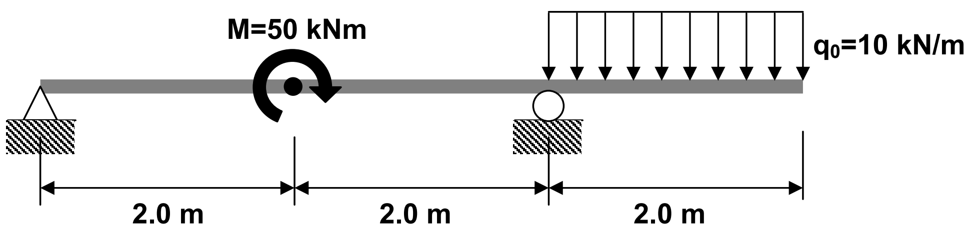
Practice Exam #2 – Fall 2012

PROBLEM #1 (33 points)

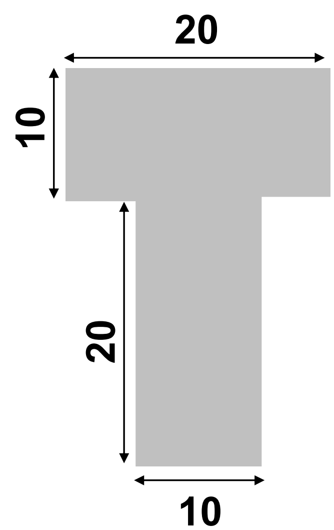
1) The beam shown below is loaded by a distributed load q_0=10 kN/m and a moment M=50 kNm.

- Determine and plot the shear force and bending moment diagrams. Your answer needs to be expressed as a V(x) and M(x) diagram with characteristic values (any local maxima and minima as well as values of discontinuities) clearly stated.



2) The beam possesses a T–shaped cross section.

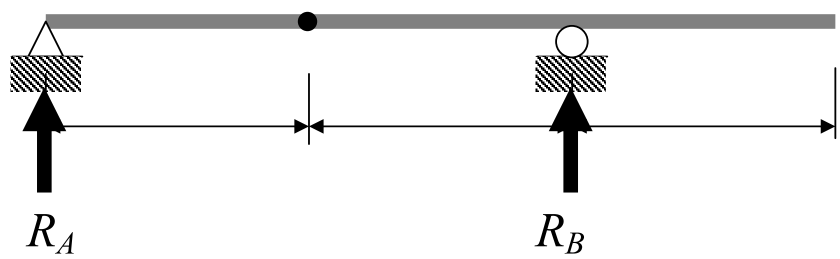
- Determine the location along the beam axis where the maximum value of flexural stress occurs and determine the maximum value of flexural stress;
- Determine the distribution of the flexural stresses on the beam cross-section. Document your answer in a drawing that depicts the distribution of these stresses in the beam cross-section and indicate minimum and maximum values.

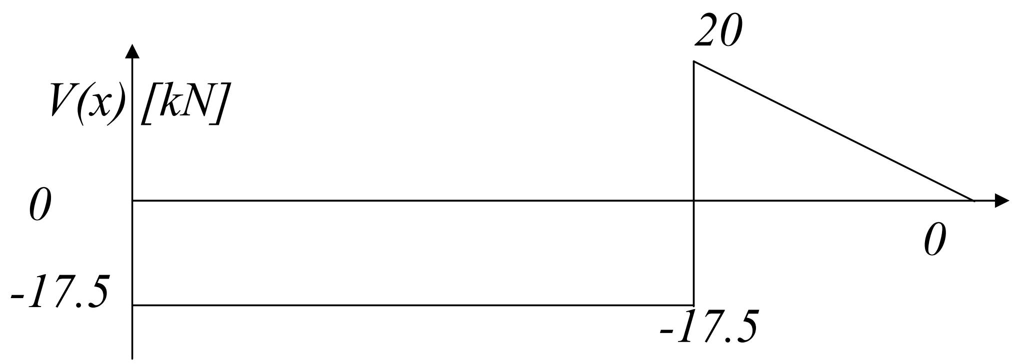


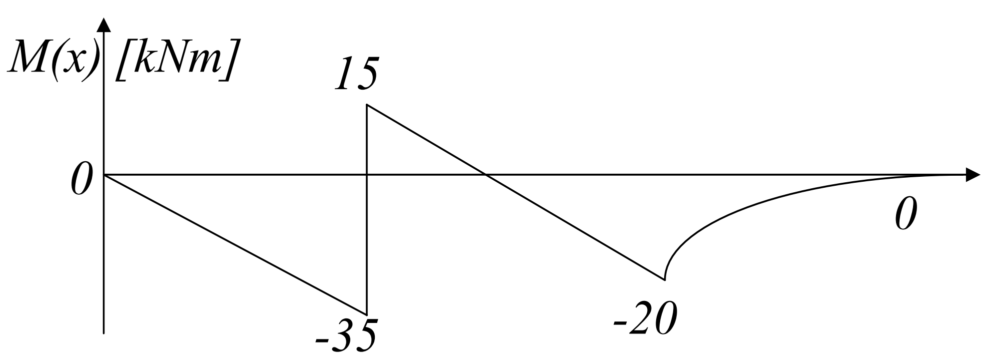
Reaction Forces

$R_A = -17.5kN$

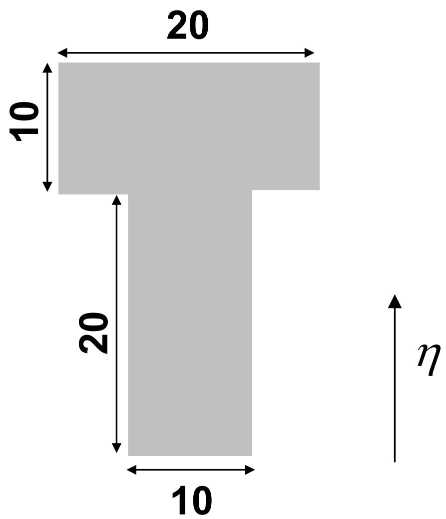
$R_B = 37.5kN$







$$\max[M(x)] = -35kNm$$
$$at \quad x = 2.0m$$



Centroid:

$$\bar{\eta} = \frac{25(20\times10)+10(20\times10)}{2(20\times10)} = 17.5mm$$

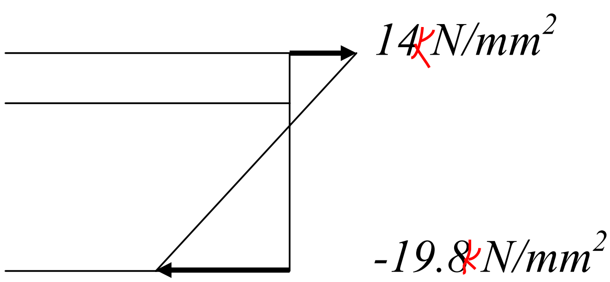
Second area moment:

$$I = \frac{10\times20^3}{12} + 7.5^2(10\times20) + \frac{20\times10^3}{12} + 7.5^2(10\times20) = 30832mm^4$$

Flexural Stresses:

$$\sigma_x = -\frac{(-35kNm)(-17.5mm)}{30832mm^4} = -19.8\frac{kN}{mm^2}$$

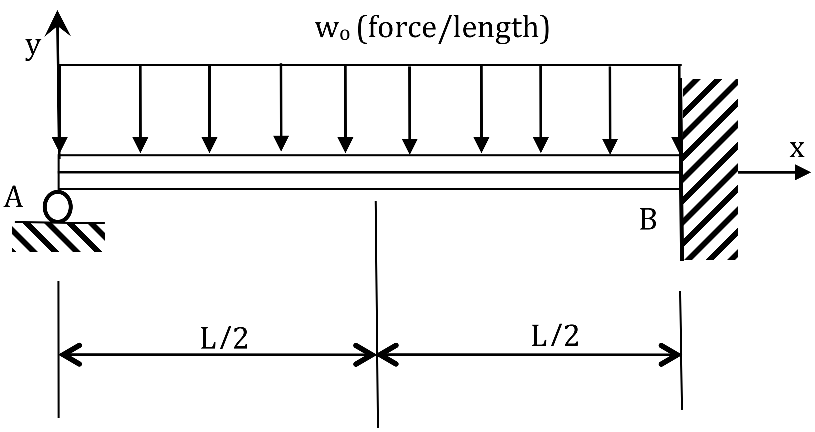
$$\sigma_x = -\frac{(-35kNm)(12.5mm)}{30832mm^4} = 14\frac{kN}{mm^2}$$

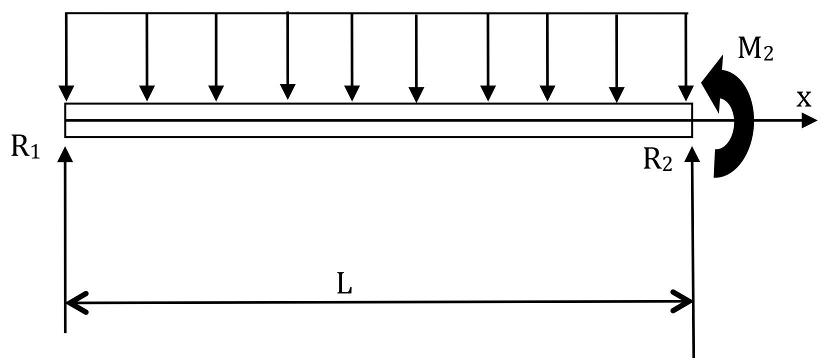


Problem #2

The beam shown below has a constant cross sectional area and modulus of elasticity (EI = constant). It is supported by a roller at A and fixed into the wall at B. The beam is loaded vertically downward by a distributed load w_o as shown. Determine:

- The reaction forces and moment at the supports





$$\sum F_y = 0 \rightarrow R_1 + R_2 = w_0 L$$
$$\sum M_{R_2} = 0 \rightarrow R_1 * L - w_0 L^2 / 2 - M_2 = 0$$
$$2 \; equations \; and \; three \; unknowns : R_1, R_2, M_2$$
$$q(x) = -w_0$$
$$V(x) = -w_0 x + R_1$$
$$M(x) = -w_0 \frac{x^2}{2} + R_1 x + M_1$$
$$M_1 = 0$$
$$EIv'(x) = \frac{R_1}{2} x^2 - \frac{w_0}{6} x^3 + C_1$$
$$EIv(x) = \frac{R_1}{6} x^3 - \frac{w_0}{24} x^4 + C_1 x + C_2$$
Boundary Conditions are:

1. $x = 0, v = 0$
2. $x = L, v = 0$
3. $x = L, v' = 0$

using boundary condition 1 $\rightarrow C_2 = 0$

using boundary condition 3 $\rightarrow C_1 = -\frac{R_1}{2} L^2 + \frac{w_0}{6} L^3$

Substitute for C_1 from above in deflection equation and then use boundary condition 3, we obtain

$$R_1 = \frac{3 w_0}{8} L$$

Substitute for R_1 in sum of forces in the y direction results in:

$$R_2 = \frac{5 w_0}{8} L$$

Substitute for R_1 in sum of the moments at the wall results in:

$$M_2 = -\frac{w_0}{8} L^2$$

Problem #3

At a point in a structure the stress state is determined to be plane stress and characterized by $\boldsymbol{\sigma_x}$ **= 20 MPa**, $\boldsymbol{\sigma_y}$ **= 40 MPa**, $\boldsymbol{\tau_{xy}}$ **= 50 MPa.**

Determine:

- The values of the first and second principal stresses; and draw a properly oriented material element for this stress state;
- The value of the maximum in-plane shear stress; and draw a properly oriented material element for this stress state;
- Draw Mohr's circle for the stress state, and on Mohr's circle mark the locations representing the stress state in (x-y) coordinates, mark the locations representing the principal stresses and the maximum shear stress.

SOLUTION: Principal stresses are given by

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{20+40}{2} \pm \sqrt{\left(\frac{20-40}{2}\right)^2 + (50)^2} = 30 \pm \sqrt{10^2 + 50^2} = 30 \pm 50.9$$

$$\sigma_1 = 30 + 50.9 = 81\,MPa$$

$$\sigma_2 = 30 - 50.9 = -21\,MPa$$

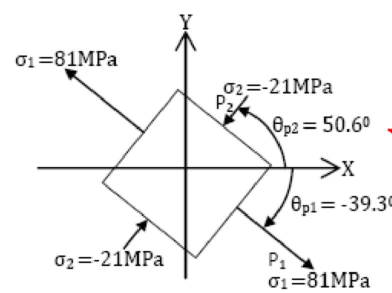
The orientation of the principal stress plane is given by:

$$\tan(2\theta_p) = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$, substituting the values we get

$$\tan(2\theta_p) = \frac{50}{(20-40)/2} = -5$$

$\theta_{p1} = \cancel{-39.3^0}$ 50.6^0

$\theta_{p2} = \theta_{p1} + 90^0 = -39.3^0 + 90 = \cancel{50.6^0}$ -39.3^0



Switch

Having found the principal stress values and the orientation of the principal plane, a material element can be drawn as shown on the side.

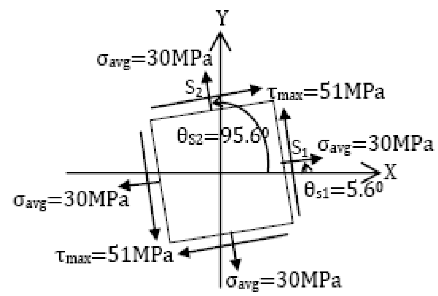
The value of maximum in-plane shear stress can be calculated using the following:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{20-40}{2}\right)^2 + 50^2}$$

$$\tau_{max} = 50.99\,MPa \approx 51\,MPa$$

The orientation of the maximum in-plane shear stress is given by:



$$\tan(2\theta_s) = \frac{-\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\tau_{xy}}$$

$$\tan(2\theta_s) = \frac{-\left(\frac{20-40}{2}\right)}{50} = \frac{1}{5}$$

$$\theta_{s1} = 5.6^0$$

$$\theta_{s2} = \theta_{s1} + 90^0 = 5.6 + 90 = 95.6^0$$

On the plane of maximum in-plane shear stress the normal stresses are the average normal stress given by $\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{20+40}{2} = 30\,MPa$. With this we can plot the material element for the maximum in-plane shear stress as shown on the side.

Now Mohr circle can be drawn as shown below: