PROBLEM #1 (22 points)

A solid brass core is connected to a hollow rod made of aluminum. Both are attached at each end to a rigid plate as shown in Fig. 1. The moduli of aluminum and brass are E_A=11,000 ksi, E_B=15,000 ksi, and the coefficients of thermal expansion are $\alpha_A = 13\times10^{-6}$/°F, $\alpha_B = 12\times10^{-6}$/°F, respetively. The cross section areas of aluminum and brass are A_A=2 in^2 and A_B=1 in^2, respectively. The core and the sleeve are stress free at the reference temperature. If the core is heated by 100°F and the temperature of the sleeve is held constant, calculate:

1) The axial force and average axial stress (*x* direction) in each member.
2) The elongation of the composite bar in the *x* direction.
3) What should be the magnitude of the force *F* in Fig. 1 to prevent the elongation of the composite bar in the *x* direction?

Hints: Ignore the radial expansion due to Poisson's ratio effect.
For 1) and 2) use F=0

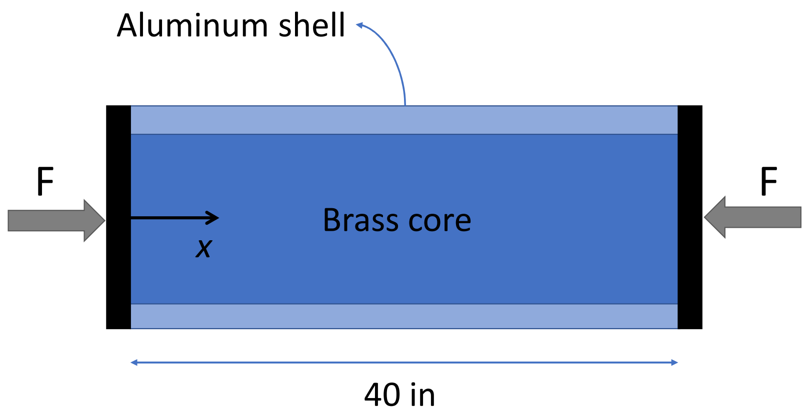
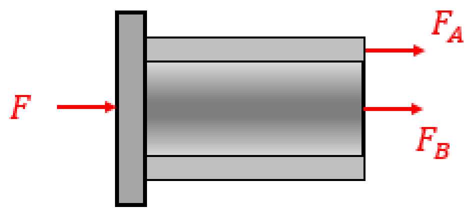


Fig. 1

Solution:



a) $F = 0$

From the FBD,

$$\Sigma F_x = F_A + F_B = 0 \Rightarrow F_A = -F_B \tag{1.1}$$

From the force-elongation equation,

$$e_A = \frac{F_A L_A}{A_A E_A} \tag{1.2}$$

$$e_B = \frac{F_B L_B}{A_B E_B} + \alpha_B \Delta T L_B \tag{1.3}$$

The compatibility condition,

$$e_A = e_B \tag{1.4}$$

Combine (1.1)-(1.4) to get,

$$F_A = 10702\ lb$$
$$F_B = -10702\ lb$$
$$\sigma_A = 5351\ psi$$
$$\sigma_B = 10702\ psi$$

b) From the compatibility equation,

$$e = e_A = e_B \tag{1.5}$$

$$e_A = \frac{F_A L_A}{A_A E_A} = 0.01946\ in \tag{1.6}$$

c) Use the same FBD,

$$\Sigma F_x = F + F_A + F_B = 0 \Rightarrow F_A + F_B = -F \tag{1.7}$$

The compatibility equation becomes,

$$e_A = e_B = 0 \tag{1.8}$$

Combine (1.2), (1.3), (1.7), (1.8) to get,

$$F_A = \frac{e_A A_A E_A}{L_A} = 0\ lb$$
$$F_B = -\alpha_B \Delta T A_B E_B = -18000\ lb$$
$$F = -F_B = 18000\ lb$$

ME 323 Examination # 1 **Name ____________________________________**
(Print) (Last) (First)

February 15, 2017 **Instructor _______________________________**

<u>PROBLEM #2 (28 points)</u>

A shaft is made up of three components: solid circular shafts (1) of length $2L$ and diameter d; solid circular shaft (2) of length L and diameter $2d$; and *tubular* shaft (3) of length L, outer diameter $3d$ and inner diameter $2d$. All three components are made of the same material having a shear modulus of G. Shafts (1) and (2) are connected by a thin, rigid connector C, whereas shafts (2) and (3) are connected by a rigid connector D. Shaft (1) is rigidly attached to ground at B. Shaft (3) is rigidly attached to ground at H. Torques T and $2T$ are applied to C and D, respectively, as shown in the Fig. 2 (a).

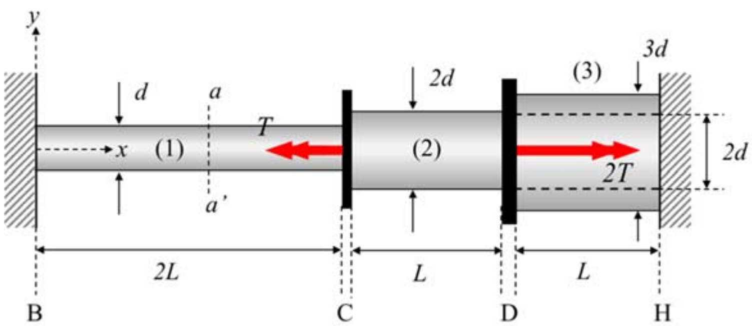


Fig. 2 (a)

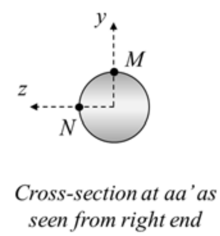


Fig. 2 (b)

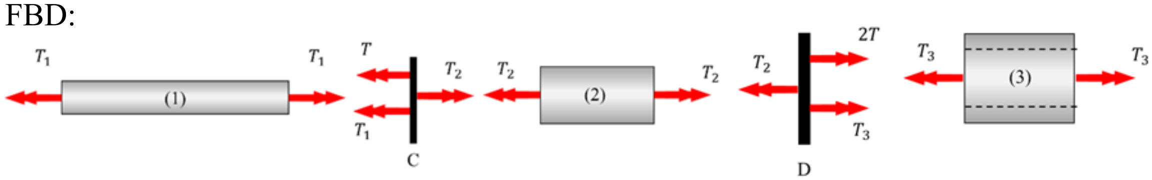
Fig. 2 (c)

1) Determine the torque carried by each of the shaft components.
2) Determine the angle of rotation at connector C.
3) Show the stress element for the points M (x, $0.5d$, 0) and N (x, 0, $0.5d$), whose locations on shaft (1) are also shown on Fig. 2 (b). Indicate both magnitudes and axes corresponding to the state of stress on Fig. 2 (c).
Express your final answers in terms of T, L, d and G.

Solution:

a) 1. Equilibrium.

FBD:



$$\text{C: } T_2 - T_1 - T = 0 \qquad (2.1)$$

$$\text{D: } T_3 + 2T - T_2 = 0 \qquad (2.2)$$

2. Torque/Rotation

$$\Delta\Phi_1 = \frac{T_1 \cdot 2L}{G \cdot Ip_1}, Ip_1 = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32}d^4 \quad \Rightarrow \Delta\Phi_1 = 64 \cdot \frac{T_1 \cdot L}{G\pi d^4} \qquad (2.3)$$

$$\Delta\Phi_2 = \frac{T_2 \cdot L}{G \cdot Ip_2}, Ip_2 = \frac{\pi}{2}\left(\frac{2d}{2}\right)^4 = \frac{\pi}{2}d^4 \quad \Rightarrow \Delta\Phi_2 = 2 \cdot \frac{T_2 \cdot L}{G\pi d^4} \qquad (2.4)$$

$$\Delta\Phi_3 = \frac{T_3 \cdot L}{G \cdot Ip_3}, Ip_3 = \frac{\pi}{2}[\left(\frac{3d}{2}\right)^4 - \left(\frac{2d}{2}\right)^4] = \frac{65}{32}\pi d^4 \quad \Rightarrow \Delta\Phi_3 = \frac{32}{65} \cdot \frac{T_3 \cdot L}{G\pi d^4} \qquad (2.5)$$

3. Compatibility

$$\Phi_C = \Phi_B + \Delta\Phi_1 = \Delta\Phi_1\ (\Phi_B = 0),\ \Phi_D = \Phi_C + \Delta\Phi_2 = \Delta\Phi_1 + \Delta\Phi_2,\ \Phi_H = \Phi_D + \Delta\Phi_3 = 0$$

$$\Rightarrow \Delta\Phi_1 + \Delta\Phi_2 + \Delta\Phi_3 = 0 \qquad (2.6)$$

From (2.3) to (2.6), we have $64 \cdot \frac{T_1 \cdot L}{G\pi d^4} + 2 \cdot \frac{T_2 \cdot L}{G\pi d^4} + \frac{32}{65} \cdot \frac{T_3 \cdot L}{G\pi d^4} = 0$

$$\Rightarrow 64\mathrm{T}_1 + 2T_2 + \frac{32}{65}T_3 = 0 \qquad (2.7)$$

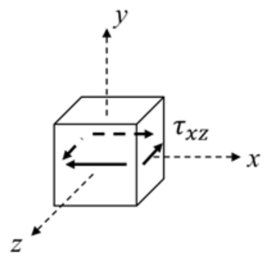
4. Solve

With (2.1), (2.2) and (2.7), we can solve

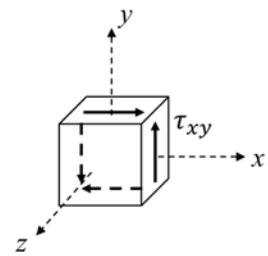
$$\begin{cases} T_1 = -0.023T \\ T_2 = 0.977T \\ T_3 = -1.023T \end{cases}$$

b) $\Phi_C = \Delta\Phi_1 = 64 \cdot \frac{T_1 \cdot L}{G\pi d^4} = 64 \cdot \frac{-0.023T \cdot L}{G\pi d^4} = -0.468\frac{T \cdot L}{Gd^4}$ (radians)

c) Since the elements M and N are both at the circumference of the cross section at aa’, and a negative torque (T_1) has been applied, their states of stress are shown as below:



element at "M"



element at "N"

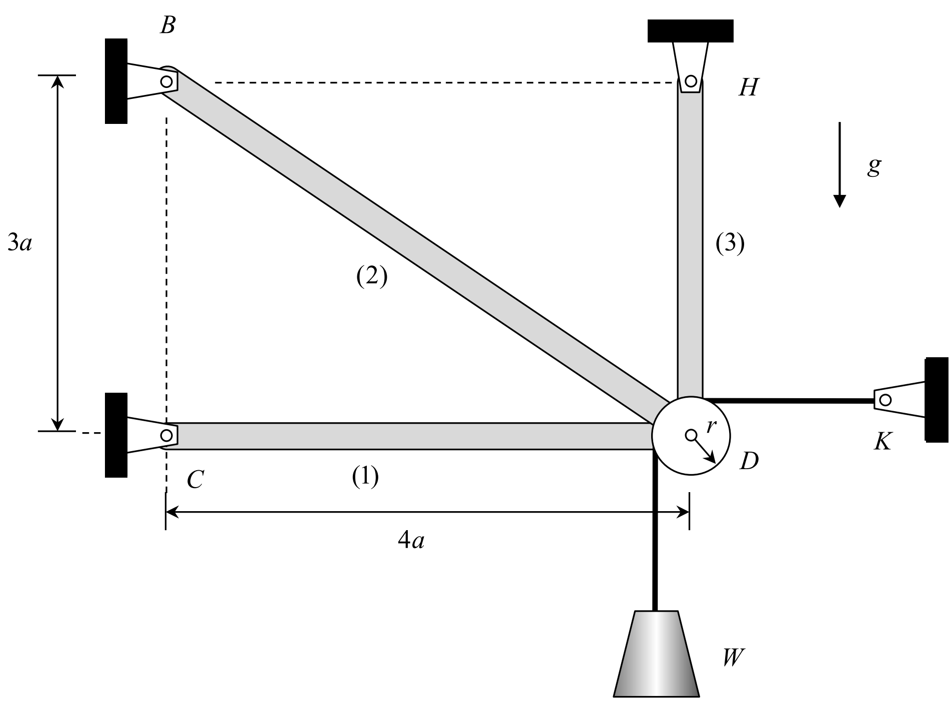
The magnitude of shear stresses $|\tau_{xz}| = |\tau_{xy}| = \left|\frac{T_1 \cdot \frac{d}{2}}{Ip_1}\right| = \left|\frac{-0.023T \cdot \frac{d}{2}}{\frac{\pi d^4}{32}}\right| = 0.117\frac{T}{d^3}$

PROBLEM #3 (26 points)

A truss is made up of members (1)-(3), with each member having a cross-sectional area of A and being made up of a material with a Young's modulus of E. Members (1)-(3) are connected to ground at pins C, B and H, respectively, and are connected together at pin D. A frictionless pulley is also attached to pin D. A block of weight W is supported by a cable that is pulled over the pulley and attached to ground at K. Pins B, C and H each have a cross-sectional diameter of d, and have a "*single-sided*" connection to ground. The pins are made of a material having a shear yield strength of τ_Y .

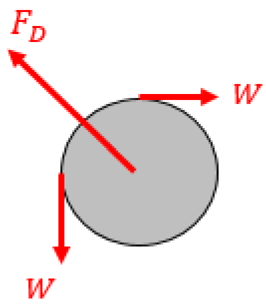
a) Determine the load carried by each member of the truss. Express your final answers in terms of, at most, A, E, W and a.

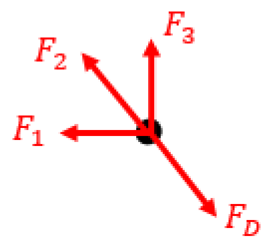
b) Determine the largest W that can be supported by the truss without failure at pins B, C or H. Use a factor of safety of FS= 2.



Solution:

a)





From the FBD,

$$F_D = \sqrt{2}w$$

$$\Sigma F_x = 0 => F_1 + 0.8F_2 = w \tag{3.1}$$

$$\Sigma F_y = 0 => 0.6F_2 + F_3 = w \tag{3.2}$$

For each truss,

$$\theta_1 = 0, \qquad \theta_2 = 323°, \qquad \theta_3 = 270°, \tag{3.3}$$

From the force-elongation equation,

$$e_1 = \frac{F_1 L_1}{E_1 A_1} = \frac{F_1 4a}{EA} \tag{3.4}$$

$$e_2 = \frac{F_2 L_2}{E_2 A_2} = \frac{F_1 5a}{EA} \tag{3.5}$$

$$e_3 = \frac{F_3 L_3}{E_3 A_3} = \frac{F_1 3a}{EA} \tag{3.6}$$

Compatibility conditions,

$$e_1 = u_D \cos\theta_1 + v_D \sin\theta_1 = u_D \tag{3.7a}$$

$$e_2 = u_D \cos\theta_2 + v_D \sin\theta_2 = 0.8u_D - 0.6v_D \tag{3.7b}$$

$$e_3 = u_D \cos\theta_3 + v_D \sin\theta_3 = -v_D \tag{3.7c}$$

From (3.7a) to (3.7c) to get,

$$e_2 = 0.8e_1 + 0.6e_3 \tag{3.8}$$

Combine (3.4) to (3.6) with (3.8) to get,

$$5F_2 = \frac{16}{5}F_1 + \frac{9}{5}F_3 \tag{3.9}$$

Solve (3.1), (3.2) and (3.9) to get,

$$F_1 = 0.537w$$
$$F_2 = 0.578w$$
$$F_3 = 0.653w$$

b) From the failure theory,

$$FS = \frac{\tau_y}{\tau_{\text{allow}}}$$

The problem is for the single pin situation,

$$V = F$$

For point B,

$$\tau_B = \frac{F_2}{A} = \frac{0.578w}{\frac{\pi d^2}{4}} = \frac{\tau_y}{2} => (w_B)_{\text{allow}} = 0.678\tau_y d^2$$

For point C,

$$\tau_C = \frac{F_1}{A} = \frac{0.537w}{\frac{\pi d^2}{4}} = \frac{\tau_y}{2} \Rightarrow (w_C)_{\text{allow}} = 0.731\tau_y d^2$$

For point H,

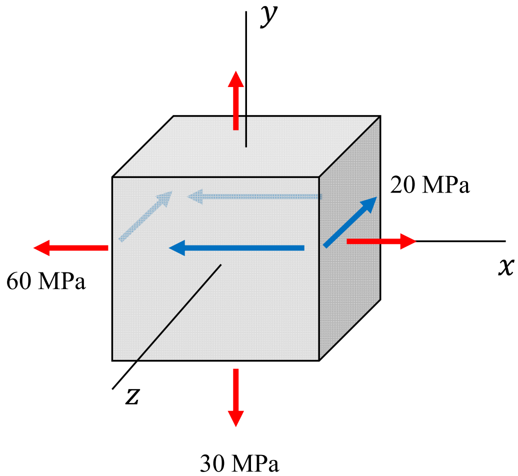
$$\tau_H = \frac{F_3}{A} = \frac{0.653w}{\frac{\pi d^2}{4}} = \frac{\tau_y}{2} \Rightarrow (w_H)_{\text{allow}} = 0.601\tau_y d^2$$

Thus,

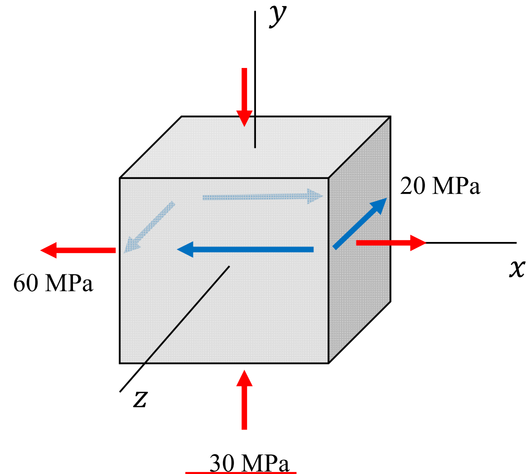
$$w_{\text{allow}} = \text{smallest } w = (w_H)_{\text{allow}} = 0.601\tau_y d^2$$

PROBLEM #4 (24 Points):

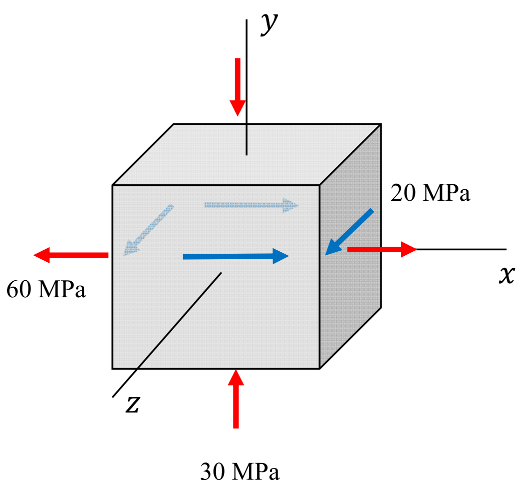
4.1. A material point in a steel machine is subjected to the following stress state: $\sigma_x = 60\text{ MPa}$, $\sigma_y = -30\text{ MPa}$, and $\tau_{xz} = -20\text{ MPa}$. Which of the following stress elements represents the correct stress state of the material point?



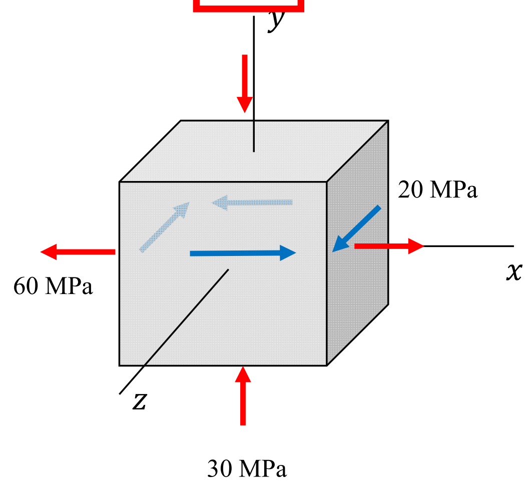
(a)



(b)



(c)



(d)

Using E=210 GPa and ν=0.3 for steel, $G = \dfrac{E}{2(1+\nu)}$, determine the strain components:

$$\varepsilon_x = \frac{\sigma_x - \nu\sigma_y}{E} = 0.00033$$

$$\varepsilon_y = \frac{\sigma_y - \nu\sigma_x}{E} = -0.00023$$

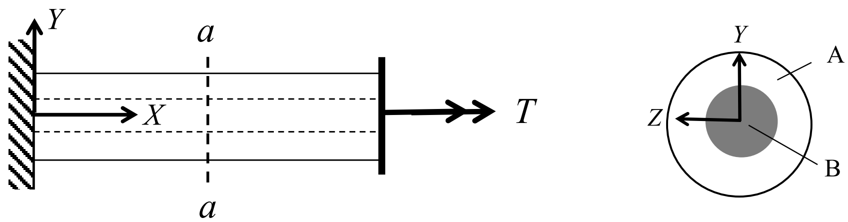
$$\varepsilon_z = \frac{-\nu(\sigma_x + \sigma_y)}{E} = -0.00043$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$

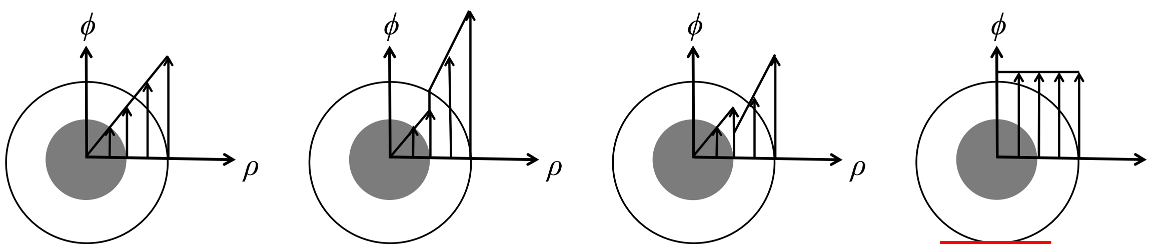
$$\gamma_{yz} = \frac{\tau_{yz}}{G} = 0$$

$$\gamma_{xz} = \frac{\tau_{xz}}{G} = -0.00025$$

4.2. A bimetallic bar with circular cross section consists of a shell A and a core B. The bimetallic bar is subjected to a torque T. The shear moduli of the core and shell are known to be $G_A = 2G_B$, and polar moment of inertia $I_{PA} = 0.5\, I_{PB}$.

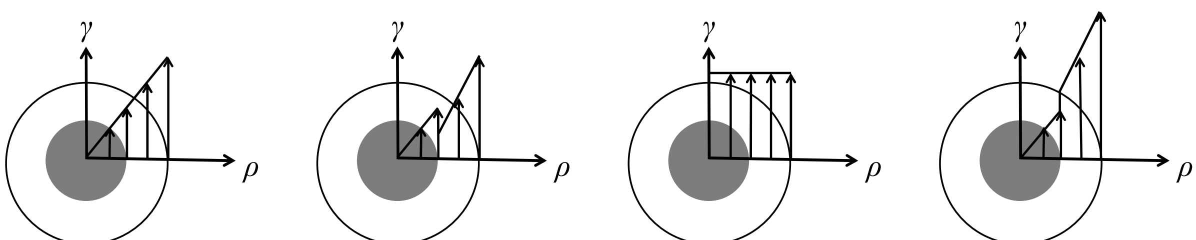


Which figure shows the correct distribution of the twist angle in the cross section *aa*?



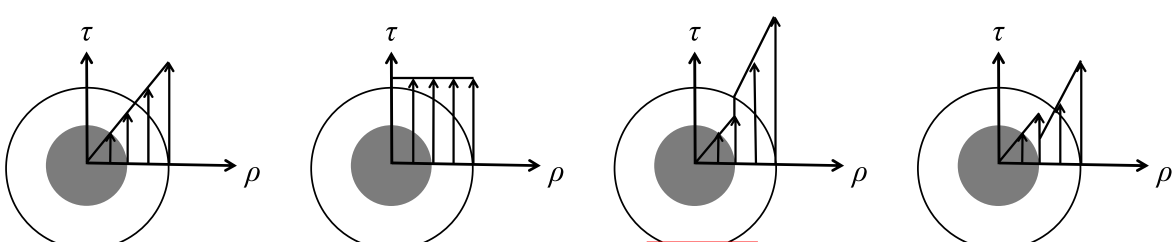
(a) (b) (c) (d)

Which figure shows the correct distribution of the shear strain in the cross section *aa*?



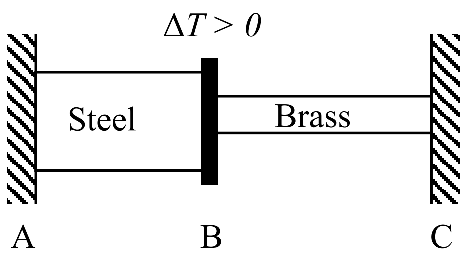
(a) (b) (c) (d)

Which figure shows the correct distribution of the shear stress in the cross section *aa*?

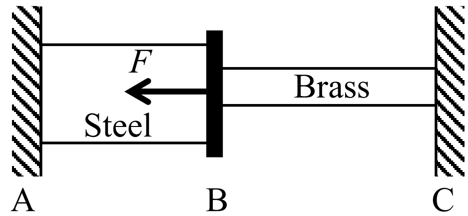


(a) (b) (c) (d)

4.3. A stepped rod is composed of steel and brass which are joined by a rigid connector at B and are fixed at the two ends. The coefficients of thermal expansion for steel and brass are α_S and α_B ($\alpha_s > \alpha_B$), respectively. Consider two load conditions shown below, the rod is homogeneously heated with a temperature increase of ΔT in (1), and a force F is applied at the rigid connector while temperature is held constant in (2). Which of the following statements are correct? (More than one item can be selected)



(1)



(2)

(a) Both configurations (1) and (2) are statically indeterminate structures.

(b) (1) is a statically determinate structure and (2) is a statically indeterminate structure.

(c) The forces F_S (steel) and F_B (brass) in (1) are both zero

(d) The forces F_S (steel) and F_B (brass) in (1) are non-zero and are equal.

(e) The forces F_S (steel) and F_B (brass) in (1) are equal in magnitude and of opposite signs.

(f) The strains ε_S (steel) and ε_B (brass) in (1) are both zero.

(g) The strains ε_S (steel) and ε_B (brass) in (1) are non-zero and are equal.

(h) The strains ε_S (steel) and ε_B (brass) in (1) are equal in magnitude and of opposite signs.

(i) The stresses σ_S (steel) and σ_B (brass) in (1) are both compressive.

(j) The stresses σ_S (steel) and σ_B (brass) in (1) are equal in magnitude and of opposite signs.

(k) The forces F_S (steel) and F_B (brass) in (2) are non-zero and are equal.

(l) The forces F_S (steel) and F_B (brass) in (2) are both compressive.

(m) The elongations e_S (steel) and e_B (brass) in (2) are equal.

(n) The elongations e_S (steel) and e_B (brass) in (2) are equal in magnitude and of opposite signs.

(o) The stresses σ_S (steel) and σ_B (brass) in (2) are equal in magnitude and of opposite signs.

(p) The solution of the stresses σ_S (steel) and σ_B (brass) in (2) depends on the cross sections, lengths, and elastsic moduli of steel and brass.