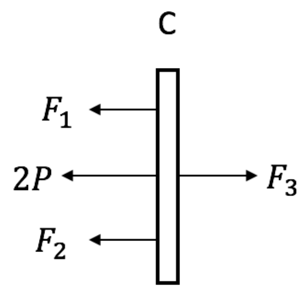
Problem 1:

FBD:



Equilibrium equation:

$$F_3 - F_1 - F_2 - 2P = 0 \qquad (1)$$

$$e_1 = \frac{F_1 L_1}{EA_1} + \alpha \Delta T L_1 = \frac{2F_1 L}{E\ \frac{1}{4}\pi d^2} + 2\alpha \Delta T L = \frac{8F_1 L}{E\pi d^2} + 2\alpha \Delta T L$$

$$e_2 = \frac{F_2 L_2}{EA_2} = \frac{F_2 L}{E\ \frac{1}{4}\pi(9d^2 - 4d^2)} = \frac{4F_2 L}{5E\pi d^2}$$

$$e_3 = \frac{F_3 L_3}{EA_3} = \frac{F_3 L}{E\ \frac{1}{4}\pi(4d^2 - d^2)} = \frac{4F_3 L}{3E\pi d^2}$$

Compatibility equations:

$$e_1 = e_2$$

$$\frac{8F_1 L}{E\pi d^2} + 2\alpha \Delta T L = \frac{4F_2 L}{5E\pi d^2}$$

$$F_1 = \frac{1}{10}F_2 - \frac{1}{4}\alpha \Delta T E\pi d^2$$

$$e_2 + e_3 = 0$$

$$\frac{4F_2 L}{5E\pi d^2} + \frac{4F_3 L}{3E\pi d^2} = 0$$

$$F_3 = -\frac{3}{5}F_2$$

solve equation (1):

$$F_1 = -\frac{4}{17}\alpha \Delta T E \pi d^2 - \frac{2}{17}P$$

$$F_2 = \frac{5}{34}\alpha \Delta T E \pi d^2 - \frac{20}{17}P$$

$$F_3 = -\frac{3}{34}\alpha \Delta T E \pi d^2 + \frac{12}{17}P$$

Axial stresses:

$$\sigma_1 = \frac{F_1}{A_1} = \frac{-\frac{4}{17}\alpha \Delta T E \pi d^2 - \frac{2}{17}P}{\frac{1}{4}\pi d^2} = \frac{-16\alpha \Delta T E \pi d^2 - 8P}{17\pi d^2}$$

$$\sigma_2 = \frac{F_2}{A_2} = \frac{\frac{5}{34}\alpha \Delta T E \pi d^2 - \frac{20}{17}P}{\frac{5}{4}\pi d^2} = \frac{2\alpha \Delta T E \pi d^2 - 16P}{17\pi d^2}$$

$$\sigma_3 = \frac{F_3}{A_3} = \frac{-\frac{3}{34}\alpha \Delta T E \pi d^2 + \frac{12}{17}P}{\frac{3}{4}\pi d^2} = \frac{-2\alpha \Delta T E \pi d^2 + 16P}{17\pi d^2}$$

Displacement of the connector C:

$$u_c = 0 + e_2 = \frac{4F_2 L}{5E\pi d^2} = \frac{2}{17}\alpha \Delta T L - \frac{16PL}{17E\pi d^2}$$

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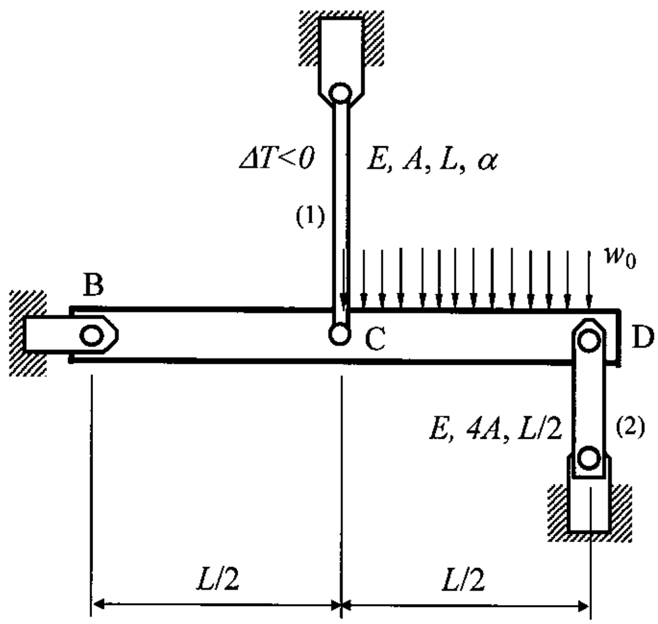
PROBLEM # 2 (25 points)

The rigid bar BCD is supported by the two elastic elements (1) and (2) as shown in the figure below. Elastic element (1) has a length L, cross-sectional area A, modulus of elasticity E and coefficient of thermal expansion α. Elastic element (2) has a length L/2, cross-sectional area 4A and modulus E. A distributed load w_0 is applied between CD of the bar. At the same time, element (1) is being **cooled** by a temperature change ΔT, while element (2) does not change temperature. Determine:

a) The axial force experienced by element (1).
b) The axial stress experienced by (1). Indicate if it is in tension or compression.
c) The axial force experienced by element (2).
d) The axial stress experienced by (2) and whether it is in tension or compression.

Use the following numerical values:
$E=30\times10^3$ ksi, $\alpha=10^{-6}$/ F, $\Delta T=-30$ F, L=12 in., A=0.5 in^2, $w_0=150$ lb/in



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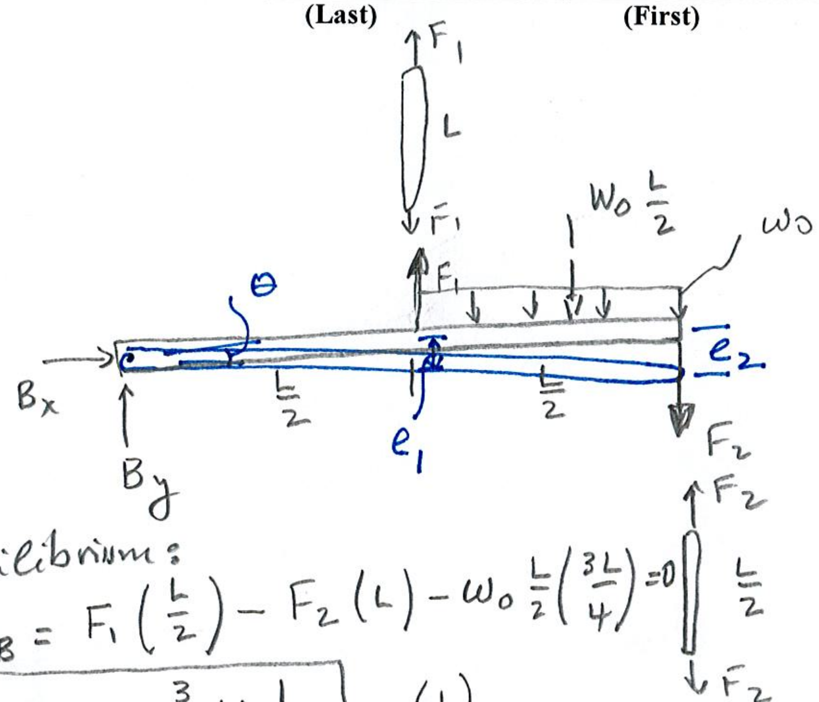
PROBLEM # 2 (cont.)



Equilibrium:

$$\Sigma M_B = F_1\left(\frac{L}{2}\right) - F_2(L) - w_0\frac{L}{2}\left(\frac{3L}{4}\right) = 0$$

$$\boxed{F_1 - 2F_2 = \frac{3}{4}w_0 L} \quad (1)$$

Elongations:

$$e_1 = \frac{F_1 L}{EA} + \alpha \Delta T L \quad (2)$$

$$e_2 = \frac{F_2\left(\frac{L}{2}\right)}{E(4A)} = \frac{F_2 L}{8A} \quad (3)$$

Compatibility:

$$\tan\theta \approx \theta = \frac{e_1}{\left(\frac{L}{2}\right)} = -\frac{e_2}{L} \Rightarrow \boxed{2e_1 = -e_2} \quad (4)$$

Replace (2) & (3) in (4)

$$2\frac{F_1 L}{EA} + 2\alpha\Delta T L = -\frac{F_2 L}{8EA}$$

or $F_2 = -16F_1 - 16EA\alpha\Delta T$

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PROBLEM # 2 (cont.)

Replace F_2 in (1)

$$F_1 + 32F_1 + 32\,EA\alpha\Delta T = \frac{3}{4} w_o L$$

Solve for F_1

$$F_1 = 0.023\, w_o L - 0.9697\, EA\alpha\Delta T$$
$$= 0.023(150)(12) - 0.9697(30\times10^{6})(1\times10^{-6})(0.5)(-30)$$

$$\boxed{F_1 = 477.3\,\text{lbs}} \qquad \boxed{\sigma_1 = \frac{F_1}{A} = 954.6\,\text{psi (T)}}$$

$$F_2 = -16(477.3) - 16(30\times10^{6})(1\times10^{-6})(-30)(0.5)$$

$$\boxed{F_2 = -436.2\,\text{lbs}} \qquad \boxed{\sigma_2 = \frac{F_2}{4A} = -218.1\,\text{psi (C)}}$$

Name (Print) ______________________________
(Last) **(First)**

PROBLEM # 3 (25 points)

Shafts (1) and (2) in Fig. 3(a) are connected by a rigid connector at B, and are fixed to the rigid walls at the ends A and C. Both shafts have length L and shear modulus G. Shaft (1) has a solid cross section of diameter $2d$, while shaft (2) has a hollow cross section of outer diameter $2d$ and inner diameter d as shown in Figs. 3(b) and 3(c). An external torque T_B is applied at the connect B.

(a) Determine if the assembly is statically determinate or indeterminate.
(b) Determine the internal torque carried by each shaft.
(c) Consider the points M and N on the outer radius of shaft (1) and (2), respectively. The cross-section views are shown in Figs. 3(b) and 3(c). Determine the state of stress at M and N.
(d) Draw the stress elements to represent the state of stress at M and N with clear labels of the stress components.

Express your results in terms of T_B, d, L, G, and π.

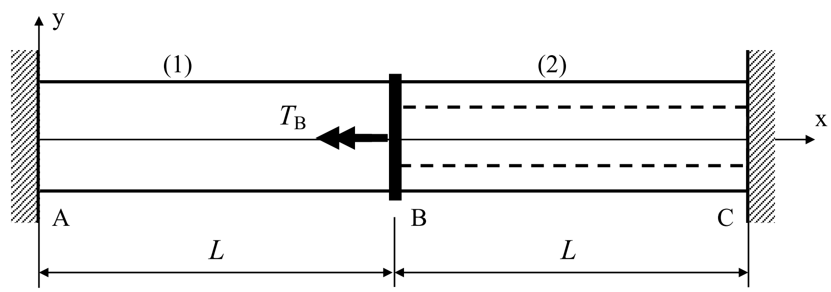


Fig. 3(a)

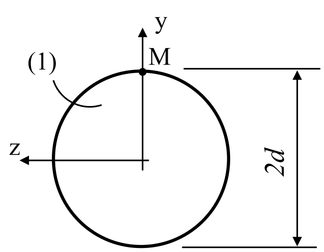


Fig. 3(b)

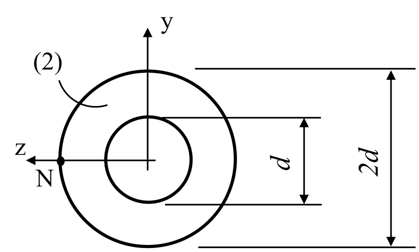
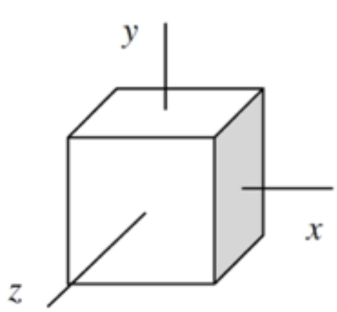
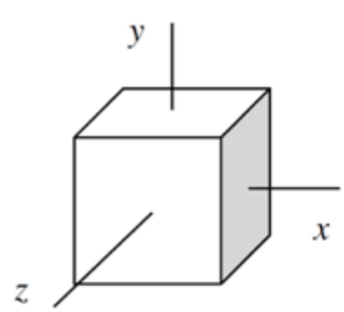


Fig. 3(c)



stress element at M



stress element at N

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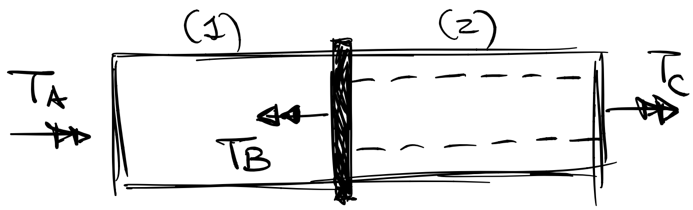
PROBLEM # 3 (cont.)

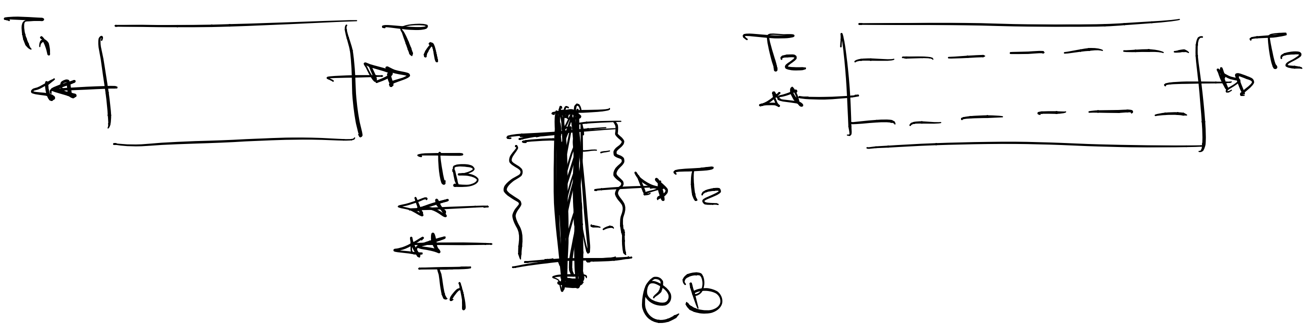
FBD:



$\sum T = 0 = T_C + T_A - T_B$ ⊕

$\Rightarrow$ statically indeterminate.

- Assume all members under positive torque. FBD



- Equilibrium: $\sum T = 0 = T_2 - T_1 - T_B$ ⊕

- Torque-twist behavior: $\phi_1 = \dfrac{T_1 L}{G I_{p1}}$ w/ $I_{p1} = \dfrac{\pi (2d)^4}{32} = \dfrac{\pi d^4}{2}$

$\phi_2 = \dfrac{T_2 L}{G I_{p2}}$ w/ $I_{p2} = \dfrac{\pi \left[(2d)^4 - d^4\right]}{32} = \dfrac{\pi 15 d^4}{32}$

- Compatibility conditions:

$$\phi_1 = \phi_B - \phi_A \quad ; \quad \phi_2 = \phi_C - \phi_B \quad ; \quad \phi_A = \phi_C = 0$$

$$\Rightarrow \quad \phi_1 = -\phi_2$$

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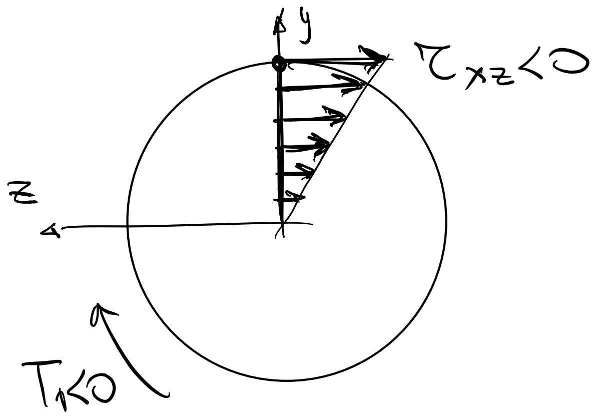
PROBLEM # 3 (cont.)

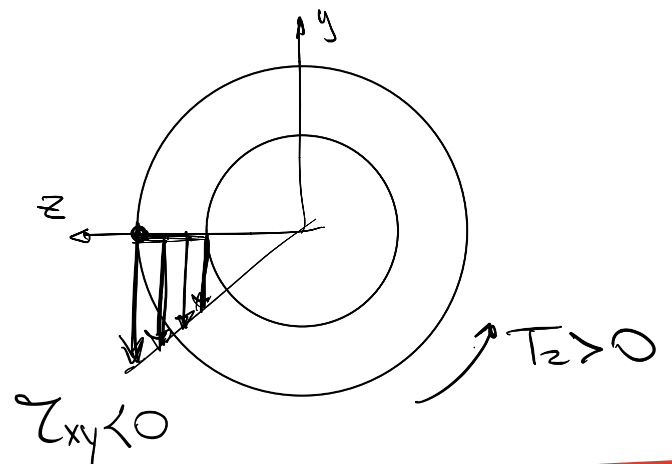
- Solve for T_1 and T_2

$$T_2 - T_1 - T_B = 0 \quad \& \quad \frac{T_1 L}{G\pi d^4/2} = -\frac{T_2 L}{G\pi 15 d^4/32}$$

$$\Downarrow$$

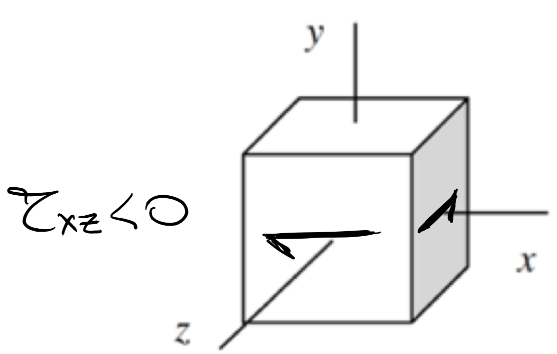
$$\boxed{\begin{aligned} T_2 &= \frac{15}{31} T_B \\ T_1 &= -\frac{16}{31} T_B \end{aligned}} \Leftarrow T_1 = -T_2 \frac{16}{15}$$

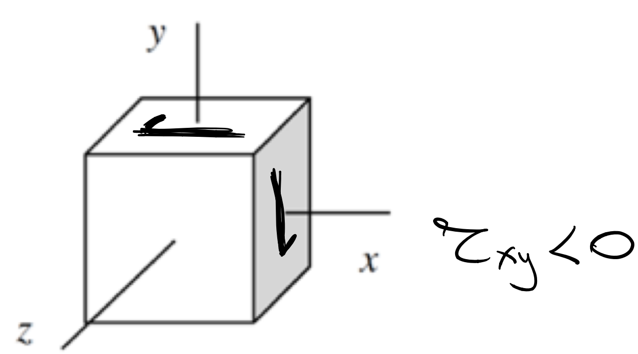




$$\boxed{\tau_{xz} = -\frac{d\,(16T_B/31)}{\pi d^4/2} = -\frac{32}{31}\frac{T_B}{\pi d^3}}$$

$$\boxed{\tau_{xy} = -\frac{d\,(15T_B/31)}{\pi\, 15 d^4/32} = -\frac{32}{31}\frac{T_B}{\pi d^3}}$$

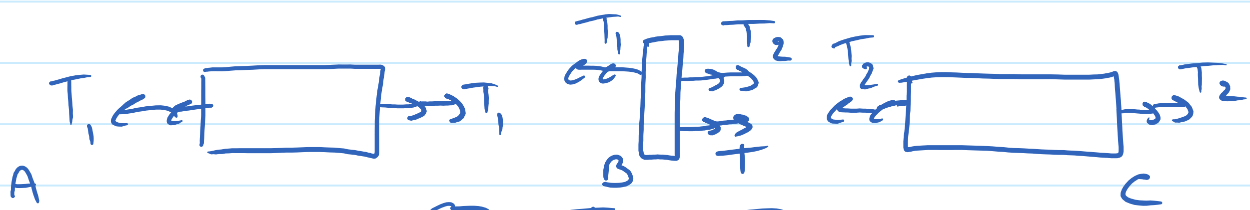




Problem 4

Part- A

FBD:



$$\Sigma T_x : T_2 + T = T_1$$

Compatibility eqn:

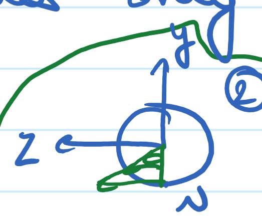
$$\Delta\phi_1 + \Delta\phi_2 = 0$$

$$\Rightarrow \frac{T_1 L}{G_1 I_{p_1}} + \frac{T_2 L}{G_2 I_{p_2}} = 0 \quad \Rightarrow T_2\left[\frac{1}{G_1 I_{p_1}} + \frac{1}{G_2 I_{p_2}}\right] = \frac{-T}{G_1 I_{p_1}}$$

$$\Rightarrow T_2 = \frac{-T G_2 I_{p_2}}{G_1 I_{p_1} + G_2 I_{p_2}} \quad < 0$$

$$\Rightarrow T_1 = \frac{T G_1 I_{p_1}}{G_1 I_{p_1} + G_2 I_{p_2}} \quad > 0$$

The given loading condition creates only shear stress on x-face

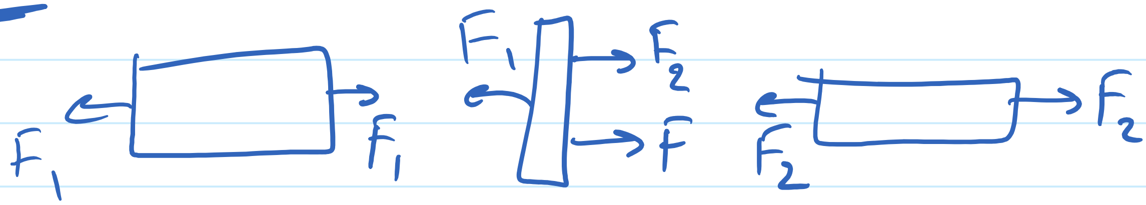


$$\therefore \sigma_x = \sigma_y = \sigma_z = \tau_{yz} = 0$$

$$\tau = \frac{T\rho}{I_p} \; ; \; \therefore \tau_{xz,M} = 0 \; (\because \rho = 0) \; ; \; \tau_{xz} > 0$$

Part B

FBD:



ΣF_x: $F + F_2 = F_1$

Compatibility eqn:

$$e_1 + e_2 = 0 \Rightarrow \frac{F_1 L}{E_1 A_1} + \frac{F_2 L}{E_2 A_2} = 0$$

$$\Rightarrow F_2 = \frac{-F E_2 A_2}{E_1 A_1 + E_2 A_2} < 0$$

$$F_1 = \frac{F E_1 A_1}{E_1 A_1 + E_2 A_2} > 0$$

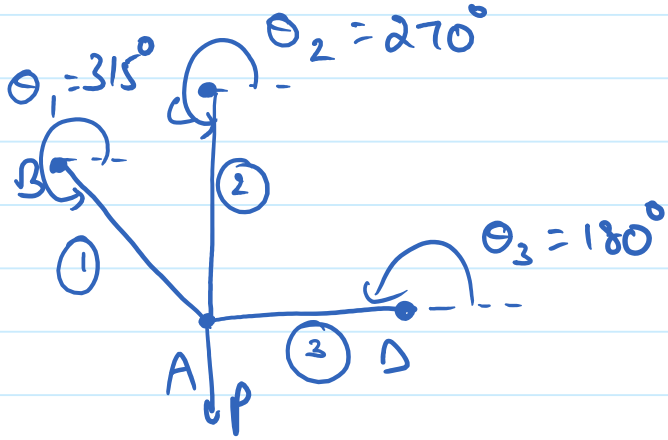
The loading condition does not create any shear stresses & also no axial stress in y & z dir.

$$\therefore \tau_{xy} = \tau_{yz} = \tau_{xz} = \sigma_y = \sigma_z = 0$$

$$\therefore \sigma_{x,M} > 0 \quad ; \quad \sigma_{x,N} < 0$$

Part C

a)



Compatibility conds:

$$e = u_A \cos\theta + v_A \sin\theta$$

$$\therefore\ e_1 = \frac{u_A}{\sqrt{2}} - \frac{v_A}{\sqrt{2}} \ ; \quad e_2 = -v_A \ ; \quad e_3 = -u_A$$

$$\Rightarrow e_1 = \frac{1}{\sqrt{2}}(e_2 - e_3)$$

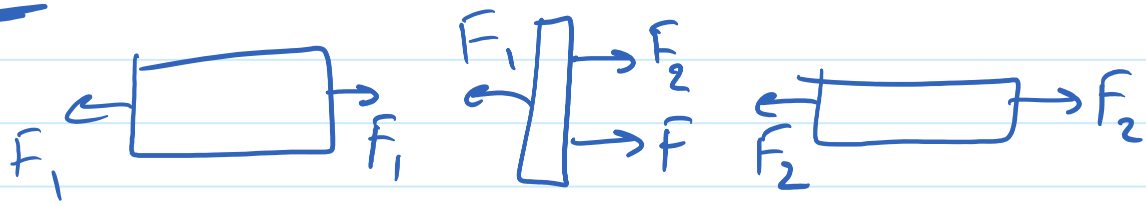
$$\boxed{\therefore\ a = \frac{1}{\sqrt{2}} = -b}$$

b) True

Because its indeterminate & needs compatibility conds. to solve for internal reactions.

Part B

FBD:



$$\Sigma F_x : F + F_2 = F_1$$

Compatibility eqn :

$$e_1 + e_2 = 0 \Rightarrow \frac{F_1 L}{E_1 A_1} + \frac{F_2 L}{E_2 A_2} = 0$$

$$\Rightarrow F_2 = \frac{-F E_2 A_2}{E_1 A_1 + E_2 A_2} < 0$$

$$F_1 = \frac{F E_1 A_1}{E_1 A_1 + E_2 A_2} > 0$$

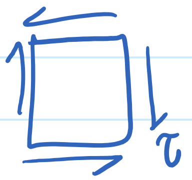
The loading condition does not create any shear stresses & also no axial stress in y & z dir.

$$\boxed{\begin{gathered} \therefore \tau_{xy} = \tau_{yz} = \tau_{xz} = \sigma_y = \sigma_z = 0 \\ \therefore \sigma_{x,M} > 0 \; ; \; \sigma_{x,N} < 0 \end{gathered}}$$

Part D

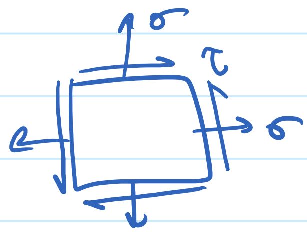
For the following states of stress, only non-zero values are shown:

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right)$$
$$\vdots$$
$$\gamma_{xy} = \frac{\tau_{xy}}{G}; \quad G = \frac{E}{2(1+\nu)}$$

a) 

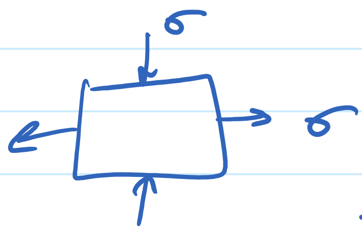
$\tau_{xy} = \tau_{yx} < 0$

$\therefore \gamma_{xy} < 0$

b) 

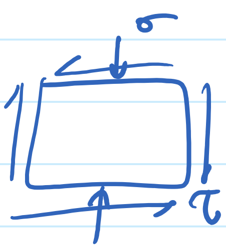
$\sigma_x = \sigma_y > 0; \; \tau_{xy} = \tau_{yx} > 0$

$\therefore \varepsilon_x > 0, \; \varepsilon_y > 0, \; \varepsilon_z < 0, \; \gamma_{xy} > 0$

c) 

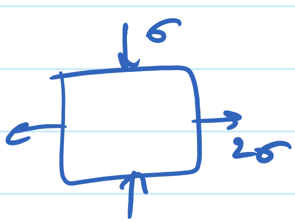
$\sigma_x > 0, \; \sigma_y < 0$

$\therefore \varepsilon_x > 0, \; \varepsilon_y < 0$

d) 

$\sigma_y < 0, \; \tau_{xy} = \tau_{yx} < 0$

$\therefore \varepsilon_x > 0, \; \varepsilon_y < 0, \; \varepsilon_z > 0, \; \gamma_{xy} < 0$

e) 

$\sigma_x > 0, \; \sigma_y < 0$

$\varepsilon_x > 0, \; \varepsilon_y < 0, \; \varepsilon_z < 0$

The final answers in order are:

f, c, f, d, a, f, f, b, f, e