Name
(Print)______________SOLUTION____

(Last) (First)

ME 323 - EXAM # 2
Date: November 5, 2008 – Time: 8:00 – 9:30 p.m.
Location: Lily Room 1105

Instructions:

Circle your Lecturer's name and your class meeting time.

Sadeghi Rao Siegmund

Begin each problem in the space provided on the examination sheets. If additional space is required, use the yellow paper provided.

Work on one side of each sheet only, with only one problem on a sheet.

Please remember that for you to obtain maximum credit for a problem, you must present your solution clearly.

Accordingly,

- coordinate systems must be clearly identified,
- free body diagrams must be shown,
- units must be stated,
- theories used have to be identified,
- write down clarifying remarks,
- state your assumptions, etc.

If your solution cannot be followed, it will be assumed that it is in error.

When handing in the test, make sure that ALL SHEETS are in the correct sequential order. Remove the staple and restaple, if necessary.

Prob. 1 ______________________

Prob. 2 ______________________

Prob. 3 ______________________

Total ________________________

Relevant Equations

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu(\varepsilon_y + \varepsilon_z)\right]$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_y + \nu(\varepsilon_z + \varepsilon_x)\right]$$

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_z + \nu(\varepsilon_x + \varepsilon_y)\right]$$

$$e = \frac{FL}{EA} + L\alpha\Delta T$$

$$FS = \frac{Failure\ Stress}{Allowable\ Stress}, \frac{Yield\ Strength}{State\ of\ Stress}$$

$$\sigma(x,y) = \frac{-E(x)y}{\rho(x)} = \frac{-M(x)y}{I_z}$$

$$I_z = \int_A y^2 dA$$

$$I = \frac{bh^3}{12} \qquad \text{(rectangle)}$$

$$I = \sum\left[I_i' + y_{c-c}^2 A_i\right]$$

$$\tau = \frac{VQ}{Ib}$$

$$Q(y) = \int_{A'} \eta dA = A'\bar{y}'$$

$$EIv'' = M$$

$$EIv''' = V$$

$$EIv'''' = p$$

$$\tau = Gr\frac{\phi}{L}$$

$$\tau = \frac{Tr}{I_p}$$

$$\phi = \frac{TL}{GI_p}$$

$$I_p = \frac{\pi d^4}{32}$$

$$I_p = \frac{\pi(d_o^4 - d_i^4)}{32}$$

$$\langle x-a\rangle^n = \begin{cases} 0 & for\ x < a \\ (x-a)^n & for\ x \geq a \end{cases} \qquad n = 0,1,2,3$$

$$\int\langle x-a\rangle^n\, dx = \begin{cases} \langle x-a\rangle^{n+1} & for\ n \leq 0 \\ \frac{1}{n+1}\langle x-a\rangle^{n+1} & for\ n > 0 \end{cases}$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2{}_{xy}} \qquad \sigma_{avg} = \left(\frac{\sigma_x + \sigma_y}{2}\right)$$

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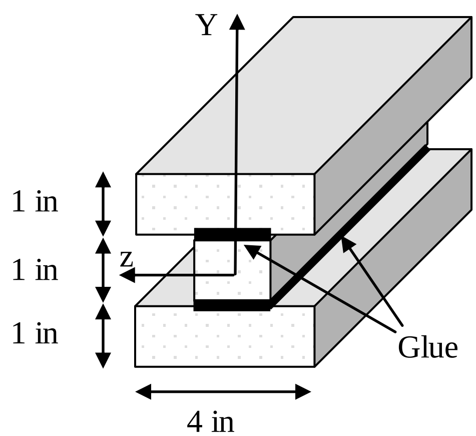
PROBLEM #1 (33 points):

(23 points) Two wood boards, each of 1 inch x 4 inch rectangular cross section and a board of 1 inch x 1 inch cross section are glued together to form an I-beam. The beam needs to withstand a vertical shear force of $F_y = 400$ lb in the +y-direction. What is the required shear strength of the glue?



(10 points) The beam is subject to a bending moment (M_{zz} = 400 in lb). Determine the normal stress at 1.0 inches above the neutral axis.

(1) Calculate I: $I = I_{outline} - I_{gaps}$

$$= \frac{4\times 3^3}{12} - 2\,\frac{1.5\times 1^3}{12} = \underline{\underline{8.75\ in^4}}$$

$Q = \bar{y}'A' = (1.0\,in)\cdot(4\,in \times 1\,in) =$
$= 4\,in^3$

$$\tau = \frac{VQ}{It} = \frac{400\,lb \cdot 4\,in^3}{8.75\,in^4 \cdot 1.0\,in} = \underline{\underline{182.8\ psi}}$$

(2) $$\sigma_x = -\frac{My}{I} = -\frac{400\,in\,lb\,(1.0\,in)}{8.75\,in^4} = \underline{\underline{-45.7\ psi}}$$

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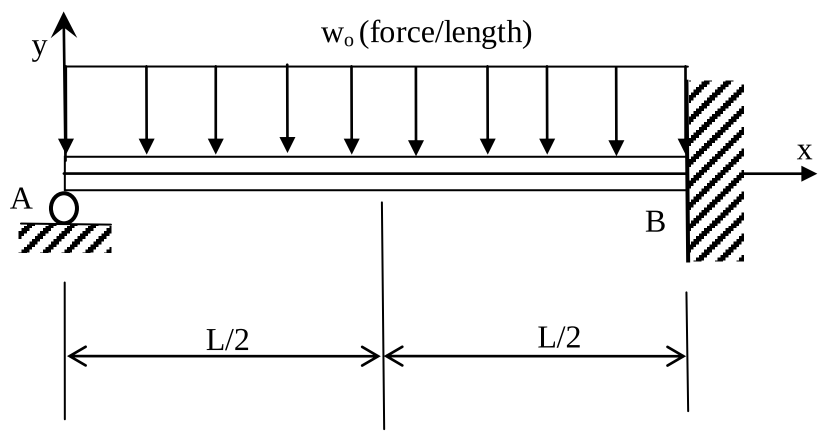
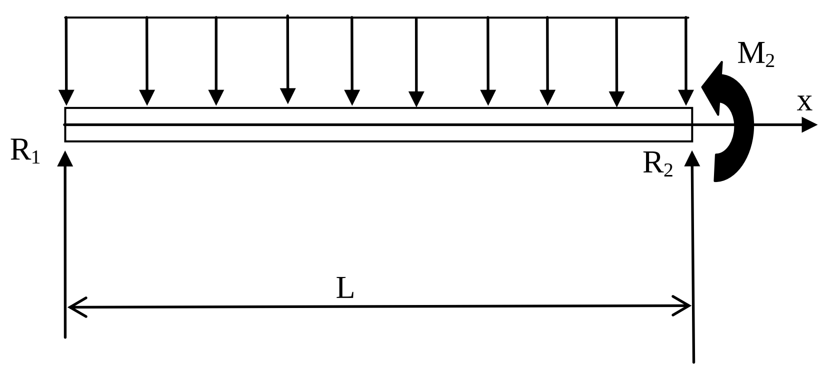
PROBLEM #2 (33 points):

The beam shown below has a constant cross sectional area and modulus of elasticity (EI = constant). It is supported by a roller at A and fixed into the wall at B. The beam is loaded vertically downward by a distributed load w_o as shown. Determine:

- The reaction forces and moment at the supports





$\sum F_y = 0 \rightarrow R_1 + R_2 = w_0 L$

$\sum M_{R_2} = 0 \rightarrow R_1 * L - w_0 L^2 / 2 - M_2 = 0$

$2 \; equations \; and \; three \; unknowns : R_1, R_2, M_2$

$q(x) = R_1 < x >^{-1} - w_0 < x >^0$

$V(x) = R_1 < x >^0 - w_0 < x >^1$

$$M(x) = R_1 < x >^1 - \frac{w_0}{2} < x >^2$$

$$EIv^{'}(x) = \frac{R_1}{2} < x >^2 - \frac{w_0}{6} < x >^3 + C_1$$

$$EIv(x) = \frac{R_1}{6} < x >^3 - \frac{w_0}{24} < x >^4 + C_1 x + C_2$$

Boundary Conditions are :

1. $x = 0, v = 0$
2. $x = L, v = 0$
3. $x = L, v^{'} = 0$

using boundary condition 1 $\rightarrow C_2 = 0$

using boundary condition 3 $\rightarrow C_1 = -\frac{R_1}{2} L^2 + \frac{w_0}{6} L^3$

Substitute for C_1 from above in deflection equation and then use boundary condition 3, we obtain

$$R_1 = \frac{3w_0}{8} L$$

Substitute for R_1 in sum of forces in the y direction results in:

$$R_2 = \frac{5w_0}{8} L$$

Substitute for R_1 in sum of the moments at the wall results in:

$$M_2 = -\frac{w_0}{8} L^2$$

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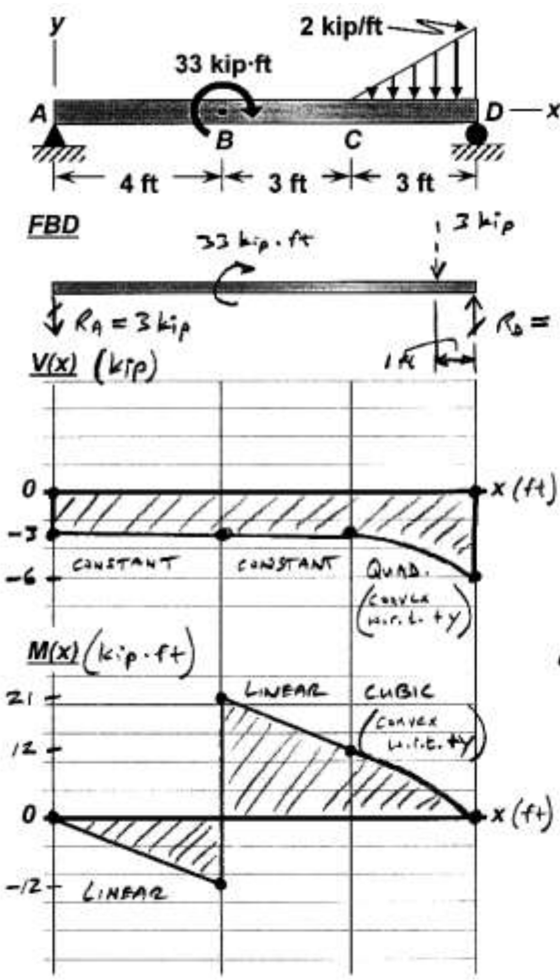
PROBLEM #3 (34 points):

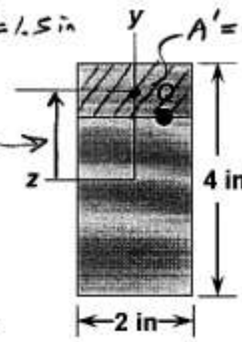
For the cantilever beam shown below, determine:

1. (2 pt) The free body diagram for the beam (in the space provided below).
2. (2 pt) All support reactions.
3. (22 pt) Shear and moment diagrams for the entire beam (include units for graph axes).
   NOTE: Exact expressions are **not** necessary, but relevant interval descriptors are required, i.e. endpoints and functional forms (e.g. constant, linear, quadratic, etc.).
4. (8 pt) Determine the normal and shear stresses at point Q, which is located at x = 7 ft, y = 1 in, and z = -0.5 in, as shown on the cross section diagram of the beam below.



$\bar{y}' = 1.5\text{ in}$

$A' = (2\text{ in})(1\text{ in}) = 2\text{ in}^2$



$$I_z = \frac{bh^3}{12} = \frac{(2\text{ in})(4\text{ in})^3}{12} = 10.67\text{ in}^4$$

STRESSES @ Q (7 ft, 1 in, -0.5 in)

$$\sigma_x = \frac{-M_z y}{I_z} = \frac{-(12\text{ kip}\cdot\text{ft})(12\,\frac{\text{in}}{\text{ft}})(1\text{ in})}{10.67\text{ in}^4}$$

$$\boxed{\sigma_x = 13.50\text{ ksi (C)}}$$

$$\tau_{xy} = \frac{|V||Q|}{I_z\, t} = \frac{(3\text{ kip})(1.5\text{ in})(2\text{ in}^2)}{(10.67\text{ in}^4)(2\text{ in})}$$

$$\boxed{\tau_{xy} = 421\text{ psi (in } -y \text{ dir)}}$$