Name: __Solution__________________ Instructor: Siegmund Sarkar Susilo
(Print) (Last) (First) (Circle one)

Solution

ME 323 EXAM #2
FALL SEMESTER 2010
8:00 PM – 9:30 PM
Nov. 2, 2010

Instructions

1. Begin each problem in the space provided on the examination sheets. If additional space is required, use the paper provided. Work on one side of each sheet only, with only one problem on a sheet.

2. Each problem is of value as indicated below.

3. To obtain maximum credit for a problem, you must present your solution clearly. Accordingly:
    a. Identify coordinate systems
    b. Sketch free body diagrams
    c. State units explicitly
    d. Clarify your approach to the problem including assumptions

4. **If your solution cannot be followed, it will be assumed that it is in error**.

Prob. 1 _(33)________________________

Prob. 2 _(34)________________________

Prob. 3 _(33)________________________

Total (100)

Name: __Solution__________________ Instructor: Siegmund Sarkar Susilo
(Print) (Last) (First) (Circle one)

$\sigma = F_n / A \quad \tau_{avg} = V / A \quad F.S. = F_{fail} / F_{allow} \quad F.S. = \sigma_{fail} / \sigma_{allow} \quad F.S. = \tau_{fail} / \tau_{allow}$

$\varepsilon_{avg} = (\Delta s' - \Delta s) / \Delta s = \delta / L_0 \quad \gamma = (\pi / 2) - \theta' \quad \sigma = E\varepsilon \quad \nu = -\varepsilon_{lat} / \varepsilon_{long} \quad \tau = G\gamma$

$$\delta_{th} = \alpha(\Delta T)L \quad \delta_{th} = \int_0^L \alpha(\Delta T)dx$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T \,, \varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T \,, \gamma_{xy} = \frac{1}{G}\tau_{xy} \quad , \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad , \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\delta = \frac{F_i L_0}{EA} \quad \delta = \int_0^L \frac{F_i(x)}{E(x)A(x)}dx \quad u_B = u_A + \delta_{AB}$$

$$\phi\rho = \gamma L \quad \tau = Gc\frac{\phi}{L} \quad \tau = \frac{T_i\rho}{J} \quad \phi = \frac{T_i L}{GJ} \quad \phi = \int_0^L \frac{T_i(x)}{G(x)J(x)}dx \quad \phi_B = \phi_A + \phi_{AB}$$

$$J = \frac{\pi c^4}{2} \ldots bar \quad J = \frac{\pi({c_o}^4 - {c_i}^4)}{2} \cdots tube$$

$$\frac{dV}{dx} = p(x), \frac{dM}{dx} = V(x), \Delta V = P, \Delta M = -M_0$$

$$\sigma(x, y) = \frac{-E(x)y}{\rho(x)} = \frac{-M(x)y}{I_z},$$

rectangle: $I = (bh^3)/12$, circle: $I = (\pi r^4)/4$,

semicircle : $I = (\pi r^4)/8$, centroid at $(4r)/(3\pi)$ from diameter

$$\bar{y} = \frac{\sum_i \bar{y}_i A_i}{\sum_i A_i} \qquad I = \sum_i I_{0,i} + {d_i}^2 A_i$$

$$\tau(x, y) = \frac{V(x)Q(y)}{I_z b}, \quad q = \frac{VQ}{I}, \quad Q(y) = \int_{A'} \eta dA = \bar{y}' A'$$

spherical PV : $\sigma_s = \dfrac{pr}{2t}$; cylindrical PV : $\sigma_h = \dfrac{pr}{t}$; $\sigma_a = \dfrac{pr}{2t}$;

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta_{xx'} + \tau_{xy}\sin 2\theta_{xx'} \quad \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta_{xx'} - \tau_{xy}\sin 2\theta_{xx'}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta_{xx'} + \tau_{xy}\cos 2\theta_{xx'} \quad \sigma_1 = \sigma_{avg} + R\,, \ \sigma_2 = \sigma_{avg} - R\,, \quad \tau_{\max} = R$$

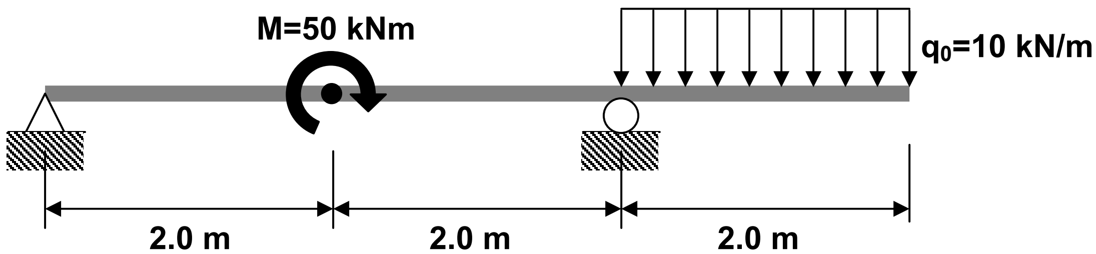
$$\sigma_{avg} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}\,, \ \tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)} \quad \tan 2\theta_s = \frac{-\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\tau_{xy}}$$

Name: __Solution__________________ Instructor: Siegmund Sarkar Susilo
(Print) (Last) (First) (Circle one)

PROBLEM #1 (33 points)

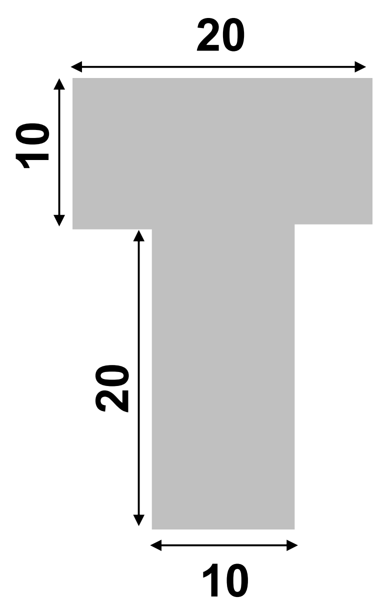
1) The beam shown below is loaded by a distributed load q_0=10 kN/m and a moment M=50 kNm.

- Determine and plot the shear force and bending moment diagrams. Your answer needs to be expressed as a V(x) and M(x) diagram with characteristic values (any local maxima and minima as well as values of discontinuities) clearly stated.



2) The beam possesses a T–shaped cross section.

- Determine the location along the beam axis where the maximum value of flexural stress occurs and determine the maximum value of flexural stress;
- Determine the distribution of the flexural stresses on the beam cross-section. Document your answer in a drawing that depicts the distribution of these stresses in the beam cross-section and indicate minimum and maximum values.

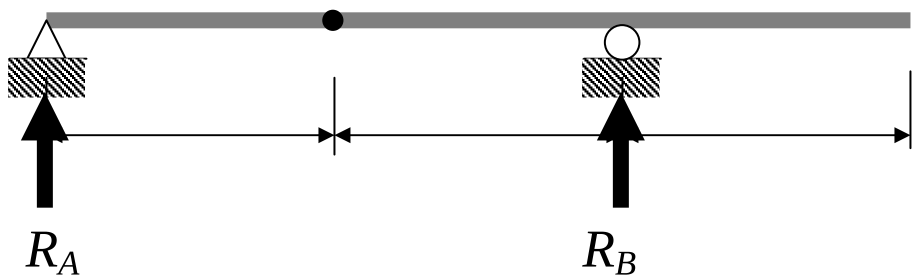


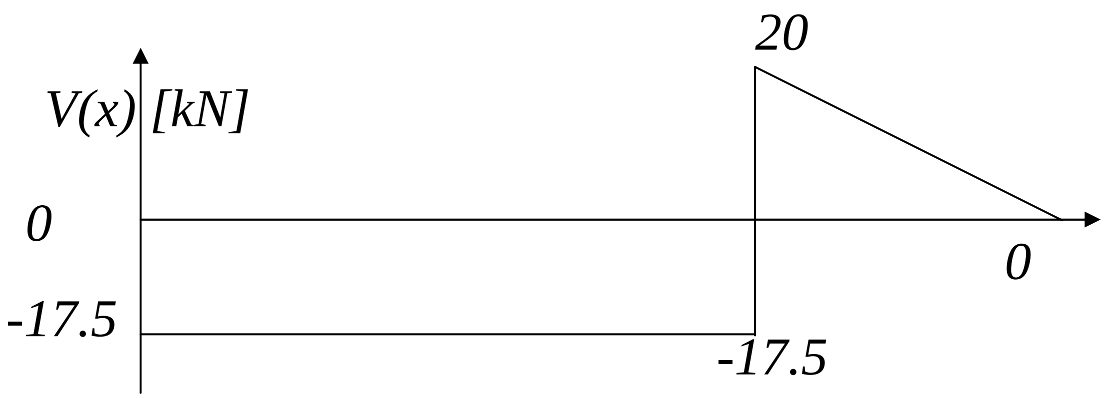
Name: __Solution___________________ Instructor: Siegmund Sarkar Susilo
(Print) (Last) (First) (Circle one)

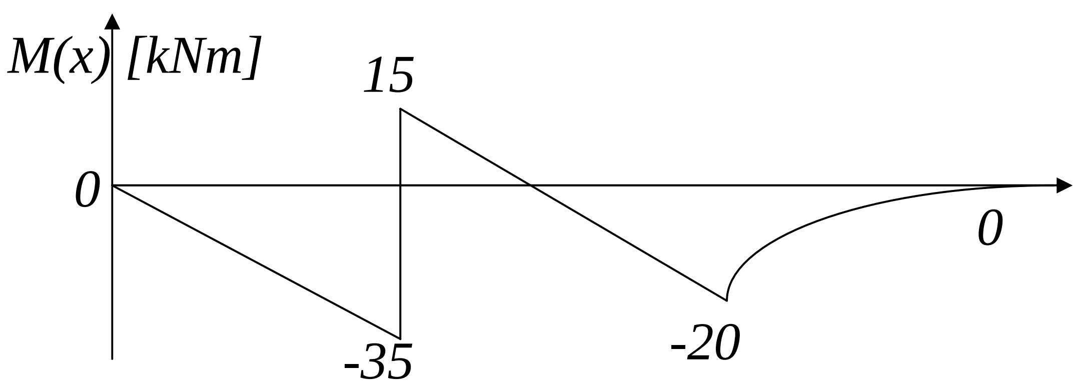
Reaction Forces

$R_A = -17.5kN$

$R_B = 37.5kN$

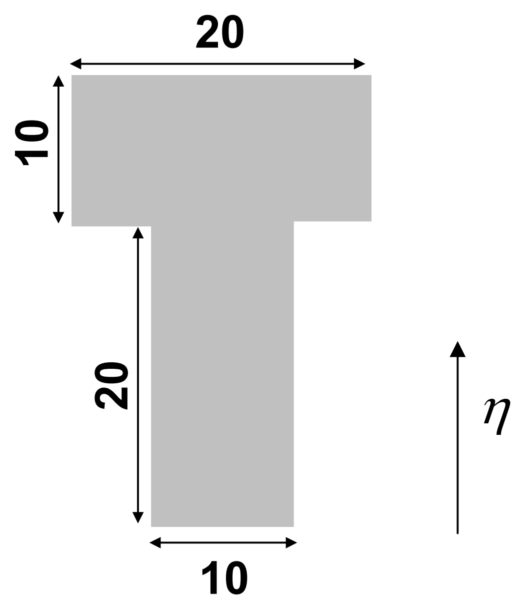






Name: __Solution___________________ Instructor: Siegmund Sarkar Susilo
(Print) (Last) (First) (Circle one)

$$\max[M(x)] = -35kNm$$
$$at \quad x = 2.0m$$



Centroid:

$$\bar{\eta} = \frac{25(20\times 10) + 10(20\times 10)}{2(20\times 10)} = 17.5mm$$

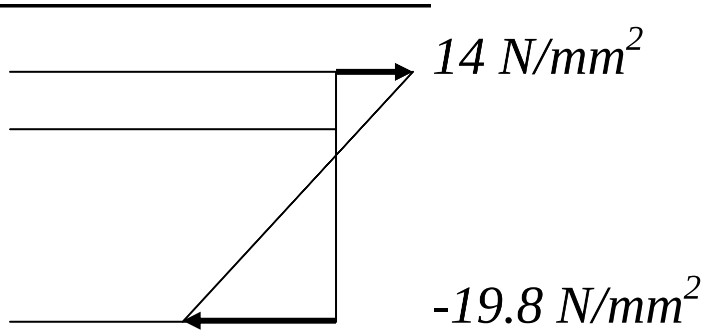
Second area moment:

$$I = \frac{10\times 20^3}{12} + 7.5^2(10\times 20) + \frac{20\times 10^3}{12} + 7.5^2(10\times 20) = 30832mm^4$$

Flexural Stresses:

$$\sigma_x = -\frac{(-35kNm)(-17.5mm)}{30832mm^4} = -19.8\frac{N}{mm^2}$$

$$\sigma_x = -\frac{(-35kNm)(12.5mm)}{30832mm^4} = 14\frac{N}{mm^2}$$

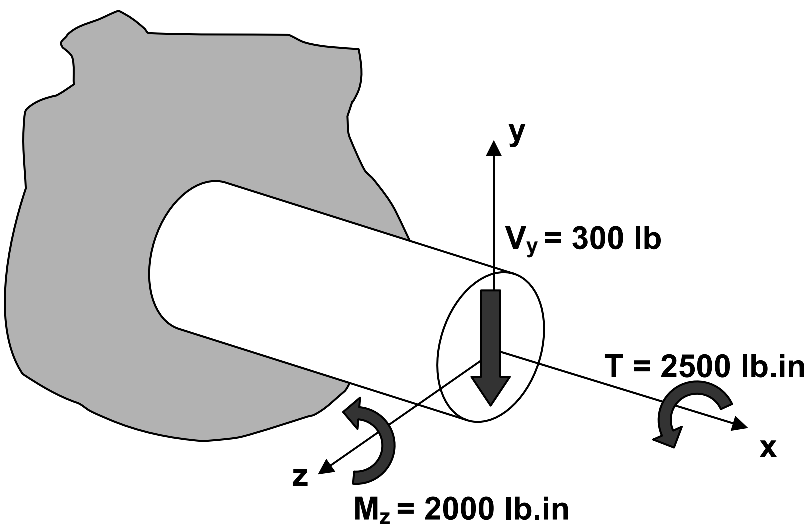


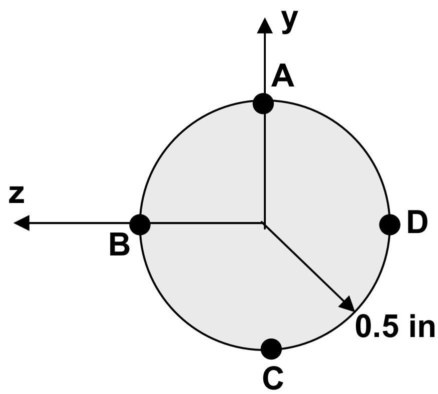
Name: __Solution__________________ Instructor: Siegmund Sarkar Susilo
(Print) (Last) (First) (Circle one)

PROBLEM #2 (34 points)

At a particular cross section of a shaft the internal resultants were determined to consist of a torque (**T = 2500 lb.in**), a bending moment (**M_z = 2000 lb.in**) and a shear force (**V_y = 300 lb**). The shaft possesses a circular cross-section with radius **r = 0.5 in.**

- Determine the states of stress at locations A,B,C,D on the cross-section. Document your answer by drawing the respective material elements at each location with the stress components clearly indicated by vectors and stress magnitudes.



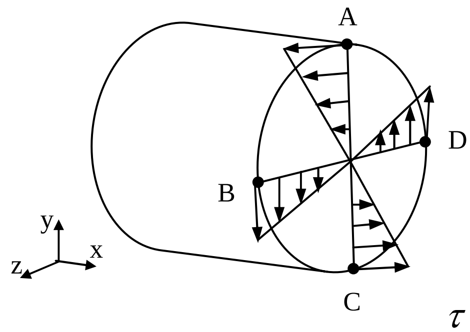


The stresses at points A, B, C, and D will be due to

- torsion

Name: __Solution__________________ Instructor: Siegmund Sarkar Susilo
(Print) (Last) (First) (Circle one)

- bending moment
- shear force



$$\tau = \frac{T_x \rho}{J_{xx}}$$

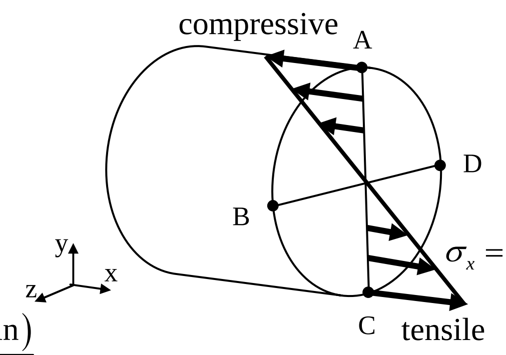
Torsion

$T = T_x = 2500$ lb.in

$J_{xx} = \frac{\pi}{2} c^4 = 98.17 \times 10^{-3}\ \text{in}^4$

$\tau = \frac{T_x c}{J_{xx}} = \frac{(2500\ \text{lb.in})(0.5\ \text{in})}{(98.17\ \text{in}^4)} = 12.73$ ksi (CCW)

@ A, $\tau_{xz} = \tau = 12.73$ ksi (towards +Z)
@ B, $\tau_{xy} = \tau = 12.73$ ksi (towards -Y)
@ C, $\tau_{xz} = \tau = 12.73$ ksi (towards -Z)
@ D, $\tau_{xy} = \tau = 12.73$ ksi (towards +Y)



$$\sigma_x = \frac{-M_z y}{I_{zz}}$$

Bending moment

$M_z = 2000$ lb.in

$I_{zz} = \frac{\pi}{4} c^4 = 49.09 \times 10^{-3}\ \text{in}^4$

@A, $\sigma_x = \frac{-M_z y}{I_{zz}} = \frac{-(2500\ \text{lb.in})(+0.5\ \text{in})}{(49.09 \times 10^{-3}\ \text{in}^4)}$

$= -20.37$ ksi (compressive)

@B, $\sigma_x = \frac{-M_z y}{I_{zz}} = \frac{-(2500\ \text{lb.in})(0\ \text{in})}{(49.09 \times 10^{-3}\ \text{in}^4)} = 0$ ksi

@C, $\sigma_x = \frac{-M_z y}{I_{zz}} = \frac{-(2500\ \text{lb.in})(-0.5\ \text{in})}{(49.09 \times 10^{-3}\ \text{in}^4)}$

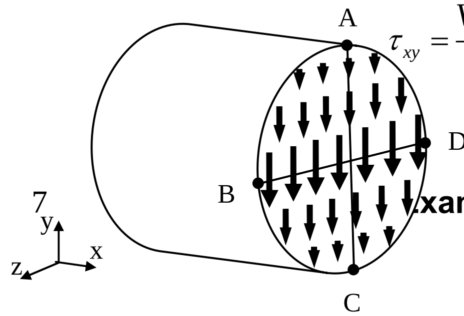
$= +20.37$ ksi (tensile)

@B, $\sigma_x = \frac{-M_z y}{I_{zz}} = \frac{-(2500\ \text{lb.in})(0\ \text{in})}{(49.09 \times 10^{-3}\ \text{in}^4)} = 0$ ksi

Shear force

$V_y = 300$ lb

$$\tau_{xy} = \frac{V_y Q_{zz}}{I_{zz} t}$$



Name: __Solution__________________ Instructor: Siegmund Sarkar Susilo
(Print) (Last) (First) (Circle one)

$$I_{zz} = \frac{\pi}{4}c^4 = 49.09\times10^{-3}\ \text{in}^4$$

$$\text{@A \& C,}\quad Q_{zz}\big|_{y=\pm0.5\ \text{in}} = 0\ \text{in}^3 \quad\Rightarrow\quad \tau_{xy} = 0\ \text{ksi}$$

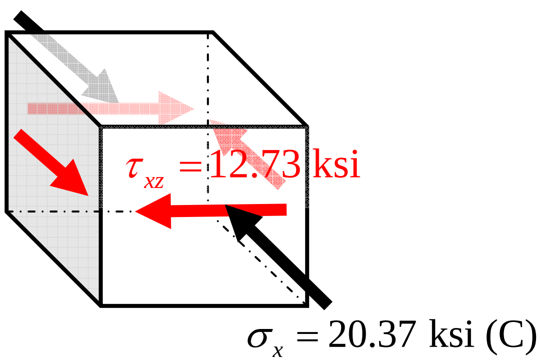
$$\text{@B \& D,}\quad Q_{zz}\big|_{y=0\ \text{in}} = \sum \bar{y}_i^{'} A_i^{'} = \left(\frac{4r}{3\pi}\right)\left(\frac{\pi}{2}r^2\right) = 8.33\times10^{-3}\ \text{in}^3$$

$$\Rightarrow\quad \tau_{xy} = \frac{V_y Q_{zz}}{I_{zz}t} = \frac{(300\ \text{lb})(8.33\times10^{-3}\ \text{in}^3)}{(49.09\times10^{-3}\ \text{in}^4)(2\times0.5\ \text{in})}$$

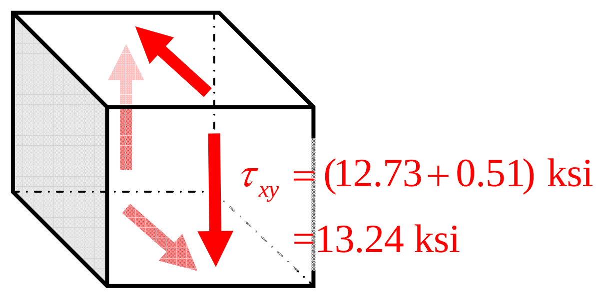
$$= 0.51\ \text{ksi (downwards)}$$

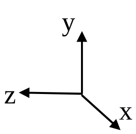
The combined state of stress at points A, B, C, and D will be the resultant **linear superposition** of the individual stress values due to torsion, bending, and shear force.

Point element A

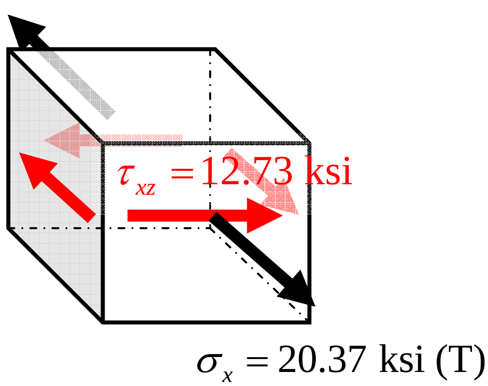


Point element B

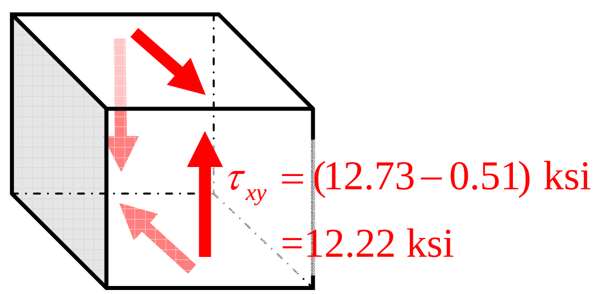




Point element C



Point element D



PROBLEM #3 (33 points)

Name: __Solution__________________ Instructor: Siegmund Sarkar Susilo
(Print) (Last) (First) (Circle one)

At a point in a structure the stress state is determined to be plane stress and characterized by $\boldsymbol{\sigma_x = 20}$ **MPa**, $\boldsymbol{\sigma_y = 40}$ **MPa**, $\boldsymbol{\tau_{xy} = 50}$ **MPa**.

Determine:

- The values of the first and second principal stresses; and draw a properly oriented material element for this stress state;
- The value of the maximum in-plane shear stress; and draw a properly oriented material element for this stress state;
- Draw Mohr's circle for the stress state, and on Mohr's circle mark the locations representing the stress state in (x-y) coordinates, mark the locations representing the principal stresses and the maximum shear stress.

<u>SOLUTION:</u> Principal stresses are given by

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{20+40}{2} \pm \sqrt{\left(\frac{20-40}{2}\right)^2 + (50)^2} = 30 \pm \sqrt{10^2 + 50^2} = 30 \pm 50.9$$

$$\sigma_1 = 30 + 50.9 = 81\,MPa$$

$$\sigma_2 = 30 - 50.9 = -21\,MPa$$

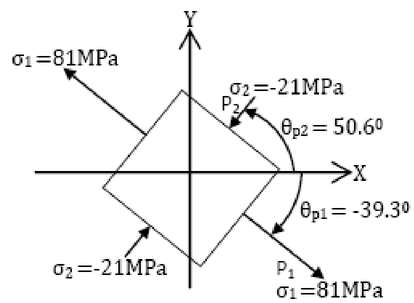
The orientation of the principal stress plane is given by:

$$\tan(2\theta_p) = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$, substituting the values we get

$$\tan(2\theta_p) = \frac{50}{(20-40)/2} = -5$$

$$\theta_{p1} = -39.3^0$$

$$\theta_{p2} = \theta_{p1} + 90^0 = -39.3^0 + 90 = 50.6^0$$



Having found the principal stress values and the orientation of the principal plane, a material element can be drawn as shown on the side.

The value of maximum in-plane shear stress can be calculated using the following:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{20-40}{2}\right)^2 + 50^2}$$

$$\tau_{max} = 50.99\,MPa \approx 51\,MPa$$

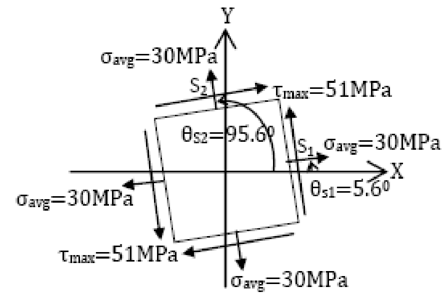
The orientation of the maximum in-plane shear stress is given by:

$$\tan(2\theta_s) = \frac{-\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\tau_{xy}}$$

$$\tan(2\theta_s) = \frac{-\left(\frac{20-40}{2}\right)}{50} = \frac{1}{5}$$

$$\theta_{s1} = 5.6^0$$

$$\theta_{s2} = \theta_{s1} + 90^0 = 5.6 + 90 = 95.6^0$$



On the plane of maximum in-plane shear stress the normal stresses are the average normal stress given by $\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{20+40}{2} = 30\,MPa$. With this we can plot the material element for the maximum in-plane shear stress as shown on the side.

Now Mohr circle can be drawn as shown below: