ME323 Midterm2 Practice Exam solutions:

PROBLEM #1

Given the plane stress state $\sigma_x = 20M\ Pa, \sigma_y = 50M\ Pa, \tau_{xy} = 10M\ Pa$ (All other stress components being zero) at a point in a body.

(a) Draw Mohr circle and represent the stress state $\sigma_x, \sigma_y, \tau_{xy}$ on Mohr circle as points X and Y correspond to the given stress state.
(b) Determine the principal stresses and the maximum in-plane shear stress; indicate them on the Mohr's circle.
(c) Draw a correctly oriented stress element for the principal stress state as well as for the maximum in-plane shear stress state (indicate rotation relative to the $x\,y$ coordinates and draw the stress vectors)

Solution:

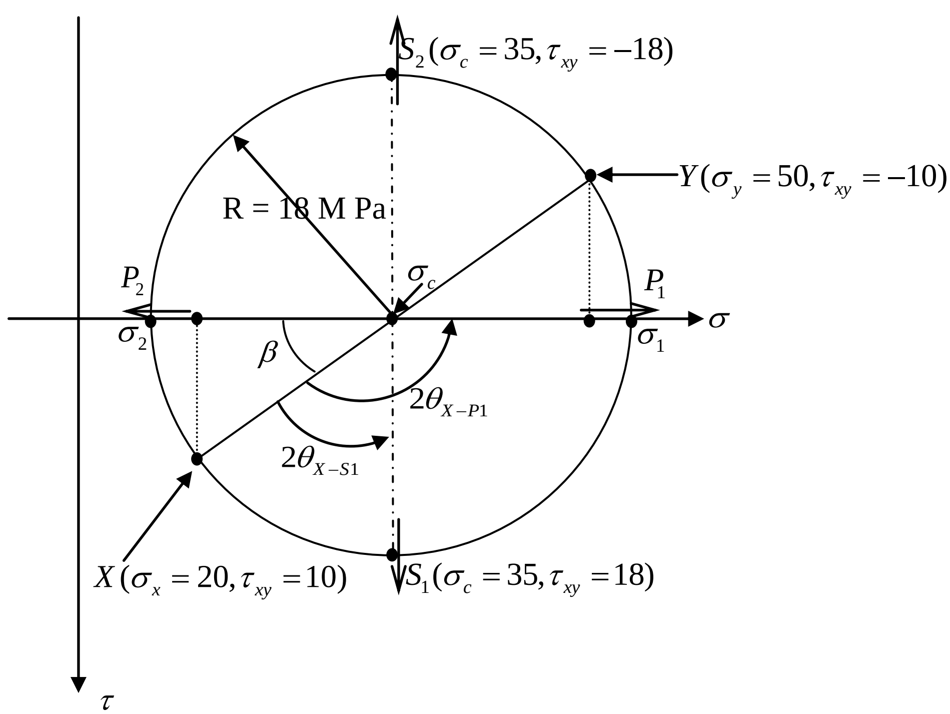
Given: $\sigma_x = 20M\ Pa, \sigma_y = 50M\ Pa, \tau_{xy} = 10\ MPa$

(a) The center and radius of Mohr circle are given by the following:

$$\sigma_c = \frac{\sigma_x + \sigma_y}{2} = \frac{(20+50)}{2} = 35\ MPa$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{20-50}{2}\right)^2 + 10^2} = 18\ MPa$$

Using the center and radius values calculated above, Mohr circle can be drawn by taking σ on the x-axis and τ on the y-axis.



(b) Principal stresses are given by:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + R = \sigma_c + R$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - R = \sigma_c - R$$

Substituting the values of σ_c, R in the above expressions we get:

$$\sigma_1 = 35 + 18 = 53M\ Pa$$

$$\sigma_2 = 35 - 18 = 17M\ Pa$$

Principal stresses, σ_1, σ_2 when plotted on the Mohr circle can be located at the diametrically end points on the σ axis.

Maximum in-plane shear stress is given by:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = R$$

$$\tau_{max} = R = 18\,MPa$$

At the location of maximum in-plane shear stress, the normal stress values are given by:

$$\sigma_{x'y}(\theta_{s1}) = \sigma_{xy'}(\theta_{s2}) = \left(\frac{\sigma_x + \sigma_y}{2}\right) = \sigma_c = 35\,MPa$$

τ_{max}, σ_c can be used to represent the maximum in-plane shear stress location on Mohr circle as shown by points S1 and S2.

(c) The orientation of the principal stress plane from X can be determined as follows: From the figure, $2\theta_{X-P1}$ is the angle between X and P1 axis measured CCW.

$\beta + 2\theta_{X-P1} = 180^o$, also we have

$$\sin(\beta) = \frac{\tau_{xy}}{R} = \frac{10}{18}$$

$$\beta = \sin^{-1}(\frac{10}{18}) = 34^o$$

$$2\theta_{X-P1} = 180^o - 34^o = 146^o$$

$$\theta_{X-P1} = 73^o$$

The orientation of the maximum in-plane shear stress plane can be determined from the Mohr circle as follows: Representing $2\theta_{X-S1}$ as the angle between X axis and S1 axis measured CCW, we have

$$\tan(2\theta_{X-S1}) = -\frac{\frac{(\sigma_x - \sigma_y)}{2}}{\tau_{xy}}$$

$$\tan(2\theta_{X-S1}) = -\frac{(20-50)/2}{10} = \frac{15}{10}$$

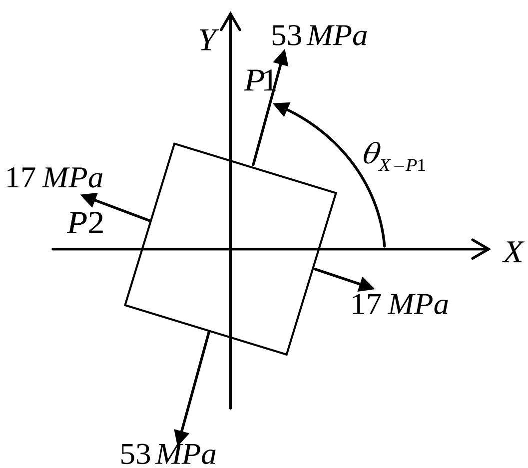
$$\theta_{X-S1} = \frac{1}{2} tan^{-1}(\frac{15}{10})$$

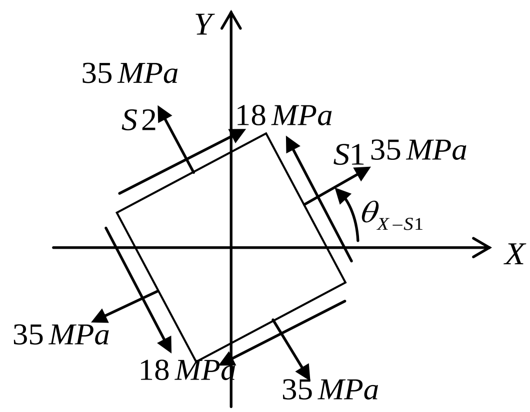
$$\theta_{X-S1} = 28^o$$

The orientation of the maximum in-plane shear stress plane can also be verified or calculated from θ_{X-P1} as follows:

$$\theta_{X-S1} = \theta_{X-P1} - 45^o = 73 - 45 = 28^o$$

Having found the orientations of the principal and maximum in-plane shear stress planes we can now draw stress elements oriented on these planes as follows:

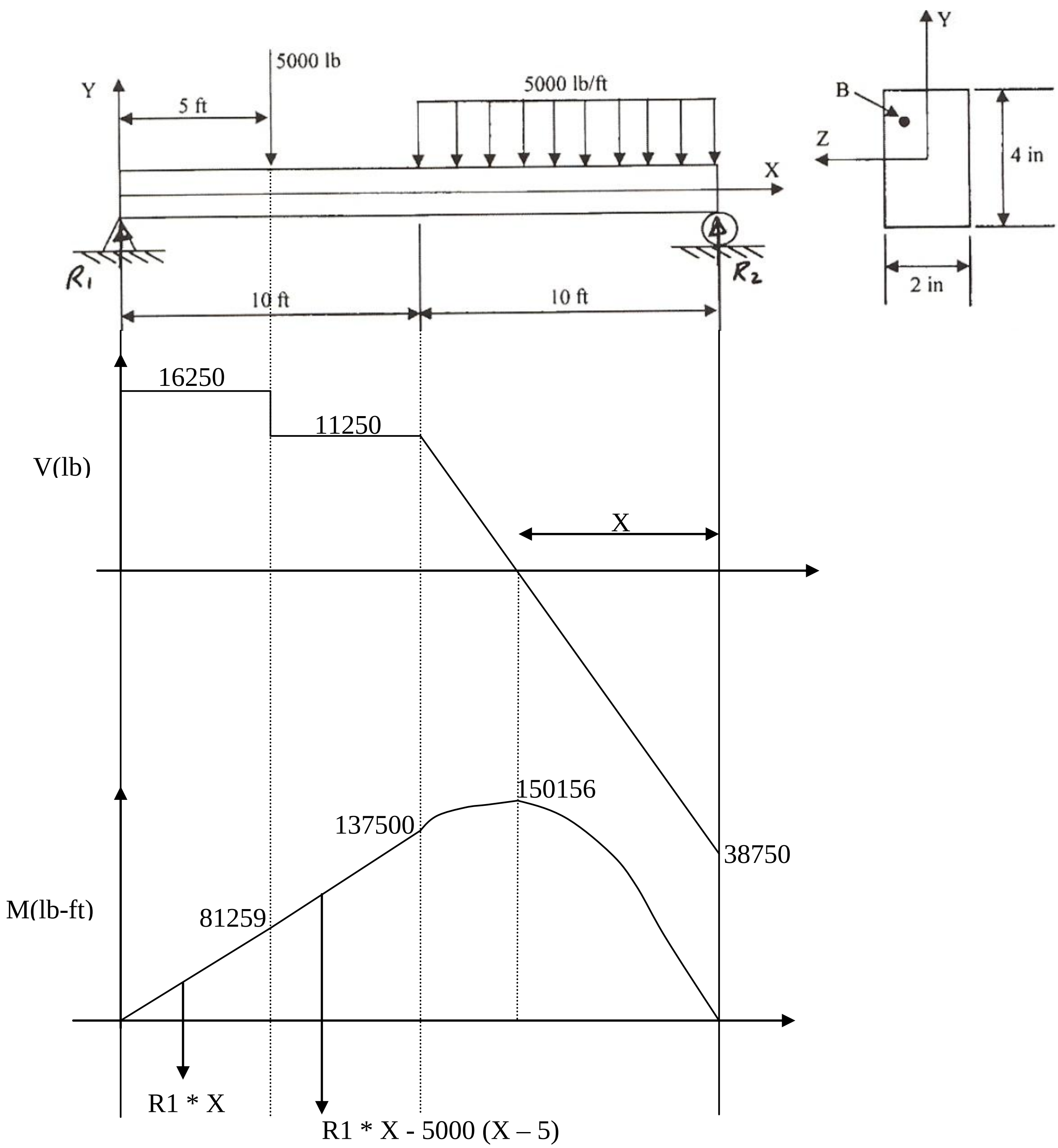




PROBLEM #2

The simply supported beam shown below has a rectangular cross section. The beam is required to support a concentrated load of 5000 lb. and a distributed load of 5000 lb/ft.

1. Draw the free body diagram of the beam
2. Determine the reaction forces at the support.
3. Draw shear and moment diagrams for the entire beam (note points will be given to the shear and moment diagrams and not to the shear and moment equations).
4. At X = 10 ft from the left end of the beam, determine the normal and shear stresses for the point B located at Z = 0.5 in and Y = 1 in. as shown on the cross section of the beam.



(1) Using the reactions at the supports the free body diagram of the beam can be drawn as shown in figure.
(2) Calculation of reactions at supports:

$$\Sigma M_{R_2} = 0$$

$$R_1 \times 20 - 5000 \times 15 - 5000 \times 10 \times 5 = 0$$

$$R_1 = \frac{(75000 + 250000)}{20}$$

$$R_1 = 16250\, lb$$

$$R_1 + R_2 = 5000 + 5000 \times 10$$

$$R_2 = 38750\, lb$$

(3) The shear force and bending moment diagrams can be plotted as shown in the figure.
The location where bending moment is maximum can be found from similar triangles as follows:

$$\frac{38750}{X} = \frac{11250}{10 - X}$$

$$38750 - 38750\, X = 11250\, X$$

$$X = 7.75\ ft$$

(4) At a location of X = 10 ft, we can read shear force (V), and bending moment (M) from the diagram as follows:

$$V = 11250\, lb$$

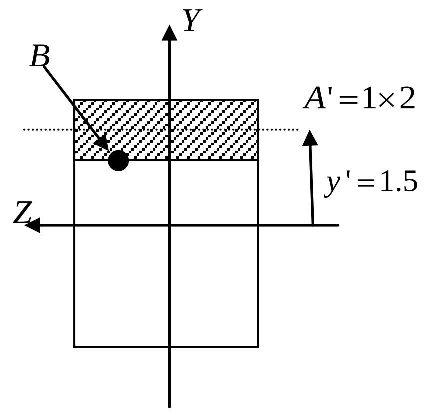
$$M = 137500\, lb\ ft$$

$$I = \frac{1}{12}(bh^3) = \frac{2 \times 4^3}{12} = 10.67 in^4$$

$$Q_{zz} = A' \times y' = 1 \times 2 \times 1.5 = 3 in^3$$

$$\sigma = \frac{M \times c}{I} = \frac{(137500 \times 12 in/\ ft) \times 1 in}{10.67} = 154639.2\ psi$$

$$\tau = \frac{V \times Q}{I \times t} = \frac{11250 \times 3}{10.67 \times 2} = 1581.5\ psi$$



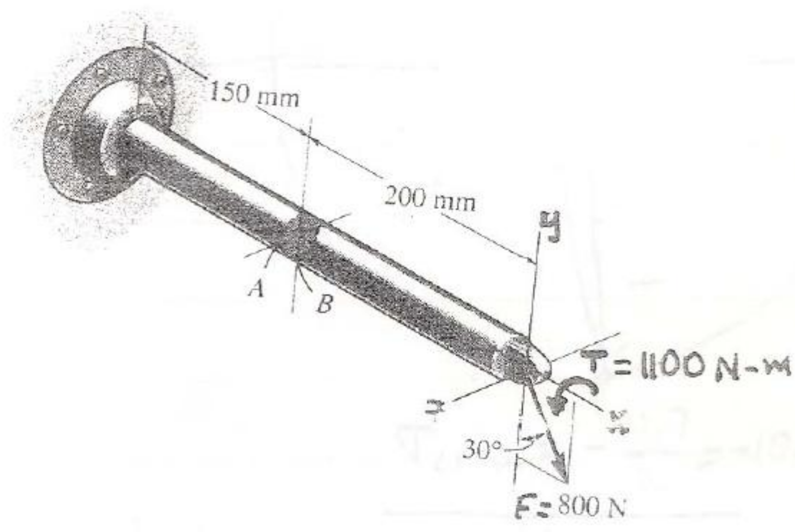
PROBLEM#3

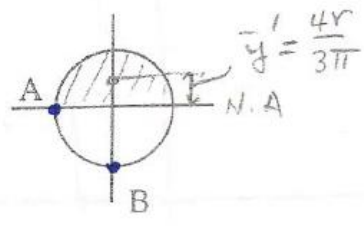
The circular structure shown has a diameter of d = 40 mm. It is subjected to a force of F = 800 N and a torque T = 1100 N-m at its end as shown.

At each of points A and B find:

(a) Stress components and show them on a material element

(b) The principal stresses using Mohr's circle:

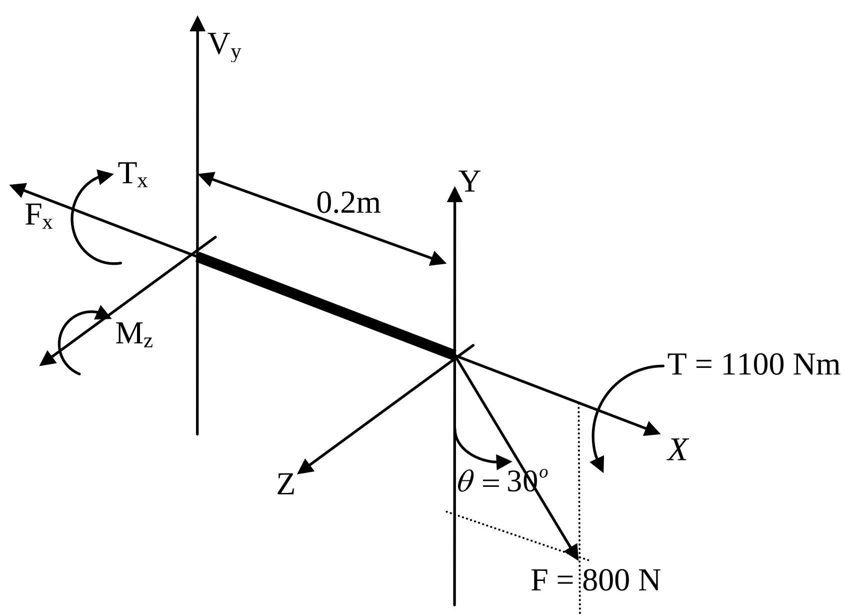




$A: \sigma_1 = 88.39MPa, \sigma_2 = -88.08MPa$

$A: \sigma_1 = 77MPa, \sigma_2 = -99MPa$

Solution:



(a) Applying equilibrium equations on the plane where A&B are located we get:

$$\Sigma F_X = 800 sin(30^o) - F_x = 0$$

$$F_x = 400\,N$$

$$\Sigma F_Y = V\text{- } 800\ \cos(30^o) = 0$$

$$V = 692.8\,N$$

$$\Sigma M_Z = \mathrm{M}_Z + 692.8 \times 0.2 = 0$$

$$M_Z = -138.56\,Nm$$

$$\Sigma M_X = -T + 1100 = 0$$

$$T = 1100\,Nm$$

Stresses due to F_x are:

$$\sigma_A = \sigma_B = \frac{F_X}{A} = \frac{400 \times 4}{\pi \times 0.04^2} = 0.318\,MPa$$

Stresses due to V_y are:

$$\tau_A = \frac{V_y \times Q}{I \times t}$$

$$Q = A' \times y' = \left(\frac{\pi \times d^2}{8}\right)\left(\frac{4 \times d}{6 \times \pi}\right) = \frac{d^3}{12}$$

$$\tau_A = \frac{(692.8) \times (0.04)^3}{12\left(\frac{\pi (0.04)^4}{64}\right)(0.04)} = 0.735$$

$$\tau_B = 0$$

Stresses due to M_z are:

$$\sigma_A = 0$$

$$\sigma_B = -\frac{My}{I} = -\frac{(-138.56)(-0.02)(64)}{\pi (0.04)^4} = -22.05\,MPa$$

Stresses due to T_x are:

$$\tau_A = \tau_B = \frac{T \times r}{J} = \frac{(1100)(0.02)32}{\pi (0.04)^4} = 87.5\,MPa$$

Total shear stress at point A = $\tau_A = \tau_A(\text{Shear Force } \mathrm{V}_y) + \tau_A(Torsion)$

$$\tau_A = 87.5 + 0.735 = 88.235\,MPa$$

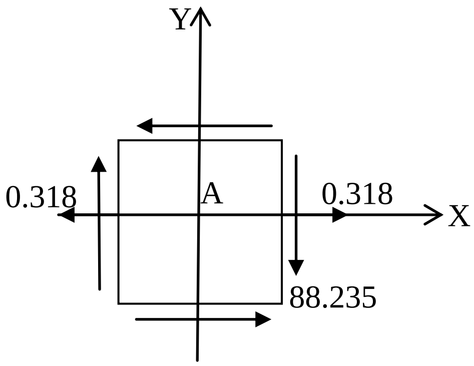
Total Normal stress at point B = $\sigma_B = \sigma_B(Axialforce\ F_X) + \sigma_B(Bending\ moment\ M_Z)$

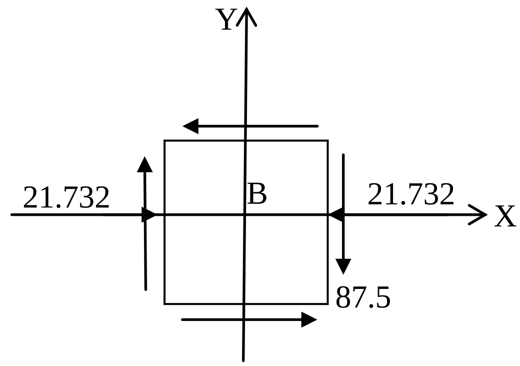
$$\sigma_B = 0.318 - 22.05 = -21.732\,MPa$$

$$\tau_{xy} = -\tau_A$$

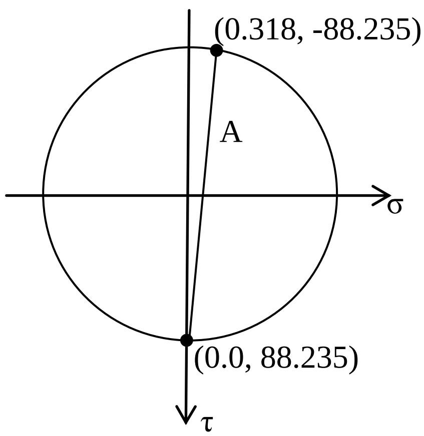
$$\tau_{xz} = -\tau_B$$

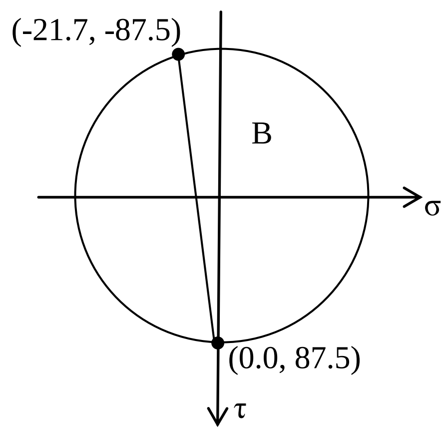
Now the stress components on the material elements can be plotted as follows:





(b)Principal stresses can be computed using Mohr's circle as follows:





$$\sigma_{center} = \frac{0.318}{2} = 0.159$$

$$R = \sqrt{(0.159)^2 + (88235)^2} = 88.235$$

$$\sigma_{center} = -\frac{21.7}{2} = -10.866$$

$$R = \sqrt{(10.866)^2 + (875)^2} = 88.172$$

Principal stresses are given by
At location A:

$$\sigma_1 = 0.159 + 88.235 = 88.39\,MPa$$

$$\sigma_2 = 0159 - 88235 = -88.08\,MPa$$

At location B:

$$\sigma_1 = -10.866 + 88.172 = 77.31\,MPa$$

$$\sigma_2 = -10.866 - 88.172 = -99.04\,MPa$$