Exam 1

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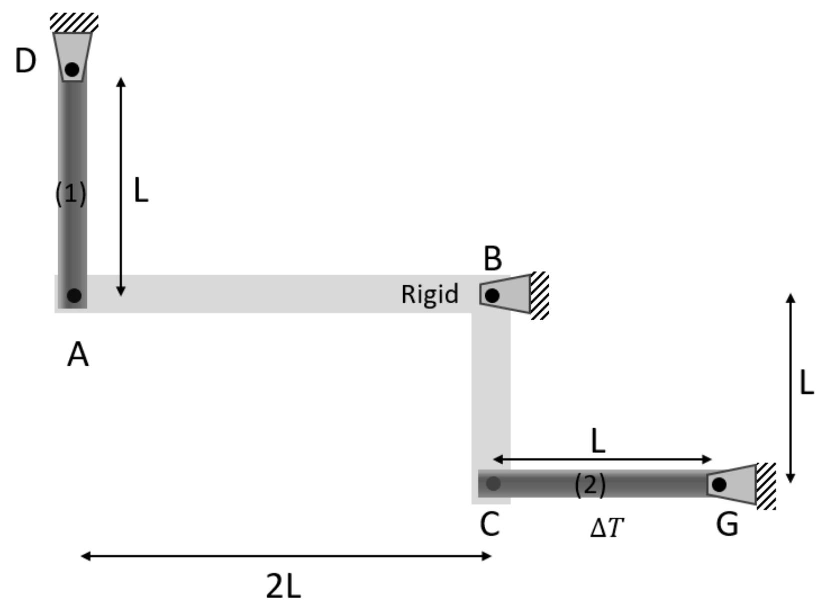
PROBLEM #1 (25 points)

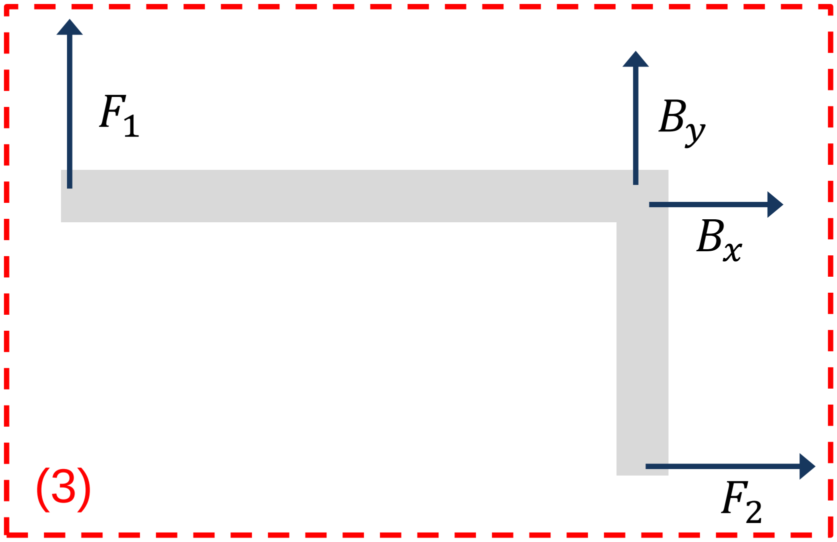
A rigid L-shaped bar ABC is connected at A to a deformable bar AD. AD has a Young's modulus of E and an area of 2A. ABC is also connected to a deformable bar at C that has a Young's modulus of 2E and an area of A. The thermal expansion coefficient of CG is α and temperature of deformable bar CG is decreased by ΔT.

	(1)	(2)
E	E	2E
A	2A	A
ΔT	0	ΔT
α	$10\text{x}10^{-6}$ °C^{-1}	$10\text{x}10^{-6}$ °C^{-1}

(a) Draw the free body diagram for the assembly.
(b) Write the equilibrium equation(s) for the assembly.
(c) Is the assembly determinate or indeterminate?
(d) Find the stresses in each member in terms of L, A, E, ΔT, and/or α.
(e) **E=60 GPa**, **A = 100 mm²**, and **L=2m**. The ultimate tensile stress of all of the materials is **σ_U=180 MPa**. Using a factor of safety of 3.0, determine the maximum temperature change (ΔT) that the assembly can withstand.



(a) FBD



(b) **Equilibrium**

$$(\Sigma M)_B = -F_1 2L + F_2 L = 0$$

$$F_2 = 2F_1$$

(4)

(c) Assembly is **indeterminate** (2 unknowns and 1 equation).

(2)

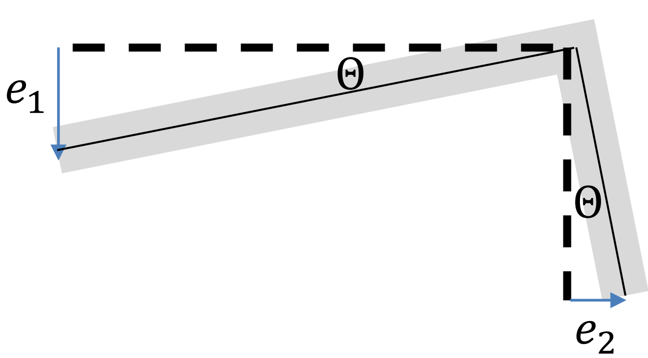
(d) **Force-Elongation**

$$e_1 = \frac{F_1 L_1}{E_1 A_1} + \alpha \Delta T_1 L_1 \qquad e_2 = \frac{F_2 L_2}{E_2 A_2} + \alpha \Delta T_2 L_2$$

$$e_1 = \frac{F_1 L}{2EA}$$ (1)

$$e_2 = \frac{F_2 L}{2EA} + \alpha \Delta T L$$ (2)

Compatibility



$$\tan(\theta) = \frac{e_1}{2L} = -\frac{e_2}{L}$$

$$e_1 = -2e_2$$ (2)

Solve

$$e_1 = -2e_2$$

$$\frac{F_1 L}{2EA} = -2\left[\frac{F_2 L}{2EA} + \alpha \Delta T L\right]$$

$$\frac{F_1 L}{2EA} = -2\left[\frac{2F_1 L}{2EA} + \alpha \Delta T L\right]$$

$$F_1 = -4F_1 - 4\alpha \Delta T E A$$

$$F_1 = -\left(\frac{4}{5}\right)\alpha \Delta T E A \qquad F_2 = -\left(\frac{8}{5}\right)\alpha \Delta T E A$$

$$\sigma_1 = -\left(\frac{2}{5}\right)\alpha \Delta T E \qquad \sigma_2 = -\left(\frac{8}{5}\right)\alpha \Delta T E$$

(2)

(e)

$$FS = \frac{\sigma_U}{\sigma_{aallow}} \longrightarrow \sigma_{allow} = \frac{\sigma_U}{3} = 160\ MPa$$

(2)

σ_2 has largest stress; will limit failure. (1)

$$\sigma_{allow} = -\left(\frac{8}{5}\right)\alpha \Delta T E$$

(1)

$$\Delta T = -\left(\frac{5}{8}\right)\left(\frac{\sigma_{allow}}{E}\right)\left(\frac{1}{\alpha}\right) = -\left(\frac{5}{8}\right)\left(\frac{60 * 10^6}{60 * 10^9}\right)\left(\frac{1}{10^5}\right) = 62.5\ °C$$

(1) (1)

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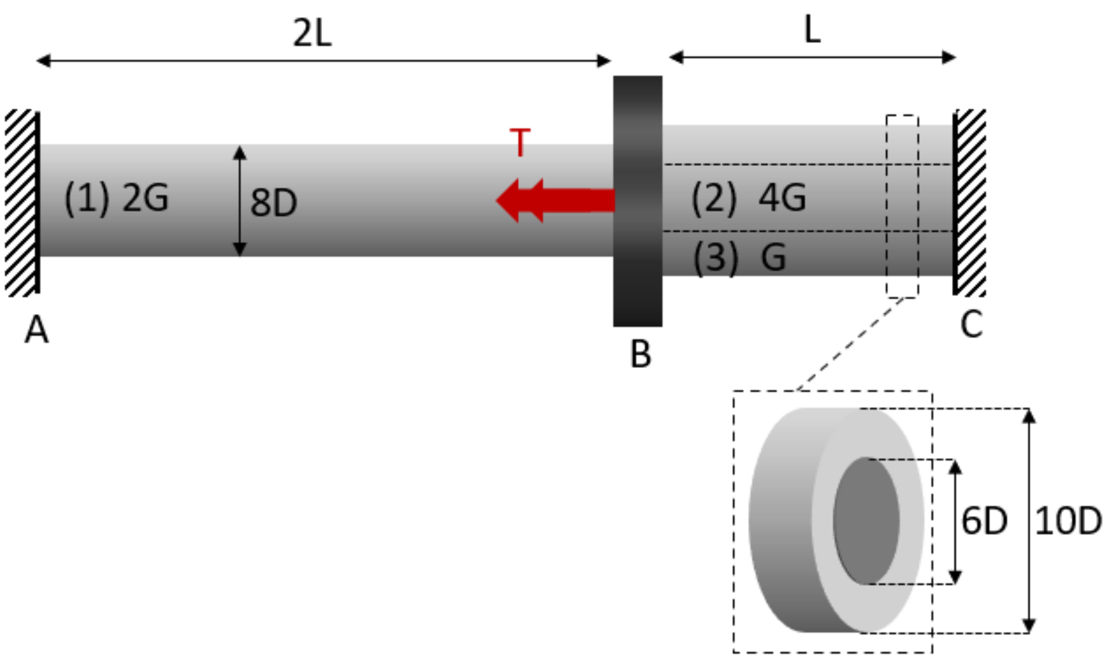
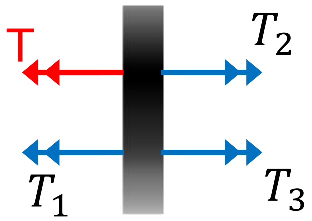
PROBLEM #2 (25 points)

A torsion assembly is rigidly secured at A and C and has an applied torque of T at rigid connector B. A solid shaft with a diameter of 8D and a shear modulus of 2G connects A and B. B and C are connected with a bimetallic torsion member with an inner metal with a diameter of 6D and a modulus of 4G and an outer metal with an outer diameter of 10D and a modulus of G.

(a) Draw the free body diagram for the assembly.
(b) Write the equilibrium equation(s) for the assembly.
(c) Is the assembly determinate or indeterminate?
(d) Find the torques in each member in terms of L, D, G, and/or T.
(e) Find the rotation of connector B in terms of L, D, G, and/or T.



(a) FBD



(2)

(b) **Equilibrium**

$$\Sigma T_B = -T - T_1 + T_2 + T_3 = 0$$

(4)

(c) Assembly is **indeterminate** (3 unknowns and 1 equation).

(1)

(d) **Torque-Twist**

$$I_{p1} = \left(\frac{\pi}{2}\right)\left(\frac{8D}{2}\right)^4 = 128\pi D^4 \qquad \Delta\phi_1 = \frac{T_1 L_1}{G_1 I_{P1}} = \frac{T_1(2L)}{2GI_{p1}} = \frac{T_1 L}{128\pi G D^4}$$

$$I_{p2} = \left(\frac{\pi}{2}\right)\left(\frac{6D}{2}\right)^4 = \frac{81}{2}\pi D^4 \qquad \Delta\phi_2 = \frac{T_2 L_2}{G_2 I_{P2}} = \frac{T_2 L}{4GI_{p2}} = \frac{T_2 L}{168\pi G D^4}$$

$$I_{p3} = \left(\frac{\pi}{2}\right)\left[\left(\frac{10D}{2}\right)^4 - \left(\frac{6D}{2}\right)^4\right] = 272\pi D^4 \qquad \Delta\phi_3 = \frac{T_3 L_3}{G_3 I_{P3}} = \frac{T_3 L}{GI_{p1}} = \frac{T_3 L}{272\pi G D^4}$$

(4)

Compatibility

$$\left.\begin{aligned} \Delta\phi_1 + \Delta\phi_2 &= 0 \\ \Delta\phi_1 + \Delta\phi_3 &= 0 \\ \Delta\phi_2 &= \Delta\phi_3 \end{aligned}\right\}$$

Degree of indeterminacy of 2; need two of these equations

(3x2)

Solve

$$\frac{T_1 L}{128\pi G D^4} + \frac{T_2 L}{168\pi G D^4} = 0$$ (2)

$$T_1 = -\frac{128 T_2}{168}$$

$$\frac{T_2 L}{168\pi G D^4} = \frac{T_3 L}{272\pi G D^4}$$

$$T_3 = \frac{272 T_2}{168}$$

$$0 = -T - T_1 + T_2 + T_3$$

$$T = -\left(-\frac{128 T_2}{168}\right) + T_2 + \frac{272 T_2}{168}$$

$$T_2 = \left(\frac{168}{562}\right) T \qquad T_1 = -\left(\frac{128}{562}\right) T \qquad T_3 = \left(\frac{272}{562}\right) T$$

(e) $$\phi_B = \Delta\phi_1 = -\Delta\phi_2 = -\Delta\phi_3$$ (2)

$$\phi_B = \frac{T_1 (2L)}{2 G I_{p1}} = \frac{T_1 L}{128\pi G D^4}$$ (1)

$$\phi_B = \left(-\frac{128}{562}\right) \frac{L}{128\pi G D^4} = \frac{-TL}{562\pi G D^4}$$ (2)

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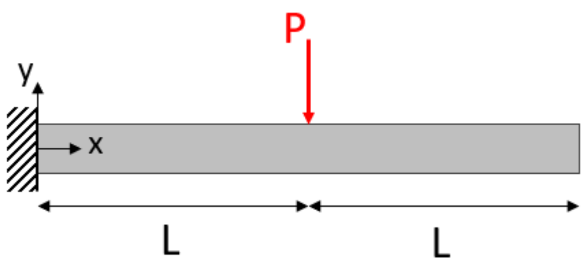
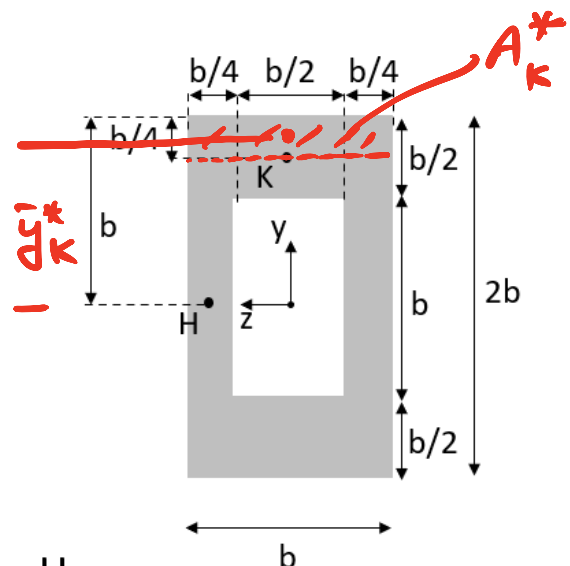
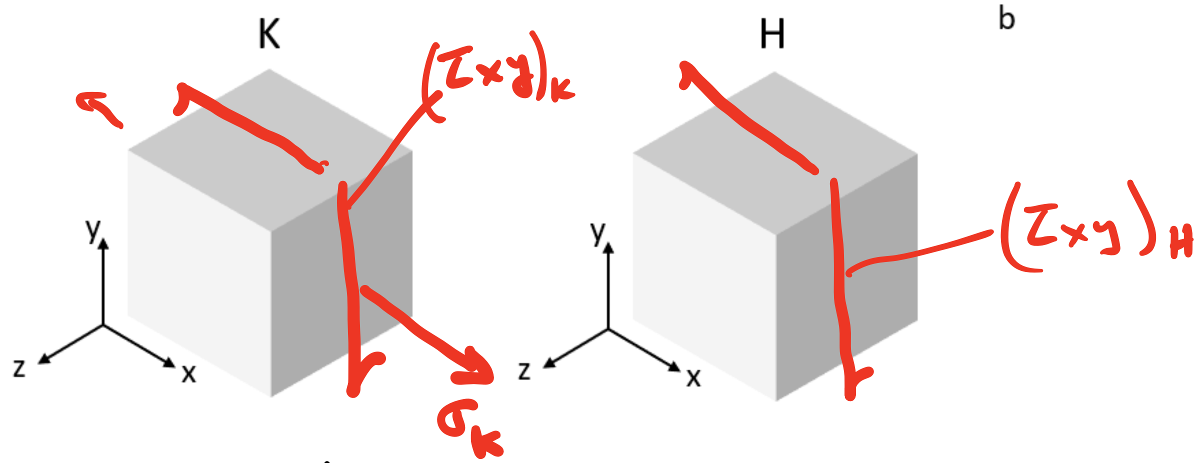
PROBLEM #3 (25 points)

The cantilever **rectangular box beam** is subjected to the loading shown below.

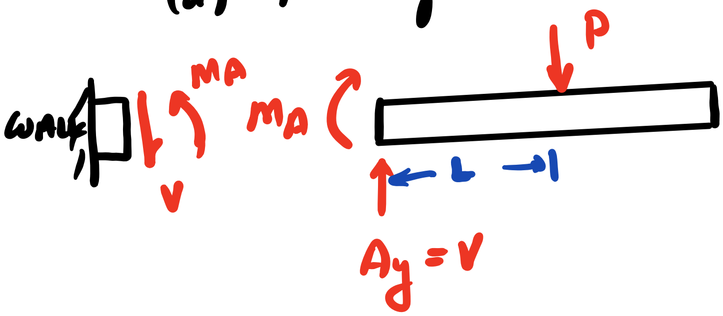
a) Determine the reactions on the beam at the wall
b) Determine the *normal stress* at the wall at point K and point H of the cross section.
c) Determine the *shear* stress at the wall at point K and point H of the cross section.
d) Show the normal and shear stresses obtained in parts (b) and (c) with the proper orientation on the stress elements shown below.







(a) FBD of beam



Equilibrium:

$$\Sigma M_A = -M_A - PL = 0$$

$$\boxed{M_A = -PL}$$

$$\Sigma F_y = V - P = 0$$

$$\boxed{V = P}$$

b) $I_{zz} = I_{\text{big rectangle}} - I_{\text{small rectangle}}$

$$= \frac{(b)(2b)^3}{12} - \frac{(b/2)(b)^3}{12} = \frac{b^4}{12}\left(8 - \frac{1}{2}\right)$$

$$\boxed{I_{zz} = \frac{5}{8} b^4}$$

$$\sigma_K = -\frac{(-PL)\left(\frac{3b}{4}\right)}{\frac{5}{8} b^4} = \boxed{\frac{6}{5}\frac{PL}{b^3} = \sigma_K}$$

$$\sigma_H = -\frac{(-PL)(0)}{\frac{5}{8} b^4} = \boxed{0 = \sigma_H}$$

c) $$\tau_K = \frac{V A_K^* \bar{y}_K^*}{I t_K}$$

$$A_K^* = (b)\left(\frac{b}{4}\right) = \frac{b^2}{4}$$

$$y_K^* = \frac{b}{2} + \frac{b}{4} + \frac{b}{8} = \frac{7b}{8}$$

$$t_K = b$$

$$\tau_K = \frac{(P)\left(\frac{b^2}{4}\right)\left(\frac{7b}{8}\right)}{\left(\frac{5}{8}b^4\right) b} = \boxed{\frac{7}{20}\frac{P}{b^2} = \tau_K}$$

$$\tau_H = \frac{V A_H^* \bar{y}_H^*}{I t_H}$$

$$A_H^* \bar{y}_H^* = \underbrace{A^*_{\text{full rect}}}\ \bar{y}^*_{\text{full rect}} - A^*_{\text{empty r.}}\ \bar{y}^*_{\text{empty r.}}$$

$$= (b)(b)\left(\frac{b}{2}\right) - \left(\frac{b}{2}\right)\left(\frac{b}{2}\right)\left(\frac{b}{4}\right) = \frac{7}{16} b^3$$

$$t_H = \frac{b}{4} + \frac{b}{4} = \frac{b}{2}$$

$$\tau_H = \frac{(P)\left(\frac{7}{16} b^3\right)}{\left(\frac{5}{8} b^4\right) \frac{b}{2}} = \boxed{\frac{7}{5} \frac{P}{b^2} = \tau_H}$$

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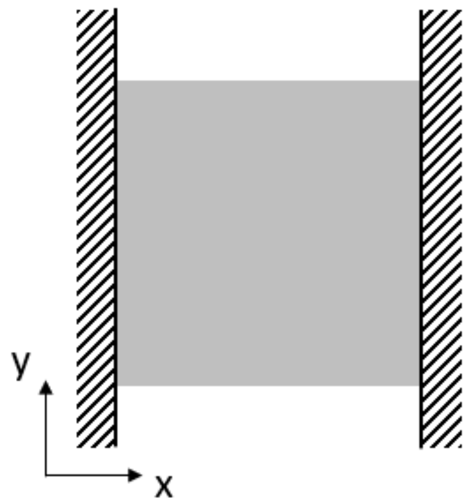
PROBLEM #4 – PART A (4 points)

A block is fully constrained in the x direction and is free to expand in the y and z (out of paper) directions. The block is initially stress free at room temperature. The temperature of the block is increased by ΔT. The coefficient of thermal expansion is α, the modulus of elasticity is E and the Poisson's ratio is ν.

Which of the following statements about stresses and strains is correct?



	σ_x	σ_y	ε_x	ε_y
(a)	$-\alpha E\Delta T$	0	$\alpha\Delta T$	$\nu\varepsilon_x$
(b)	$-\alpha E\Delta T$	0	0	$(1+\nu)\alpha\Delta T$
(c)	$-\alpha E\Delta T$	0	$-\nu\varepsilon_y$	0
(d)	None of the above			

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\cancel{\sigma_y}^{0} + \cancel{\sigma_z}^{0})\right] + \alpha\Delta T = 0$$

$$\sigma_x = -E\alpha\Delta T$$

$$\epsilon_y = \frac{1}{E}\left[\cancel{\sigma_y}^{0} - \nu(\sigma_x + \cancel{\sigma_z}^{0})\right] + \alpha\Delta T = 0$$

$$\epsilon_y = \frac{1}{\cancel{E}}\left[-\nu(-\cancel{E}\alpha\Delta T)\right] + \alpha\Delta T = \alpha\Delta T(1+\nu)$$

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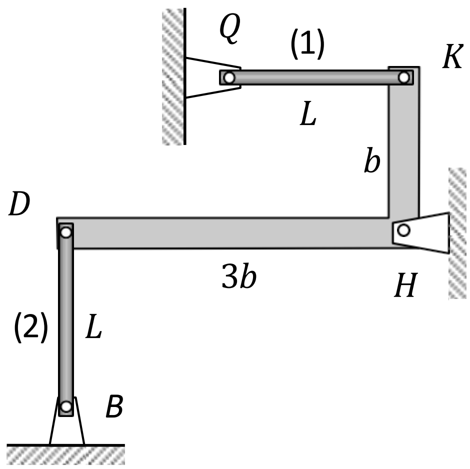
PROBLEM #4 – PART B (6 points)

The rigid, L-shaped bar DHK is pinned to ground at H, and identical elastic links (1) and (2) (having the same Young's modulus E, cross-sectional area A, length L and coefficient of thermal expansion α), are connected between D and B, and between Q and K, respectively. Links (1) and (2) are horizontal and vertical, respectively. The temperature of link (2) is ***decreased*** by an amount of ΔT, whereas the temperature of link (1) is held constant. Let ε_1 and ε_2 be the axial strains in (1) and (2), respectively, and σ_1 and σ_2 be the corresponding axial stresses in the links.



Circle the correct responses below:

2 points:

a) $|\sigma_1| > |\sigma_2|$
b) $|\sigma_1| = |\sigma_2|$
c) $|\sigma_1| < |\sigma_2|$

2 points:

a) σ_1 and ε_1 have the *same* signs
b) σ_1 and ε_1 are both *zero*
c) σ_1 and ε_1 have *opposite* signs

2 points:

a) σ_2 and ε_2 have the *same* signs
b) σ_2 and ε_2 are both *zero*
c) σ_2 and ε_2 have *opposite* signs

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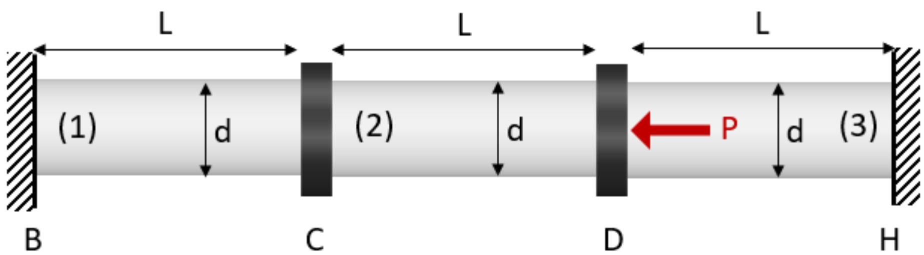
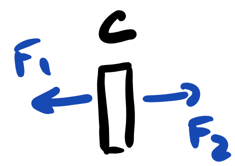
PROBLEM 4 – PART C (4 points)

A rod is made up of solid, circular cross-sectioned elements (1) and (2) and (3), with (1) and (2) joined with a rigid connector C, and (2) and (3) joined by rigid connector D. All three elements are made of the same type of steel, having a Young's modulus of $E_{steel.}$ A load P acts in the axial direction on connector D. Let F_1, F_2 and F_3 be the axial load (force) carried by, and σ_1, σ_2 and σ_3 be the axial stresses in, elements (1), (2) and (3), respectively.



Circle the correct responses below:

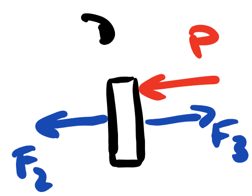
2 points:
- a) $|F_1| > |F_2|$
- b) $|F_1| = |F_2|$
- c) $|F_1| < |F_2|$



$\Sigma F_x: F_2 - F_1 = 0$

$F_1 = F_2$

2 points:
- a) $|F_2| > |F_3|$
- b) $|F_2| = |F_3|$
- c) $|F_2| < |F_3|$



$\Sigma F_x: F_3 - F_2 - P = 0$

$F_3 = F_2 + P$

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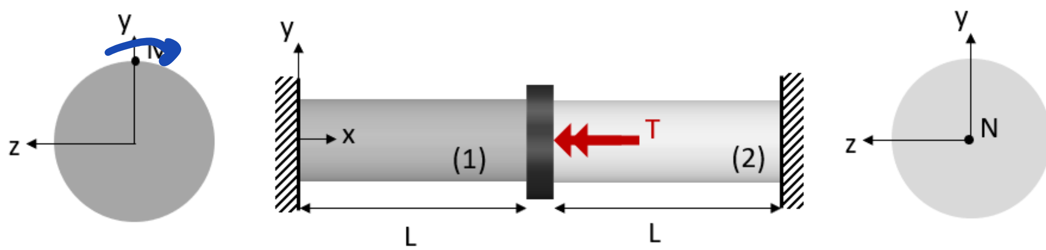
PROBLEM 4 – PART D (4 points)

For the points M and N shown below indicate whether each component of the state of stress is:

- = 0 (equal to zero)
- > 0 (greater than zero)
- < 0 (less than zero)

Elastic members (1) and (2) have different material properties, and the rigid plate that connects them is loaded with an external torque *T*.



	Point M	Point N
σ_x	0	0
σ_y	0	0
σ_z	0	0
τ_{xy}	0	0
τ_{xz}	<0	0
τ_{yz}	0	0

Fill in with '= 0', '> 0', or '< 0'

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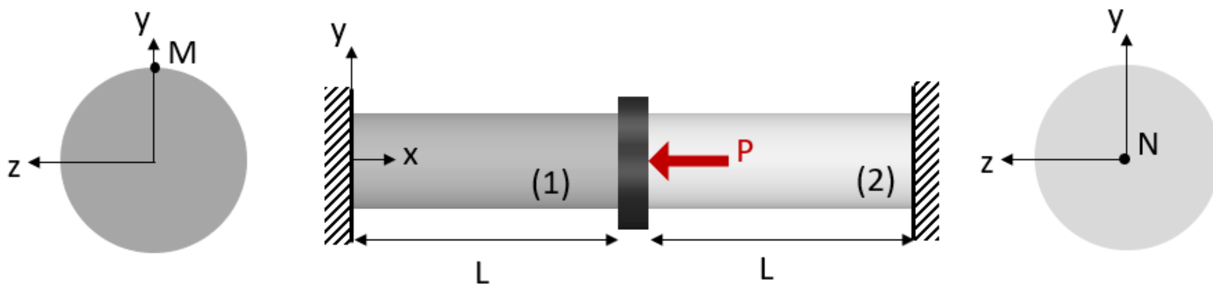
PROBLEM 4 – PART E (3 points)

For the points M and N shown below indicate whether each component of the state of stress is:

- = 0 (equal to zero)
- > 0 (greater than zero)
- < 0 (less than zero)

Elastic members (1) and (2) have different material properties, and the rigid plate that connects them is loaded with an external axial force *P*.



	Point M	Point N
σ_x	<0	>0
σ_y	0	0
σ_z	0	0
τ_{xy}	0	0
τ_{xz}	0	0
τ_{yz}	0	0

Fill in with '= 0', '> 0', or '< 0'

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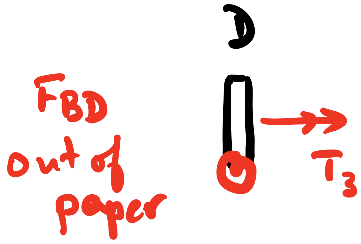
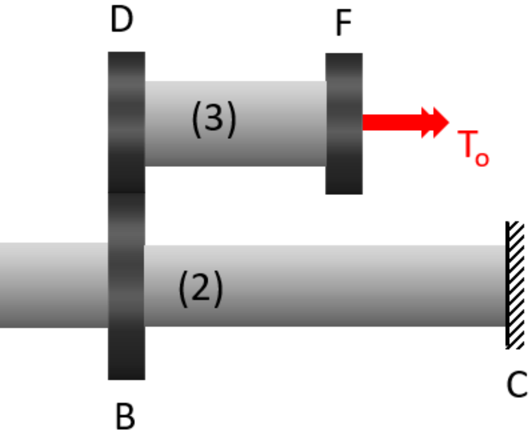
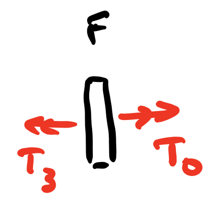
PROBLEM #4 – PART F (4 points)

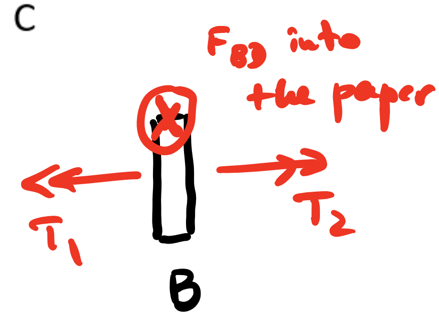
For the gear assembly shown subjected to the torque T, indicate (circle) if the equations below are True or False.

$T_3 - F_{BD} r_D = 0$









$T_2 - T_1 - F_{BD} r_B = 0$

(a) TRUE OR FALSE: $T_3 = T_0$

(b) TRUE OR FALSE: $T_2 - T_1 + F_{BD}R_B = 0$

(c) TRUE OR FALSE: $T_3 - F_{BD}R_D = 0$

(d) TRUE OR FALSE: $\Delta\phi_1 + \Delta\phi_2 = 0$

compatibility