ME 323 Examination # 1

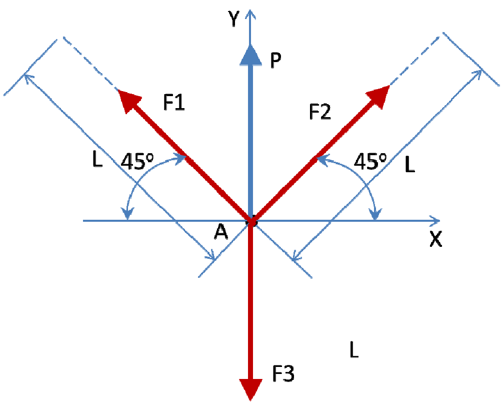
Name __________________________________
(Print) (Last) (First)

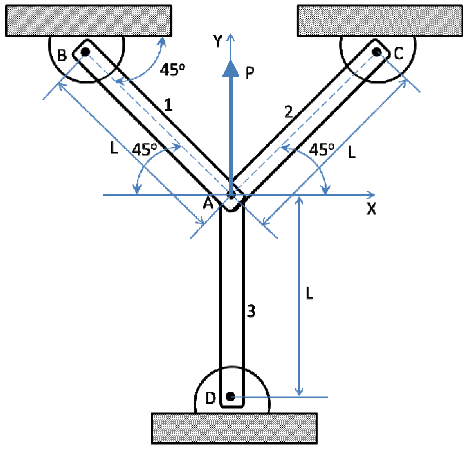
February 18, 2016

Instructor ________________________________

PROBLEM #1 (20 points)

A structure is constructed from members 1, 2 and 3, with these members made up of the same material (Young's modulus E) and having the same cross-sectional areas of A and lengths L. The temperature of member 3 is increased by ΔT and the load P acts as shown on the joint A. Draw all necessary free body diagrams and write all necessary equations needed to symbolically solve for the forces in each link (do not solve). List all knowns (i.e. dimensions, angles and values if pertinent etc.) and unknowns (i.e. all forces, displacements and values if pertinent etc.).
Note: Assume small displacement of point A





Solution:

$$\sum Fx = 0 \rightarrow -F1\cos(\pi/4) + F2\cos(\pi/4) = 0$$

$$\sum Fy = 0 \rightarrow F1\sin(\pi/4) + F2\sin(\pi/4) + P - F3 = 0$$

$$\frac{F1L1}{A1E1} = ua\cos(7\pi/4) + va\sin(7\pi/4)$$

$$\frac{F2L2}{A2E2} = ua\cos(5\pi/4) + va\sin(5\pi/4)$$

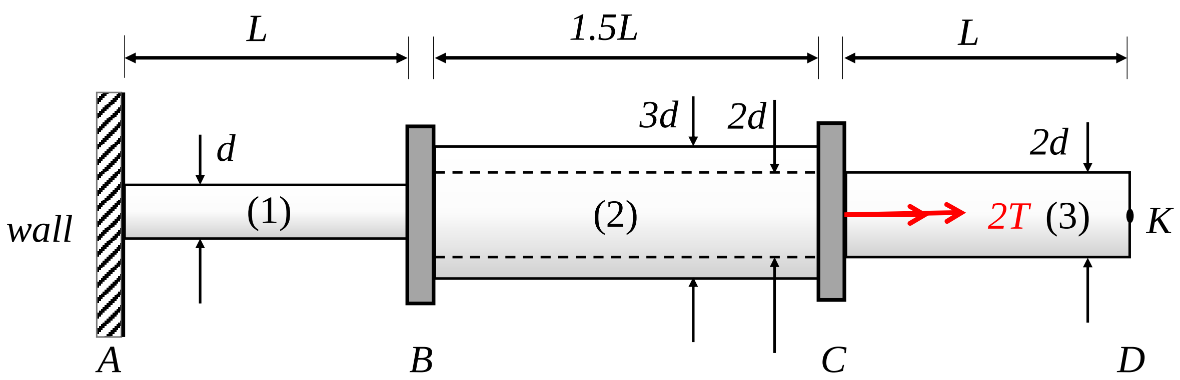
$$\frac{F3L3}{A3E3} + \alpha L\Delta T = ua\cos(\pi/2) + va\sin(\pi/2)$$

$\alpha L\Delta T$, A1, A2, A3, L1, L2, L3, E1, E2, E3, P are known
Five equations in five unknowns, F1, F2, F3, ua and va

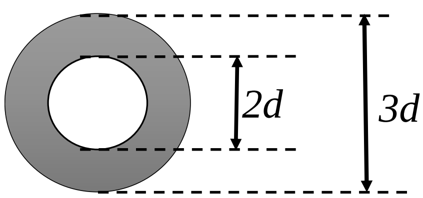
PROBLEM #2 (20 points)

Consider the stepped shaft shown in figure (a). Shafts (1) and (2) are joined by rigid connector B, while shaft (2) and (3) are joined by rigid connector C. The stepped shaft is fixed to a rigid wall at end A. Connectors C is acted upon by torque *2T* as shown. Hollow shaft (2) is of length *1.5L* with inner diameter *2d* and outer diameter *3d* (cross-section shown in figure (b)). Shafts (1) and (3) are solid, each having length *L*, and diameters *d* and 2*d* respectively. Shear modulus of the entire shaft is *G*. The length of the connectors is negligible.

a) What are the torques carried by each shaft, (1), (2), and (3)?
b) Determine the magnitude of the maximum shear stress in this structure.
c) Determine the angular rotation of connectors B and C relative to the location A.
d) Consider a point K on the outer radius of shaft (3) at D. What is the total length of the arc through which this point has travelled?

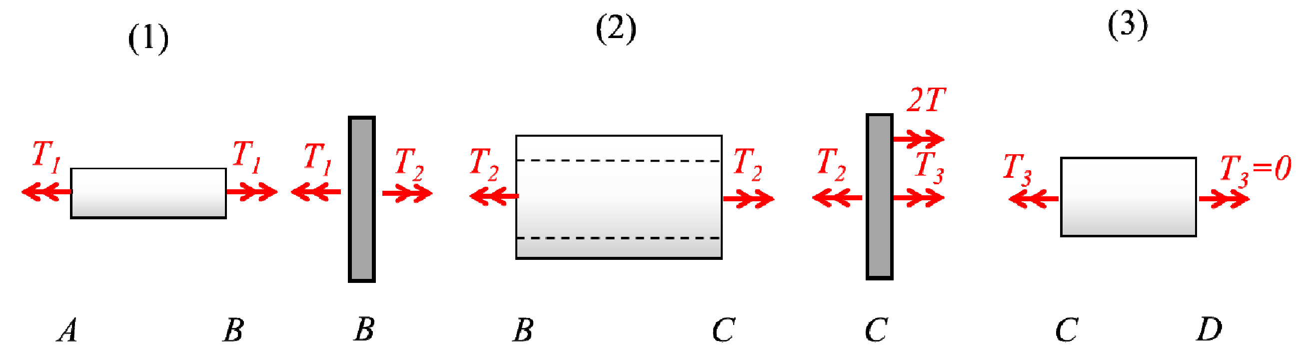


(a)



(b)

SOLUTION:



(a) *Equilibrium*:

Shaft (3) is free at end D. $\Rightarrow T_3 = 0$

Connector C: $T_3 + 2T - T_2 = 0 \Rightarrow T_2 = 2T$ (1)

Connector B: $T_2 - T_1 = 0 \Rightarrow T_1 = T_2 = 2T$ (2)

Where, T_1, T_2, T_3 are the torques carried by the three shafts.

Polar moments are represented here as J_i. (They can also be denoted as I_p)

(b) $J_1 = \frac{\pi}{32} d^4$; $J_2 = \frac{\pi}{32}\{(3d)^4 - (2d)^4\} = \frac{65\pi}{32} d^4$; $J_3 = \frac{\pi}{32}(2d)^4 = \frac{\pi}{2} d^4$

Maximum shear stress:

Shaft (1): $(\tau_1)_{max} = \frac{T_1 \rho_{max}}{J_1} = \frac{(2T)\left(\frac{d}{2}\right)}{J_1} = \frac{32T}{\pi d^3}$

Shaft (2): $(\tau_2)_{max} = \frac{T_2 \rho_{max}}{J_2} = \frac{(2T)\left(\frac{3d}{2}\right)}{J_2} = \frac{96T}{65\pi d^3}$

Therefore, maximum shear stress occurs on the outer surface of shaft (1) and its magnitude is given by:

$$\tau_{max} = 32T/\pi d^3$$

(c) $\theta_1 = \frac{T_1 L_1}{G_1 J_1} = \frac{(2T)(L)}{(G)(J_1)} = \frac{64TL}{\pi G d^4}$; $\theta_2 = \frac{T_1 L_1}{G_1 J_1} = \frac{(2T)(1.5L)}{(G)(J_2)} = \frac{96TL}{65\pi G d^4}$

, where θ_1, θ_2 are rotations between the ends of shafts (1) and (2) respectively. Note that $\theta_3 = 0$, since torque carried by shaft (3) is 0.

Compatibility:

$$\theta_A = 0 \qquad (wall) \tag{3}$$

$$\theta_B = \theta_A + \theta_1 = \theta_1 \tag{4}$$

$$\theta_C = \theta_B + \theta_2 = \theta_1 + \theta_2 \tag{5}$$

$$\theta_D = \theta_C + \theta_3 = \theta_C \tag{6}$$

From (4): $\theta_B = \frac{64TL}{\pi G d^4}$

From (5): $\theta_C = \theta_B + \theta_2 = \frac{64TL}{\pi G d^4} + \frac{96TL}{65\pi G d^4} = \frac{4256}{65}\frac{TL}{\pi G d^4} = 65.5\frac{TL}{\pi G d^4} = \theta_D$

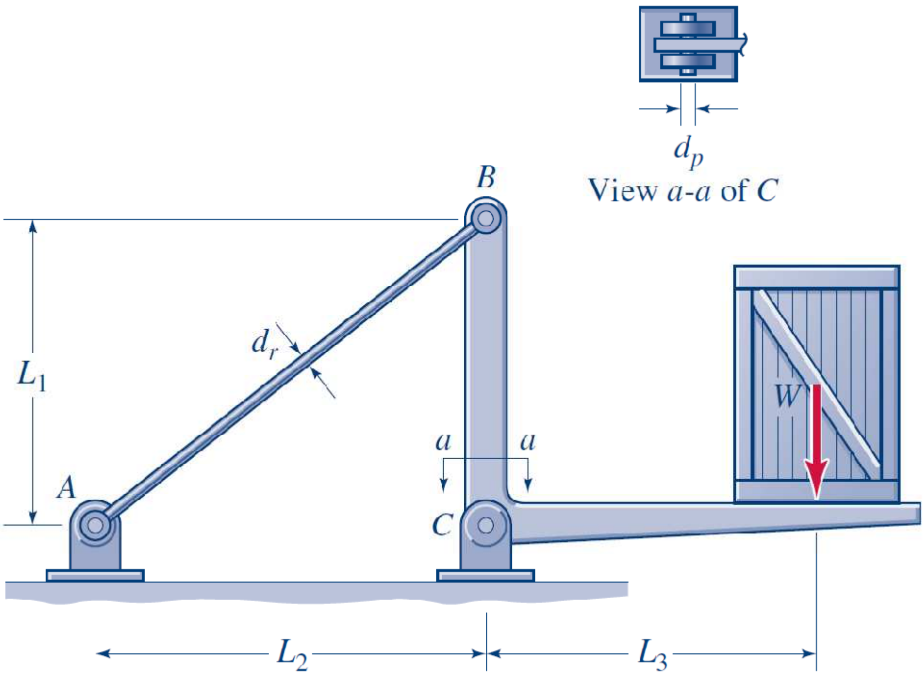
(d) Length of arc travelled by K:

$$l_K = \frac{2d}{2}\theta_D = 65.5\frac{TL}{\pi G d^3}$$

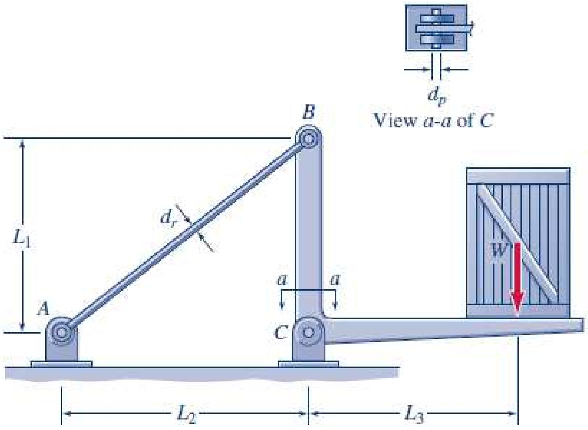
PROBLEM #3 (20 points)

The L-shaped loading frame below is supported by a shear pin (for which the ultimate shear stress is τ_u=40ksi) and by a tie-rod AB (for which the tensile yield stress is σ_y=30ksi). Both the tie-rod and the pin are to be sized with a factor of safety of $FS = 2.5$, the tie rod with respect to tensile yielding, and the shear pin with respect to ultimate shear failure. If the platform load is $W = 3\ kips$ and the frame dimensions are $L_1 = 3\ ft$, $L_2 = 4\ ft$, $L_3 = 6\ ft$,

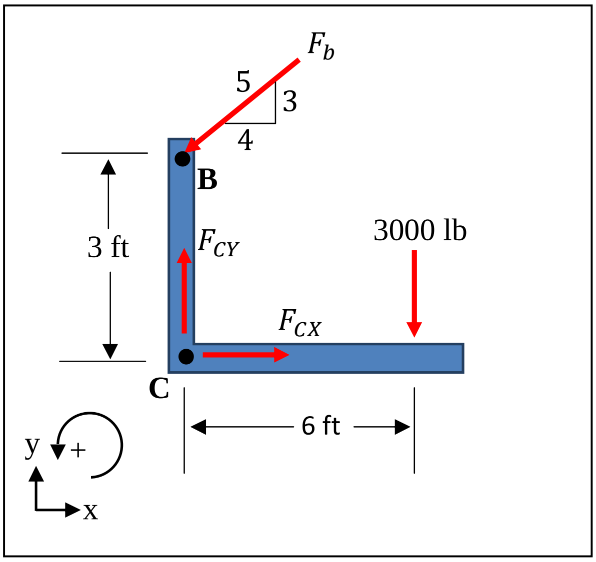
a) Determine the reaction forces in the tie-rod AB and at the pin support C.
b) Determine the minimum allowable diameters d_p of the shear pin and d_r of the tie-rod.



Solution:



FBD:



Allowable stresses:

$$\sigma_{rod,allow} = \frac{\sigma_y}{FS} = 12\ ksi$$

$$\tau_{pin,allow} = \frac{\tau_f}{FS} = 16\ ksi$$

Sum moments about C:

$$\Sigma M_C = 0 = (6\ ft)(-3000\ lb) + (3\ ft)\left(\frac{4}{5}F_b\right)$$

$$\rightarrow F_b = 7500\ lb$$

Sum forces in x and y:

$$\Sigma F_y = 0 = F_{CY} - \frac{3}{5}F_b - 3000\ lb \rightarrow F_{CY} = 7500\ lb$$

$$\Sigma F_x = 0 = F_{CX} - \frac{4}{5}F_b \rightarrow F_{CX} = 6000\ lb$$

Stresses in rod and pin, and minimum diameters:

$$\sigma_{rod,allow} = \frac{F_b}{A_r} \rightarrow 12\ ksi = \frac{7500\ lb}{\pi\left(\frac{d_r}{2}\right)^2} \rightarrow \boxed{d_r = 0.89\ in}$$

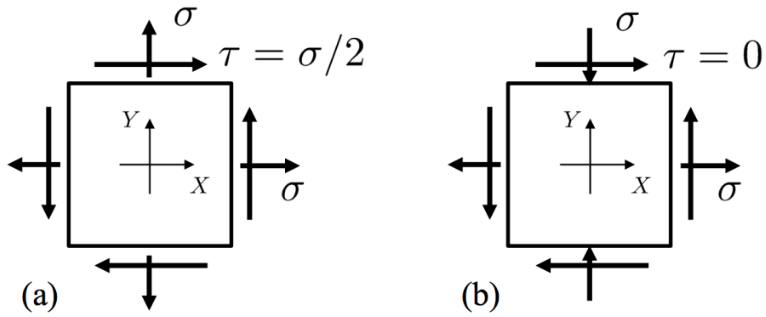
$$\sigma_{pin,allow} = \frac{F_c}{A_r} \rightarrow 16\ ksi = \frac{\sqrt{(7500\ lb)^2 + (6000\ lb)^2}}{2\left[\pi\left(\frac{d_p}{2}\right)^2\right]} \rightarrow \boxed{d_p = 0.62\ in}$$

<u>PROBLEM #4 (20 Points)</u>:

PART A – 10 points - For each state of plane stress shown below, i.e., for configurations (a) and (b), indicate whether each component of the state of strain is:

- = 0 (equal to zero)
- > 0 (greater than zero)
- < 0 (less than zero)

The material is linear elastic with Poisson's ratio ν ($0 < \nu < 0.5$), and the deformations are small.



	(a)	(b)
ϵ_x		
ϵ_y		
ϵ_z		
γ_{xy}		
γ_{xz}		
γ_{yz}		

Fill in with '= 0', '> 0', *or* '< 0'.

Solution:

Part A.

Plane stress conditions:

$$\sigma_z = \tau_{xz} = \tau_{yz} = 0$$

The needed equations are:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \upsilon\left(\sigma_y + \sigma_z\right)\right] = \frac{1}{E}\left[\sigma_x - \upsilon\sigma_y\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \upsilon(\sigma_x + \sigma_z)\right] = \frac{1}{E}\left[\sigma_y - \upsilon\sigma_x\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \upsilon\left(\sigma_y + \sigma_x\right)\right] = -\frac{\upsilon}{E}\left(\sigma_y + \sigma_x\right)$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$

$$\gamma_{xz} = \frac{1}{G}\tau_{xz} = 0$$

$$\gamma_{yz} = \frac{1}{G}\tau_{yz} = 0$$

Case a) $\sigma_x = \sigma_y = \sigma$

$$\tau_{xy} = \frac{\sigma}{2}$$

Case b)

$$\sigma_x = -\sigma_y = \sigma$$

$$\tau_{xy} = 0$$

Using these into the equations, we have

	(a)	(b)
ε_x	$= \frac{1}{E}[1-\upsilon]\sigma \ > \mathbf{0}$	$= \frac{1}{E}[1+\upsilon]\sigma \ > \mathbf{0}$
ε_y	$= \frac{1}{E}[1-\upsilon]\sigma \ > \mathbf{0}$	$=-\frac{1}{E}[1+\upsilon]\sigma \ < \mathbf{0}$
ε_z	$=-\frac{2\upsilon}{E}\sigma \ < \mathbf{0}$	$= \mathbf{0}$
γ_{xy}	$=\frac{\sigma}{2G} \ > \mathbf{0}$	$= \mathbf{0}$
γ_{xz}	$= \mathbf{0}$	$= \mathbf{0}$
γ_{yz}	$= \mathbf{0}$	$= \mathbf{0}$

Part B.

Compatibility codition $\phi_A = \phi_B$

The length L of the two segments is the same, and thus

$$\frac{\phi_A}{L} = \frac{\phi_B}{L} \quad \text{or} \quad \frac{T_A}{G_A I_A} = \frac{T_B}{G_B I_B}$$

From which, $T_A = \frac{G_A I_A}{G_B I_B} T_B$

The stress in a is then,

$$\tau_A = \frac{T_A r}{I_A} = \frac{G_A I_A}{G_B I_B I_A} T_B r = \frac{G_A}{G_B} \frac{T_B r}{I_B} = \frac{G_A}{G_B} \tau_B$$

And since $G_B = 2G_A$

$$\tau_A = \frac{1}{2} \tau_B$$

Which means that $\tau_A < \tau_B$ and the correct stress distribution is the one in **Answer 1.**

Part C.

Compatibility condition $e_A = e_B$

$$\frac{F_A L}{E_A A_A} = \frac{F_B L}{E_B A_B}$$

From which,

$$\frac{F_A}{A_A} = \frac{E_A}{E_B} \frac{F_B}{A_B}$$

Or,

$$\sigma_A = \frac{E_A}{E_B} \sigma_B$$

Since $E_A = 2E_B$, we have

$$\sigma_A = 2\sigma_B$$

Therefore, $\sigma_A > \sigma_B$ and the correct stress distribution is the one shown in **Answer 3.**

PROBLEM # 5 (20 Points):

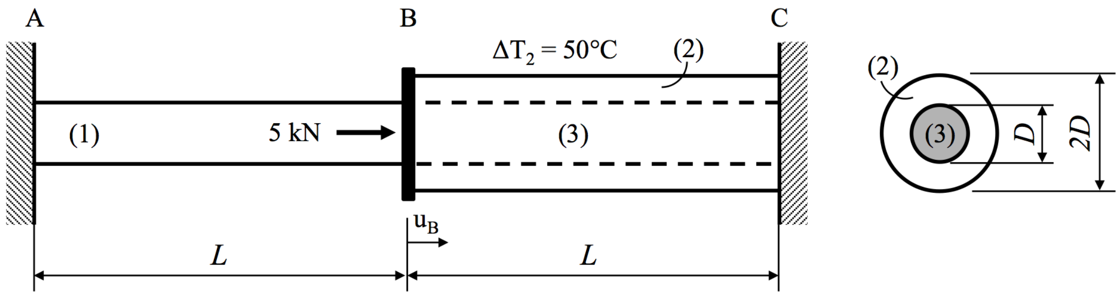
The three-part axially loaded member shown in the figure consists of a solid circular rod segment (1) with diameter *D* and Young's modulus *E*, a pipe sleeve element (2) with outer diameter *2D*, inner diameter *D* and Young's modulus *E/2*, and a cylindrical core element (3) with diameter *D* and Young's modulus *E*. All elements have length *L*. Segments (1), (2) and (3) are joined by a rigid connector at B. Ends A and C are fixed to rigid walls. The rigid connector at B is acted upon by a 5 kN axial load as shown. In addition, the temperature of element (2) is increased by 50°C, while the temperature of other two elements is kept constant. The length of the connector is negligible.

Given the following parameters:

L = 1 m, D = 25 mm, E = 210 GPa, α = 11×10-6 /°C (coefficient of linear expansion),

Determine:

a) Load carried by each segment (1), (2), and (3).
b) Stress in each segment. State whether the stress is compressive or tensile.
c) The axial displacements of the connector u_B.

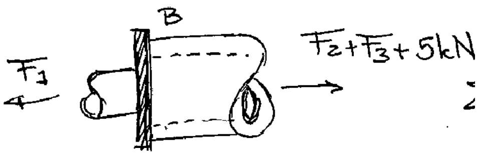


Solution:

$$A_1 = \frac{\pi D^2}{4}, \qquad E_2 = \frac{E_1}{2}, \qquad A_2 = \frac{\pi(4D^2 - D^2)}{4} = \frac{3\pi D^2}{4},$$

Equilibrium:

$$\sum F = 0 = F_2 + F_3 + 5\ kN - F_1$$



Force-displacement behavior:

$$e_1 = \frac{F_1 \mathrm{L}}{A_1 E_1}, \qquad e_2 = \frac{F_2 \mathrm{L}}{A_2 E_2} + \alpha \Delta T L, \qquad e_3 = \frac{F_3 \mathrm{L}}{A_1 E_1}$$

Compatibility:

$e_1 = u_B - u_A = u_B$

$e_2 = u_C - u_B = -u_B$

$e_3 = u_C - u_B = -u_B$

$e_1 = -e_2, \qquad e_2 = e_3$

$$\frac{F_1 \mathrm{L}}{\frac{\pi D^2}{4} E_1} = -\frac{F_2 \mathrm{L}}{\frac{3\pi D^2}{4}\frac{E_1}{2}} - \alpha \Delta T L = -\frac{F_3 \mathrm{L}}{\frac{\pi D^2}{4} E_1}$$

$F_2 + F_3 + 5\ kN - F_1 = 0$

$$\left[-\frac{F_1}{\frac{\pi D^2}{4} E_1} - \alpha \Delta T\right] \frac{3\pi D^2}{4}\frac{E_1}{2} - F_1 + 5\ kN - F_1 = 0$$

$F_1 = -22.87\ kN$

$F_2 = -50.74\ kN$

$F_1 = 22.87\ kN$

$$\sigma_1 = \frac{F_1}{\frac{\pi D^2}{4}} = -46.59\ MPa\ \ (C)$$

$$\sigma_2 = \frac{F_2}{\frac{3\pi D^2}{4}} = -34.45\ MPa\ \ (C)$$

$$\sigma_3 = \frac{F_3}{\frac{\pi D^2}{4}} = 46.59\ MPa \quad (T)$$

$$u_B = e_1 = \frac{F_1 \mathrm{L}}{\frac{\pi D^2}{4} E_1} = -0.22 mm$$