Name: ________________________Instructor: Krousgrill/Raman Siegmund Bilal
(Print) (Last) (First) (Circle your instructor)

ME 323 EXAM #2
FALL SEMESTER 2012
8:00 PM – 9:30 PM
Oct. 31, 2012

Instructions

1. Begin each problem in the space provided on the examination sheets. If additional space is required, use the paper provided. Work on one side of each sheet only, with only one problem on a sheet.

2. Each problem is of value as indicated below.

3. To obtain maximum credit for a problem, you must present your solution clearly. Accordingly:
   a. Identify coordinate systems
   b. Sketch free body diagrams
   c. State units explicitly
   d. Clarify your approach to the problem including assumptions

4. **If your solution cannot be followed, it will be assumed that it is in error**.

Prob. 1	(35)	__________
Prob. 2	(33)	__________
Prob. 3	(32)	__________
Total	**(100)**	

PROBLEM #1 - SOLUTION

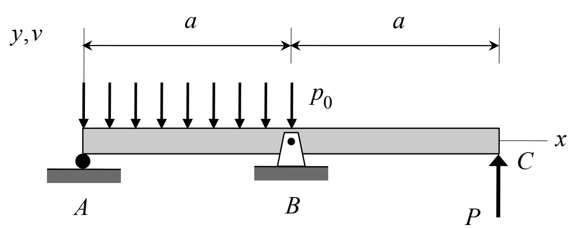
Consider the beam shown below that is supported by a roller at its left end A and a pin joint at its midpoint B. A constant, downward force/length loading p_0 acts along the left half of the beam, and an upward concentrated force P acts at its right end C.

(a) Determine the reactions for the beam.

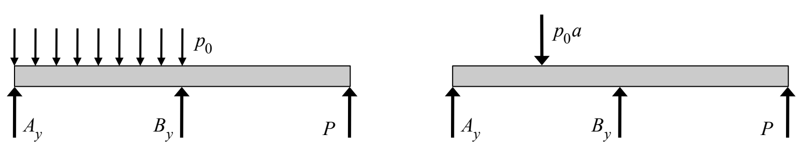
(b) Determine the deflection $v(x)$ of the neutral axis of the beam.

(c) Determine the rotations θ_A and θ_B of the beam deflection at locations A and B.

Use the following parameters in your solution: $a = 2\ ft$, $p_0 = 2\ kips/ft$ and $P = 5\ kips$. Leave the flexural rigidity EI as a variable in your solution.



Equilibrium



$$\sum M_A = B_y(a) - p_0 a(a/2) + P(2a) = 0 \quad \Rightarrow \quad B_y = p_0 a/2 - 2P = -8\,kips$$

$$\sum M_B = -A_y(a) + P(a) + p_0 a(a/2) = 0 \quad \Rightarrow \quad A_y = P + p_0 a/2 = 7\,kips$$

Deflection

<u>AB</u>: $V(x) = V(0) + \int_0^x p(s)\,ds = A_y - 2x = 7 - 2x$

$$M(x) = M(0) + \int_0^x V(s)\,ds = 0 + \int_0^x (7 - 2s)\,ds = 7x - x^2$$

$$\theta(x) = \theta(0) + \frac{1}{EI}\int_0^x M(s)\,ds = \theta_A + \frac{1}{EI}\left(\frac{7}{2}x^2 - \frac{1}{3}x^3\right)$$

$$v(x) = v(0) + \int_0^x \theta(s)\,ds = \theta_A x + \frac{1}{EI}\left(\frac{7}{6}x^3 - \frac{1}{12}x^4\right)$$

<u>B</u>: $V(2^-) = 3$; $V(2^+) = V(2^-) + B_y = -5$

$M(2) = 10$

$$\theta(2) = \theta_B = \theta_A + \frac{34}{3EI}$$

$$v(2) = 0 = 2\theta_A + \frac{8}{EI} \quad \Rightarrow \quad \boxed{\theta_A = -\frac{4}{EI}} \quad \Rightarrow \quad \boxed{\theta_B = \frac{22}{3EI}}$$

Therefore, for $0 < x < 2$: $\boxed{v(x) = \frac{1}{EI}\left(-\frac{1}{12}x^4 + \frac{7}{6}x^3 - 4x\right)}$

<u>BC</u>: $V(x) = V(2^+) + \int_2^x p(s)\,ds = -5 + 0 = -5$

$$M(x) = M(2) + \int_2^x V(s)\,ds = 10 - 5(x-2) = 20 - 5x$$

$$\theta(x) = \theta(2) + \frac{1}{EI}\int_2^x M(s)\,ds = \frac{22}{3EI} + \frac{1}{EI}\left[20(x-2) - \frac{5}{2}\left(x^2 - 2^2\right)\right]$$

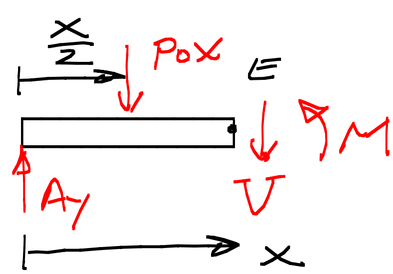
$$= \frac{1}{EI}\left(-\frac{68}{3} + 20x - \frac{5}{2}x^2\right)$$

$$v(x) = v(2) + \int_2^x \theta(s)\,ds = \frac{1}{EI}\left[-\frac{68}{3}(x-2) + 10\left(x^2 - 2^2\right) - \frac{5}{6}\left(x^3 - 2^3\right)\right]$$

$$\boxed{v(x) = \frac{1}{EI}\left(-\frac{5}{6}x^3 + 10x^2 - \frac{68}{3}x + 12\right)}$$

ALTERNATE SOLUTION

<u>AB</u>: $\sum M_E = M(x) + \left(p_0 x\right)\left(\frac{x}{2}\right) - A_y x = 0 \quad \Rightarrow \quad M(x) = -x^2 + 7x$

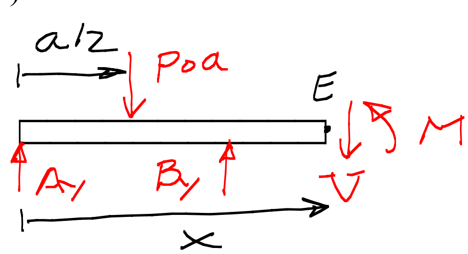


$$EI\frac{d^2 v_1}{dx^2} = M(x) = -x^2 + 7x$$

$$EI\frac{dv_1}{dx} = -\frac{x^3}{3} + \frac{7x^2}{2} + C_1$$

$$EIv_1 = -\frac{x^4}{12} + \frac{7x^3}{6} + C_1 x + C_2$$

<u>BC</u>: $\sum M_E = M(x) + \left(ap_0\right)\left(x - \frac{a}{2}\right) - A_y x - B_y(x-a) \quad \Rightarrow \quad M(x) = -5x + 20$



$$EI\frac{d^2 v_2}{dx^2} = M(x) = -5x + 20$$

$$EI\frac{dv_2}{dx} = -\frac{5x^2}{2} + 20x + C_3$$

$$EIv_2 = -\frac{5x^3}{6} + 10x^2 + C_3x + C_4$$

Apply boundary/continuity conditions

$$v_1(0) = 0 = C_2$$

$$v_1(a) = 0 = \frac{1}{EI}\left(-\frac{a^4}{12} + \frac{7a^3}{6} + C_1a + C_2\right) \Rightarrow C_1 = \frac{a^3}{12} - \frac{7a^2}{6} = -4$$

$$v_2(a) = 0 = \frac{1}{EI}\left(-\frac{5a^3}{6} + 10a^2 + C_3a + C_4\right) \Rightarrow$$

$$C_4 = \frac{5a^3}{6} - 10a^2 - C_3a = -\frac{100}{3} - 2C_3$$

$$\frac{dv_1}{dx}(a) = \frac{dv_2}{dx}(a) \Rightarrow \frac{1}{EI}\left(-\frac{a^3}{3} + \frac{7a^2}{2} + C_1\right) = \frac{1}{EI}\left(-\frac{5a^2}{2} + 20a + C_3\right) \Rightarrow$$

$$C_3 = -\frac{a^3}{3} + 6a^2 - 20a + C_1 = -\frac{68}{3}$$

Therefore,

$$C_4 = -\frac{100}{3} - 2\left(-\frac{68}{3}\right) = 12$$

Summary:

$$0 < x < a: \quad v(x) = v_1(x) = \frac{1}{EI}\left(-\frac{x^4}{12} + \frac{7x^3}{6} - 4x\right)$$

$$a < x < 2a: \quad v(x) = v_2(x) = \frac{1}{EI}\left(-\frac{5x^3}{6} + 10x^2 - \frac{68}{3}x + 12\right)$$

Also:

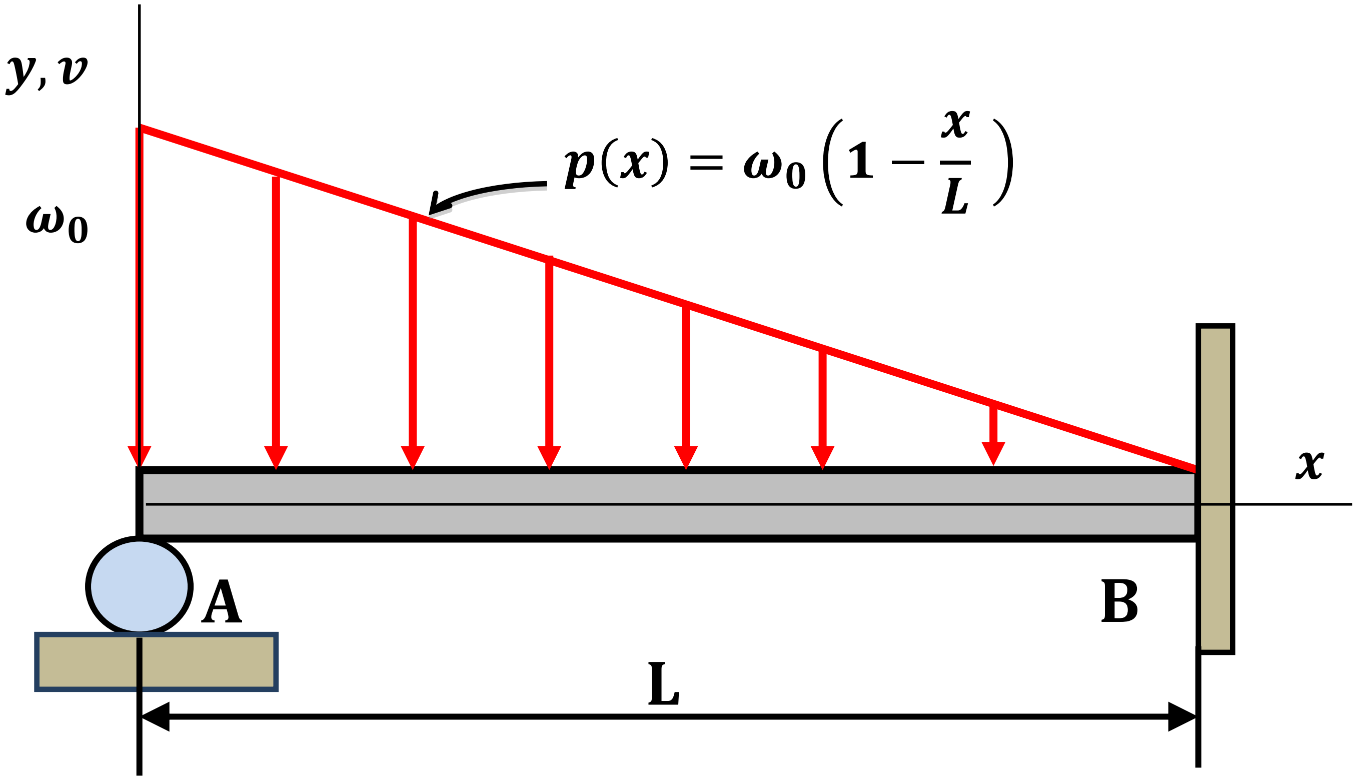
$$\theta_A = \frac{dv_1}{dx}(0) = \frac{C_1}{EI} = -\frac{4}{EI}$$

$$\theta_B = \frac{dv_1}{dx}(a) = \frac{1}{EI}\left(-\frac{a^3}{3} + \frac{7a^2}{2} + C_1\right) = \frac{22}{3EI}$$

Name: ________________________**Instructor: Krousgrill/Raman Siegmund Bilal**
(Print) (Last) (First) (Circle your instructor)

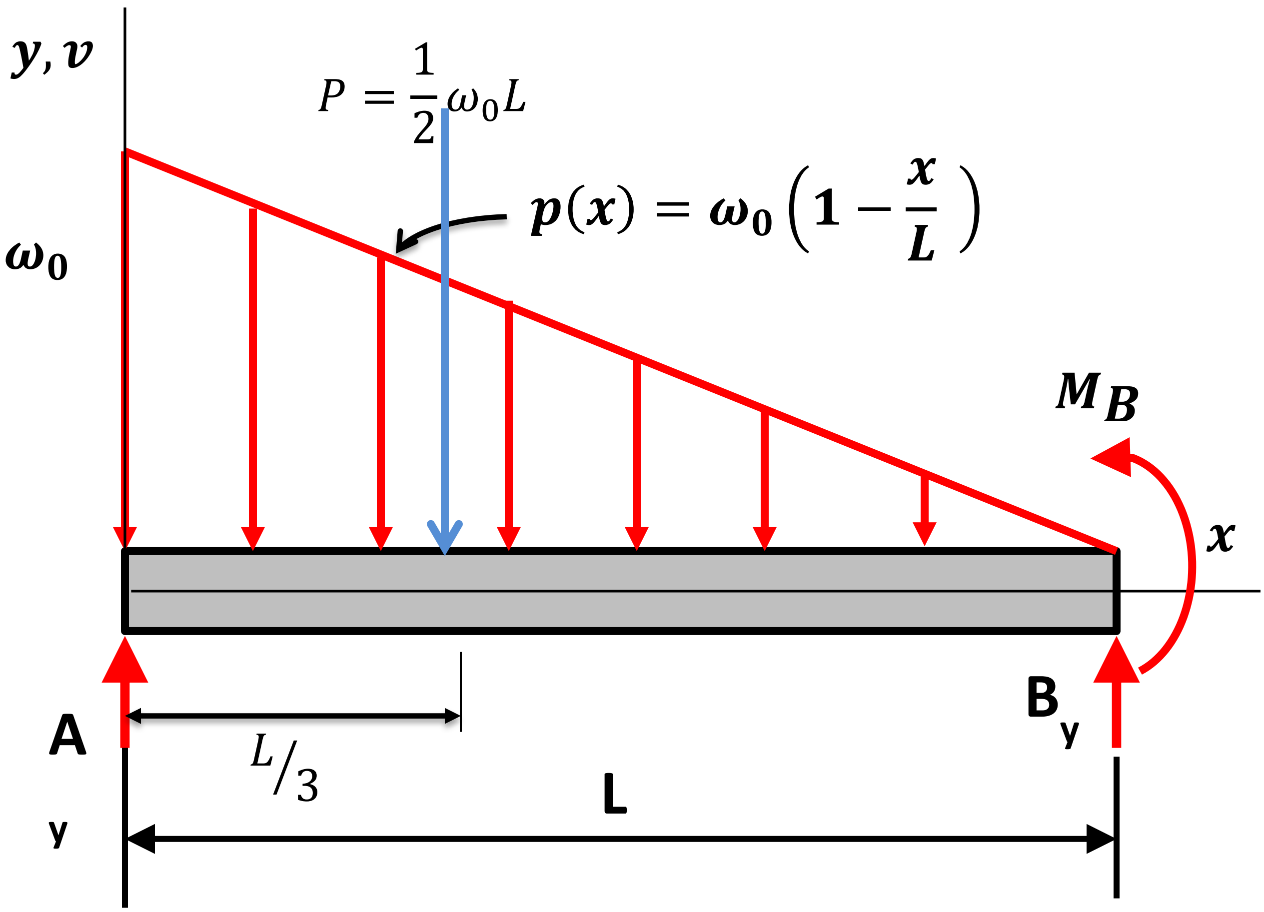
PROBLEM #2 (33 points)

A beam ***AB*** of length ***L*** is loaded and supported as shown in the Figure below. Determine the support reaction at ***A***. Assume ***EI*** is constant for the beam.



Solution:

The FBD is shown in the Figure below.



$$p(x) = -(\omega_0 - \omega_0 \frac{x}{L}), or$$

$$p(x) = \frac{\omega_0}{L}(x - L) ---> (1)$$

$$\sum F_y = 0 \Rightarrow A_y + B_y - \frac{1}{2}\omega_0 L = 0 ---> (2)$$

$$\sum M_B = 0 \Rightarrow -A_y L + M_B + \frac{1}{2}(\omega_0 L)(\frac{2}{3}L) = 0$$

$$M_B = Ay - \frac{\omega_0}{3}L^2 ---> (3)$$

Write the 4th Order deflection equation

$$(EIv'')'' = \frac{\omega_0}{L}(x-L)$$

integrating 4 times

$$(EIv'')' = \frac{\omega_0}{2L}(x-L)^2 + C_1 ---> (4)$$

$$(EIv'') = \frac{\omega_0}{6L}(x-L)^3 + C_1 x + C_2 ---> (5)$$

$$(EI\frac{dv}{dx}) = \frac{\omega_0}{24L}(x-L)^4 + \frac{C_1 x^2}{2} + C_2 x + C_3 ---> (6)$$

$$(EIv) = \frac{\omega_0}{120L}(x-L)^5 + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4 ---> (7)$$

Boundary Conditions

$(i) @ x = 0; \quad v = 0$

$(ii) @ x = 0; \quad M = 0$

$(iii) @ x = L; \quad v' = 0$

$(iv) @ x = L; \quad v = 0$

applying BC $\#1: @ x = 0; v = 0$

$$EIv(0) = 0 = \frac{\omega_0}{120L}(0-L)^5 + \frac{C_1(0)^3}{6} + \frac{C_2(0)^2}{2} + C_3(0) + C_4$$

$$\boxed{C_4 = \frac{\omega_0 L^4}{120}}$$

applying BC $\#2: @ x = L; M = 0$

$$EI\frac{d^2v(0)}{dx^2} = 0 = \frac{\omega_0}{6L}(0-L)^3 + C_1(0) + C_2$$

$$\boxed{C_2 = \frac{\omega_0 L^2}{6}}$$

$$applying\ BC\#3: @\ x = L; v' = 0$$

$$EI\frac{dv(0)}{dx} = 0 = \frac{\omega_0}{24L}(L-L)^4 + \frac{C_1(L)^2}{2} + (\frac{\omega_0 L^2}{6})L + C_3$$

$$C_3 = -\frac{C_1 L_2}{2} - \frac{\omega_0 L^3}{6}$$

$$applying\ BC\#4: @\ x = L; v = 0$$

$$EIv(0) = 0 = \frac{\omega_0}{120L}(L-L)^5 + \frac{C_1(L)^3}{6} + \frac{C_2(L)^2}{2} + C_3 L + C_4$$

$$0 = \frac{C_1 L^3}{6} + \left(\frac{\omega_0 (L)^2}{6}\right)\frac{L^2}{2} + \left(\frac{-C_1 L^2}{2} - \frac{\omega_0 L^3}{6}\right)L + C_4$$

$$\frac{C_1 L^3}{6} + \frac{\omega_0 L^4}{12} - \frac{C_1 L^3}{2} - \frac{\omega_0 L^4}{6} + \frac{\omega_0 L^4}{120} = 0$$

$$C_1 = -\frac{9}{40}\omega_0 L$$

$$applying\ BC\#5: @\ x = 0; V = \frac{dM}{dx} = \frac{d^3 v}{dx^3} = A_y\ (equal\ to\ the\ shear\ force)$$

$$(EIv'')' = \frac{\omega_0}{2L}(x-L)^2 + C_1$$

$$A_y = \frac{\omega_0}{2L}(0-L)^2 - \frac{9}{40}\omega_0 L$$

$$A_y = \frac{11}{40}\omega_0 L$$

PROBLEM #2

ALTERNATE SOLUTION

$$V(x) = V(0) + \int_0^x p(s)\,ds = A_y - w_0 \int_0^x \left(1 - \frac{s}{L}\right) ds = A_y - w_0 x + \frac{w_0 x^2}{2L}$$

$$M(x) = M(0) + \int_0^x V(s)\,ds = 0 + \int_0^x \left(A_y - w_0 s + \frac{w_0 s^2}{2L} \right) ds = A_y x - \frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L}$$

$$\theta(x) = \theta(0) + \frac{1}{EI} \int_0^x M(s)\,ds = \theta_A + \frac{1}{EI} \left(\frac{A_y x^2}{2} - \frac{w_0 x^3}{6} + \frac{w_0 x^4}{24L} \right)$$

$$v(x) = v(0) + \int_0^x \theta(s)\,ds = \theta_A x + \frac{1}{EI} \left(\frac{A_y x^3}{6} - \frac{w_0 x^4}{24} + \frac{w_0 x^5}{120L} \right)$$

Enforce boundary conditions:

$$\theta(L) = 0 = \theta_A + \frac{1}{EI} \left(\frac{A_y L^2}{2} - \frac{w_0 L^3}{6} + \frac{w_0 L^4}{24L} \right) \Rightarrow \theta_A = \frac{1}{EI} \left(-A_y + \frac{w_0 L}{4} \right) \frac{L^2}{2} \qquad (1)$$

$$v(L) = 0 = \theta_A L + \frac{1}{EI} \left(\frac{A_y L^3}{6} - \frac{w_0 L^4}{24} + \frac{w_0 L^5}{120L} \right) \Rightarrow \theta_A = \frac{1}{EI} \left(-A_y + \frac{w_0 L}{5} \right) \frac{L^2}{6} \qquad (2)$$

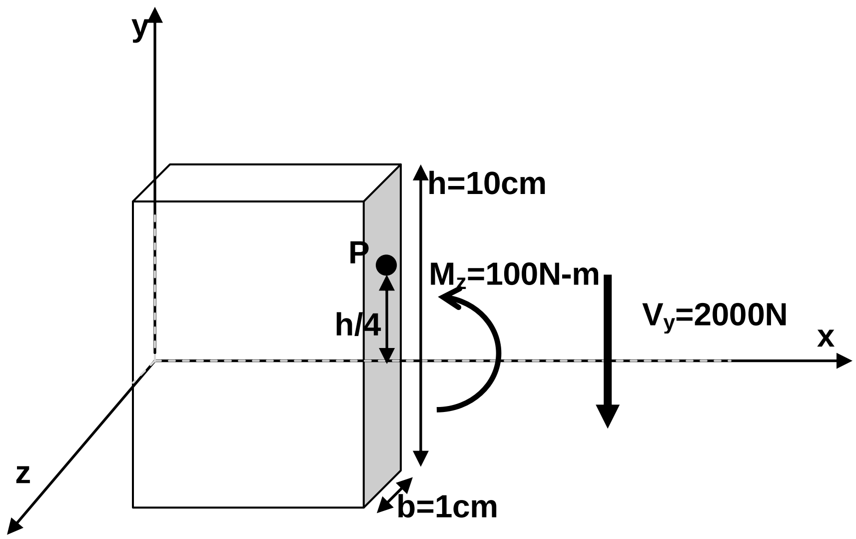
Equations (1) and (2):

$$\frac{1}{EI} \left(-A_y + \frac{w_0 L}{4} \right) \frac{L^2}{2} = \frac{1}{EI} \left(-A_y + \frac{w_0 L}{5} \right) \frac{L^2}{6} \quad \Rightarrow \quad A_y = \frac{11 w_0 L}{40}$$

Name: _________________________Instructor: Krousgrill/Raman Siegmund Bilal
(Print) (Last) (First) (Circle your instructor)

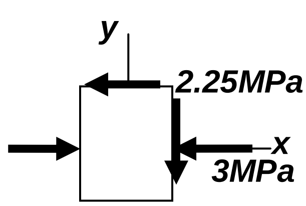
PROBLEM #3 (32 points total)

The rectangular cross section of a beam has an internal bending moment of 100 N-m and an internal shear force (downwards) of 2000 N acting on it as shown below. The cross sectional dimensions of the beam are shown below. Point P is a point located at a height h/4 up from the neutral axis.

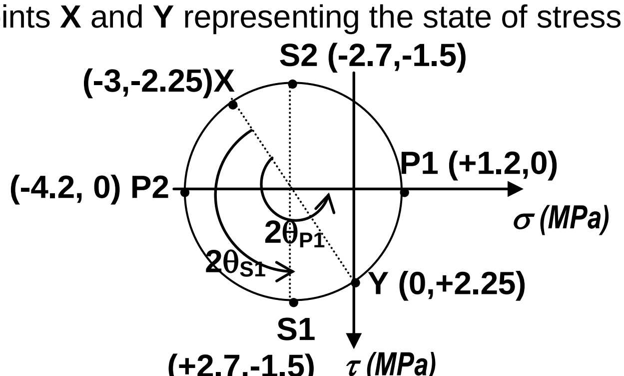


a. Find the values of the normal stresses and shear stresses $\sigma_{xx}, \tau_{xy}, \sigma_{yy}$ (in MPa) for the material element located at point P shown above. Show these stress vectors (magnitude and direction) on a properly oriented material element.

Ans:



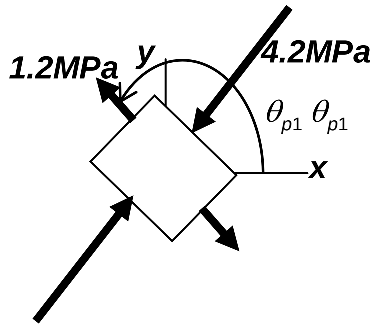
b. Sketch a Mohr's circle for this stress state of stress, indicating clearly on it the coordinates of the center the circle, the radius of the circle, the coordinates of points **X** and **Y** representing the state of stress.



c. Determine the principal stresses σ_{p1}, σ_{p2}. Depict the material element and the stress components of the principal stress state (magnitude and direction) on a properly oriented material element in the ***p*1-*p*2** configuration. Clearly indicate the transformation angle θ_{p1} relative to the **+*x***-axis.

$$2\theta_{p1} = 180 + 56.44 = 236.44$$

$$\theta_{p1} = 118.22$$



d. Determine the maximum in-plane shear stress τ_{max}. Depict the material element and the stress components of the maximum shear stress state (magnitude and direction) on a properly oriented material element in the ***s*1-*s*2** configuration. Clearly indicate the transformation angle θ_{s1} relative to the **+*x***-axis.

$$2\theta_{s1} = 90 + 56.44 = 146.44$$

$$\theta_{s1} = 73.22$$