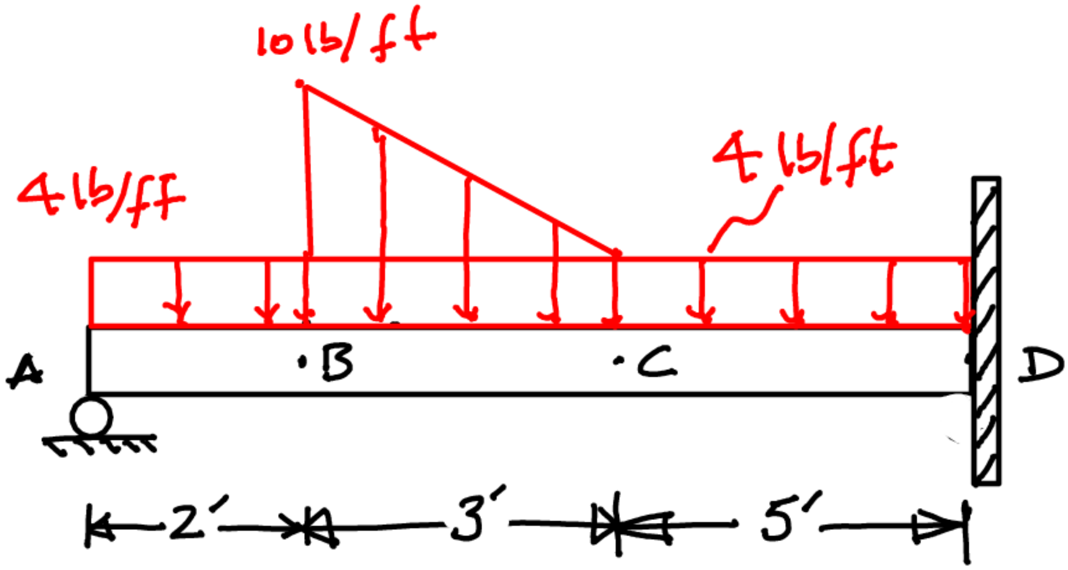
Substituting for C_1 and solving for C_2, we obtain;

$$C_2 = \frac{F_s L^3}{3} - \frac{3M_o L^2}{8}$$

$$EIv(x = 0) = \frac{F_s L^3}{3} - \frac{3M_o L^2}{8} \rightarrow v(x = 0) = \frac{F_s L^3}{3EI} - \frac{3M_o L^2}{8EI} = -\frac{F_s}{K}$$

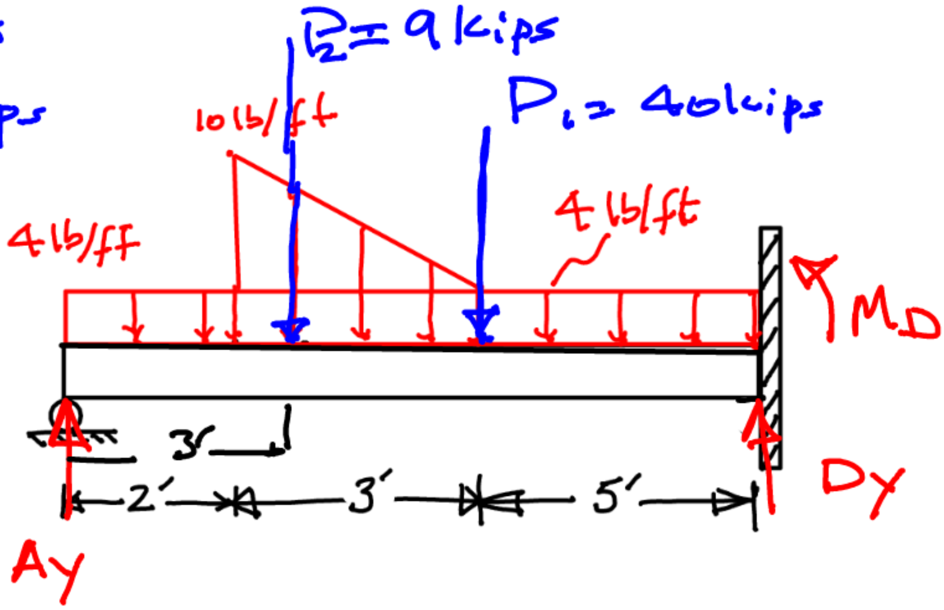
Solving for spring force, we have,

$$F_s = \frac{\frac{3M_o L^2}{8EI}}{\frac{L^3}{3EI} + \frac{1}{K}}$$



$P_1 = 4 \times 10 = 40 \text{ kips}$

$P_2 = \frac{6 \times 3}{2} = 9 \text{ kips}$



$\Sigma F_y = 0 \Rightarrow \quad A_y - 9 - 40 + D_y = 0$

$$\boxed{A_y + D_y = 49.} \longrightarrow (1)$$

$\Sigma M_D = 0 \Rightarrow$

$$-A_y(10) + 9(7) + 40 \times 5 + M_D = 0$$

$$\boxed{10A_y - M_D = 263} \longrightarrow (2)$$

$$P(x) = A_y\langle x-0\rangle^{-1} - 4\langle x-0\rangle^{0} + 4\langle x-10\rangle^{0}$$
$$-6\langle x-2\rangle^{0} + \left(-\frac{6}{3}\right)(-1)\langle x-2\rangle^{1} - \frac{6}{3}\langle x-5\rangle^{1}$$
$$-M_D\langle x-10\rangle^{-2} + D_y\langle x-10\rangle^{-1}$$

or

$$p(x) = A_y \langle x\rangle^{-1} - 4\langle x\rangle^{0} + 4\langle x-10\rangle^{0} - 6\langle x-2\rangle^{0} + 2\left[\langle x-2\rangle^{1} - \langle x-5\rangle^{1}\right] - M_D\langle x-10\rangle^{-2} + D_y\langle x-10\rangle^{-1}$$

$\therefore$ The terms (crossed out) will not be needed in the final answer.

$$p(x) = A_y \langle x\rangle^{-1} - 4\langle x\rangle^{0} - 6\langle x-2\rangle^{0} + 2\left[\langle x-2\rangle^{1} - \langle x-5\rangle^{1}\right]$$

$$V(x) = A_y \langle x\rangle^{0} - 4\langle x\rangle^{1} - 6\langle x-2\rangle^{1} + \frac{2}{2}\left[\langle x-2\rangle^{2} - \langle x-5\rangle^{2}\right] + V(0)\langle x\rangle^{0}$$

not needed. already there as A_y

$$M(x) = A_y \langle x\rangle^{1} - \frac{4}{2}\langle x\rangle^{2} - \frac{6}{2}\langle x-2\rangle^{2} + \frac{1}{3}\left[\langle x-2\rangle^{3} - \langle x-5\rangle^{3}\right] + M(0)\langle x\rangle^{0}$$

$M(0) \to 0$ (is zero)

$$EI\,V(x)' = \frac{A_y}{2}\langle x\rangle^{2} - \frac{2}{3}\langle x\rangle^{3} - \frac{3}{3}\langle x-2\rangle^{3} + \frac{1}{3.4}\left[\langle x-2\rangle^{4} - \langle x-5\rangle^{4}\right] + C_1 \quad \to (3)$$

$$EI\,V(x) = \frac{A_y}{2.3}\langle x\rangle^{3} - \frac{2}{3.4}\langle x\rangle^{4} - \frac{1}{4}\langle x-2\rangle^{4} + \frac{1}{3.4.5}\left[\langle x-2\rangle^{5} - \langle x-5\rangle^{5}\right] + C_1 x + C_2$$

or

$$EI\,V(x) = \frac{A_y}{6}\langle x\rangle^{3} - \frac{1}{6}\langle x\rangle^{4} - \frac{1}{4}\langle x-2\rangle^{4} + \frac{1}{60}\left[\langle x-2\rangle^{5} - \langle x-5\rangle^{5}\right] + C_1 x + C_2 \quad \to (4)$$

Now apply the boundary conditions.

i) at $x=0$, $V=0$

ii) at $x=L$, $V=0$

iii) at $x=L$, $V'=0$

Enforcing the boundary conditions.

i) @ $x=0$, $v=0$ $\Rightarrow$ $C_2=0$

ii) @ $x=10$, $v=0$

$$0 = \frac{A_y}{6}(10)^3 - \frac{1}{6}(10)^4 - \frac{1}{4}(8)^4 + \frac{1}{60}\left[(8)^5 - (5)^5\right] + C_1(10)$$

$$166.67 A_y + 10C_1 = +2196.62$$

or $\boxed{16.67 A_y + C_1 = +219.67}$ $\longrightarrow$ (5)

(iii) @ $x=10$, $v'=0$

$$0 = \frac{A_y}{2}(10)^2 - \frac{2}{3}(10)^3 - (8)^3 + \frac{1}{12}(8)^4 - 5^4 + C_1$$

or $\boxed{50 A_y + C_1 = 889.417}$ $\longrightarrow$ (6)

(6) − (5)

$\Rightarrow$ $33.33 A_y = 324.66$

$\Rightarrow$ $\boxed{A_y = 20.09 \text{ lb}}$

sub A_y in eq.(6), we get

$\Rightarrow$ $\boxed{C_1 = -115.083}$

sub A_y in (1) we get

$\boxed{D_y = 28.91 \text{ lbs}}$

$M_D = 10A_y - 263$

$\boxed{M_D = -62.06 \text{ lb-ft}}$

$$v(c) = \frac{1}{EI}\left[\frac{20.09}{6}(5)^3 - \frac{1}{6}(5)^4 - \frac{1}{4}(3)^4 + \frac{1}{60}(3)^5 - 115.1(5)\right.$$

$$\boxed{v(c) = \frac{-277.325}{EI}}$$

ME 323 Examination #2 **SOLUTION**

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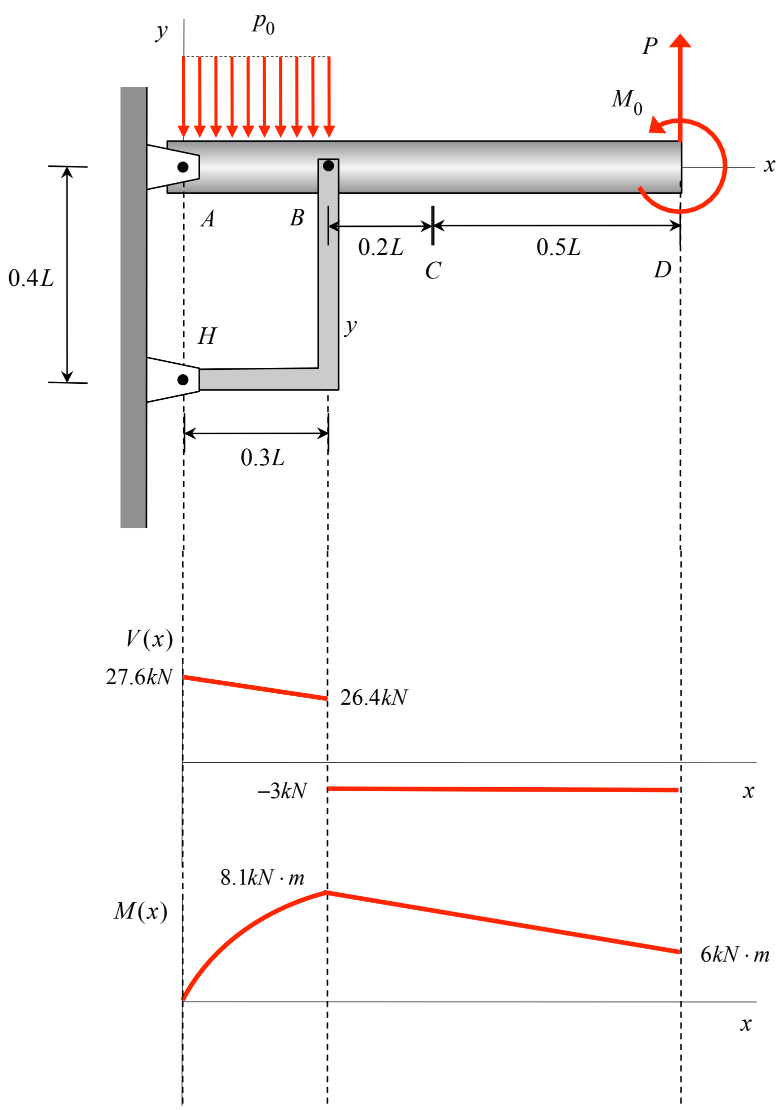
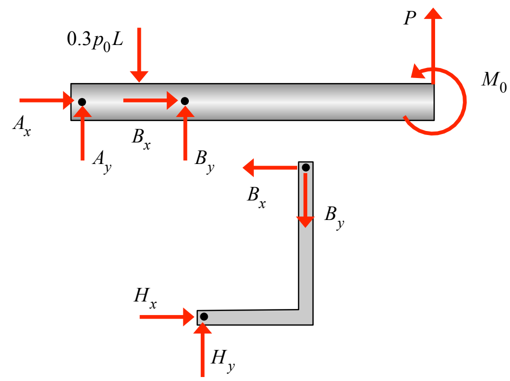
PROBLEM #3 (30 points)

Circular pipe AD has a length of L, an outer diameter of 3d and an inner diameter of d, and is made of a material having a shear modulus G and Young's modulus E. The pipe is pinned to ground at A, and is supported by the rigid, L-shaped bar BH, as shown below. Use the following in your analysis: $E = 70GPa$, $G = 26GPa$, $L = 1m$, $p_0 = 4kN/m$ $P = 3kN$, $M_0 = 6kN \cdot m$ and $d = 60\,mm$.

1. Determine the reactions on pipe AD at A and B.
2. Construct plots of the internal shear modulus V vs. x, and the internal bending moment M vs. x on the axes shown below.
3. Determine the bending stress at location "a" on a cross section of the pipe at location C.
4. Determine the shear stress at location "b" on a cross section of the pipe at location C.





Bar BH: $\sum M_H = (0.4L)B_x - (0.3L)B_y = 0 \quad \Rightarrow \quad B_x = \frac{3}{4}B_y$

Pipe AD:

$$\sum M_A = -0.3p_0L(0.15L) + B_y(0.3L) + PL + M_0 = 0 \quad \Rightarrow$$

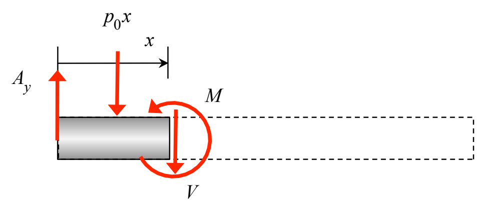
$$\boxed{B_y = \frac{15}{100}p_0L - \frac{10}{3}P - \frac{10}{3}\frac{M_0}{L} = \frac{15}{100}(4) - \frac{10}{3}(3) - \frac{10}{3}\frac{6}{1} = -29.4kN}$$

$$\boxed{B_x = \frac{3}{4}(-29.4) = -22.05kN}$$

$$\sum F_y = A_y + B_y - 0.3p_0L + P = 0 \quad \Rightarrow$$

$$\boxed{A_y = -B_y + 0.3p_0L - P = -(-29.4) + 0.3(4)(1) - 3 = 27.6kN}$$

$$\sum F_x = A_x + B_x = 0 \quad \Rightarrow \quad \boxed{A_x = -B_x = 22.05kN}$$

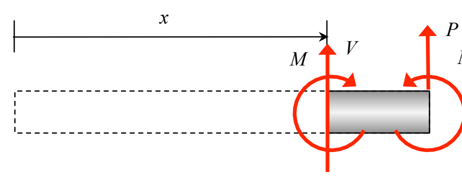


Segment AB:

$$\sum F_y = A_y - p_0x - V = 0 \quad \Rightarrow \quad \boxed{V(x) = A_y - p_0x = 27.6 - 4x}$$

$$\sum M_A = -(p_0x)\left(\frac{x}{2}\right) - Vx + M = 0 \quad \Rightarrow$$

$$\boxed{M = (p_0x)\left(\frac{x}{2}\right) + Vx = (4x)\left(\frac{x}{2}\right) + (27.6 - 4x)x = -2x^2 + 27.6x}$$



Segment BD:

$$\sum F_y = V + P = 0 \quad \Rightarrow \quad \boxed{V = -P = -3kN \quad \Rightarrow \quad V_C = -3kN}$$

$$\sum M_D = -V(L-x) + M_0 - M = 0 \quad \Rightarrow \quad \boxed{M = -V(L-x) + M_0 = -(-3)(L-x) + 6 = 9 - 3x}$$

$$\Rightarrow \quad \boxed{M_C = M(0.5L) = 7.5kN \cdot m}$$

$$\sigma_a = -\frac{M_C(3d/2)}{I} \quad ; \quad I = \frac{1}{4}\pi\left(\frac{3d}{2}\right)^4 - \frac{1}{4}\pi\left(\frac{d}{2}\right)^4 = \frac{5}{4}\pi d^4$$

$$= -\frac{M_C(3d/2)}{5\pi d^4/4} = \boxed{-\frac{6M_C}{5\pi d^3}}$$

$$\tau_b = \frac{VA^*\overline{y}^*}{It} \quad ; \quad A^*\overline{y}^* = A_o^*\overline{y}_o^* - A_i^*\overline{y}_i^* = \left[\frac{1}{2}\pi\left(\frac{3d}{2}\right)^2\right]\left(\frac{4(3d/2)}{3\pi}\right) - \left[\frac{1}{2}\pi\left(\frac{d}{2}\right)^2\right]\left(\frac{4(d/2)}{3\pi}\right) = \frac{13}{6}d^3$$

$$= \frac{V(13/6)d^3}{(5\pi d^4/4)(2d)} = \boxed{\frac{13}{15}\frac{V}{\pi d^2}}$$

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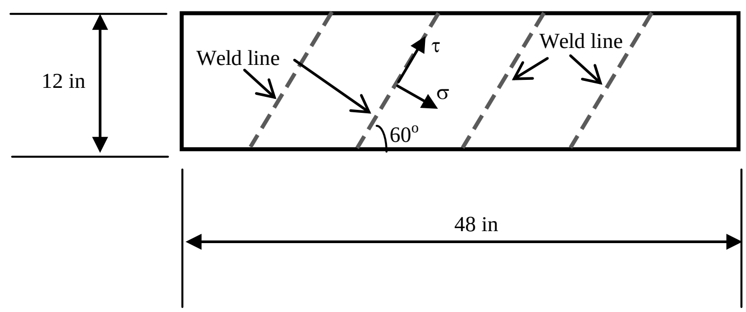
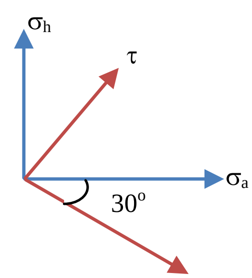
PROBLEM #4 (15 points)

A thin walled cylindrical pressure vessel has a diameter of 12 inches and length of 48 inches. The internal pressure in the vessel is 400 psi. The pressure vessel is fabricated by butt-welding a series of plates along helical arcs as shown in figure below. Determine:

1. The axial and hoop stresses
2. The normal stress perpendicular to the weld line
3. The shear stress along the weld line
4. The pressure vessel thickness, if the allowable normal and shear stresses perpendicular and along the weld line is 1200 and 500 psi respectively.





$$\sigma_a = \frac{pD}{4t} = \frac{1200}{t}$$

$$\sigma_h = \frac{pD}{2t} = \frac{2400}{t}$$

$$\theta = -30^o$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) = \frac{\frac{1200}{t} + \frac{2400}{t}}{2} + \frac{\frac{1200}{t} - \frac{2400}{t}}{2}\cos(-60)$$

therefore,

$$\sigma_n = \frac{1500}{t} \rightarrow 1200 = \frac{1500}{t} \rightarrow t = 1.25in$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin(2\theta) + \tau_{xy}\cos(2\theta) = -\left(\frac{\frac{1200}{t} - \frac{2400}{t}}{2}\right)\sin(-60)$$

$$\tau_{nt} = -\frac{519.6}{t} \rightarrow 500 = \frac{519.6}{t} \rightarrow t = 1.0392in$$

Therefore,

$$t = 1.25in$$

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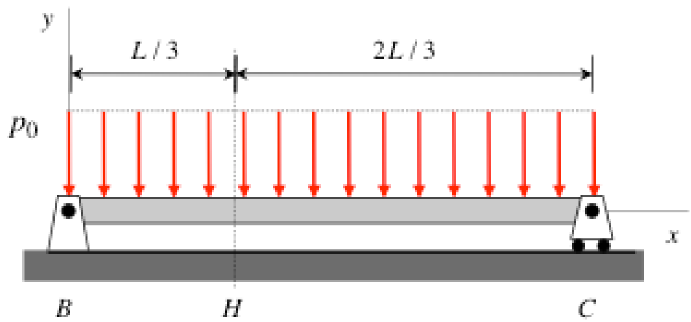
PROBLEM #5 (10 points)

Consider the simply-supported beam loaded as shown below.



PART A – 2 points

Consider two situations. One, where the beam is made from steel, having a Young's modulus of E_{st}. Two, where the beam is made from aluminum, having a Young's modulus of E_{al}, where $E_{st} > E_{al}$. Let $(\sigma_{max})_{st}$ and $(\sigma_{max})_{al}$ denote the absolute values of the maximum normal stress on the cross section at location H for the steel and aluminum beams, respectively. Circle the correct statement below related to the relative sizes of $(\sigma_{max})_{st}$ and $(\sigma_{max})_{al}$:

a) $(\sigma_{max})_{st} < (\sigma_{max})_{al}$

b) $(\sigma_{max})_{st} = (\sigma_{max})_{al}$ - This is the answer

c) $(\sigma_{max})_{st} > (\sigma_{max})_{al}$

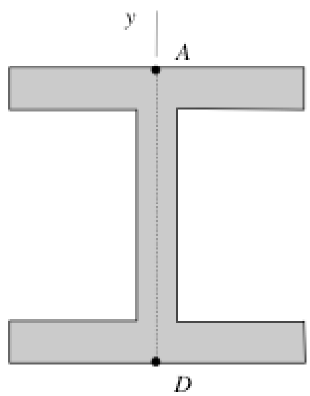
PART B – 2 points

The beam pictured above has the I-beam cross-section shown below. Let $(\sigma_{max})_A$ and $(\sigma_{max})_D$ denote the absolute values of the maximum normal stress on the cross section at location H for points A and D, respectively. Circle the correct statement below related to the relative sizes of $(\sigma_{max})_A$ and $(\sigma_{max})_D$:

a) $(\sigma_{max})_A < (\sigma_{max})_D$

b) $(\sigma_{max})_A = (\sigma_{max})_D$ - This is the answer

c) $(\sigma_{max})_A > (\sigma_{max})_D$



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PROBLEM #5 - continued

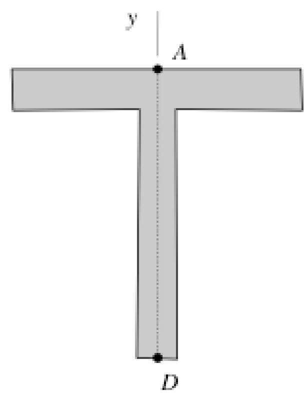
PART C – 2 points

The beam pictured above has the T-beam cross-section shown below. Let $(\sigma_{max})_A$ and $(\sigma_{max})_D$ denote the absolute values of the maximum normal stress on the cross section at location H for points A and D, respectively. Circle the correct statement below related to the relative sizes of $(\sigma_{max})_A$ and $(\sigma_{max})_D$:

a) $(\sigma_{max})_A < (\sigma_{max})_D$ - This is the answer

b) $(\sigma_{max})_A = (\sigma_{max})_D$

c) $(\sigma_{max})_A > (\sigma_{max})_D$



PART D – 4 points

The beam pictured above has the cross-section shown below. All wall thicknesses of the beam cross-section are $30\,mm$. Let τ_A denote the shear stress at point A on the cross-section of the beam at location D. Recall that the shear stress due to bending in a beam is given by $\tau = \dfrac{VQ}{It}$. For the shear stress at A, τ_A, what is the numerical value of "*t*" to be used in this equation?

The answer is 90 mm.