ME 323 Examination # 1

Name (Print) KWON (Last) JAE-SUNG (First) Y/4

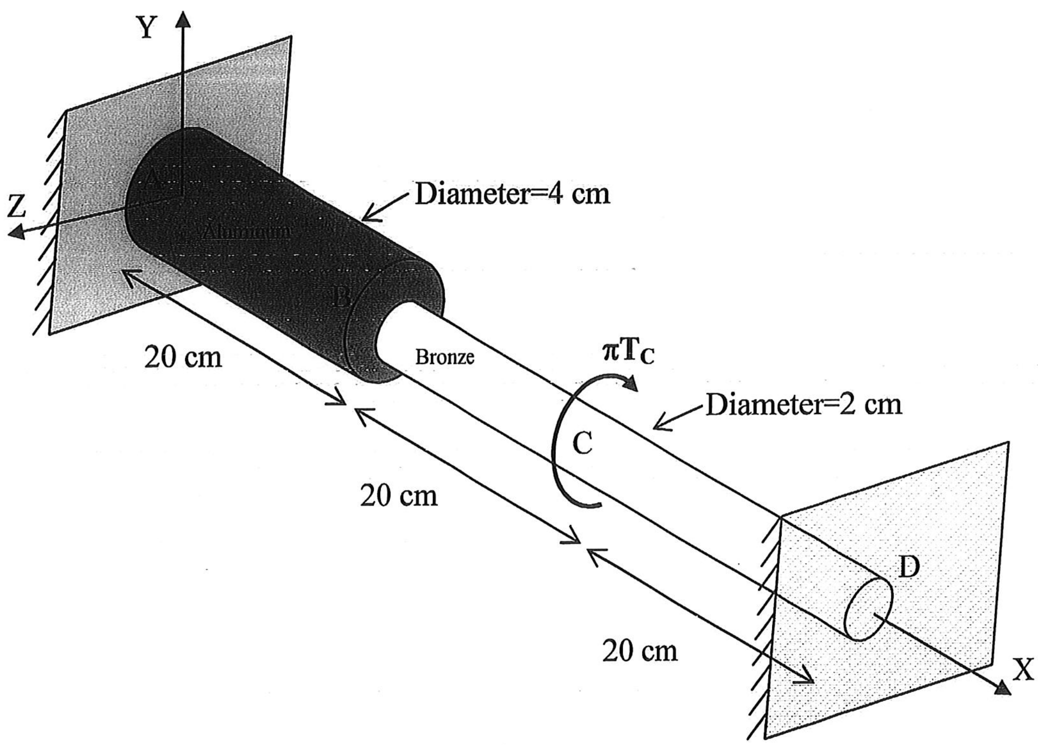
February 15, 2012

Instructor ____________________

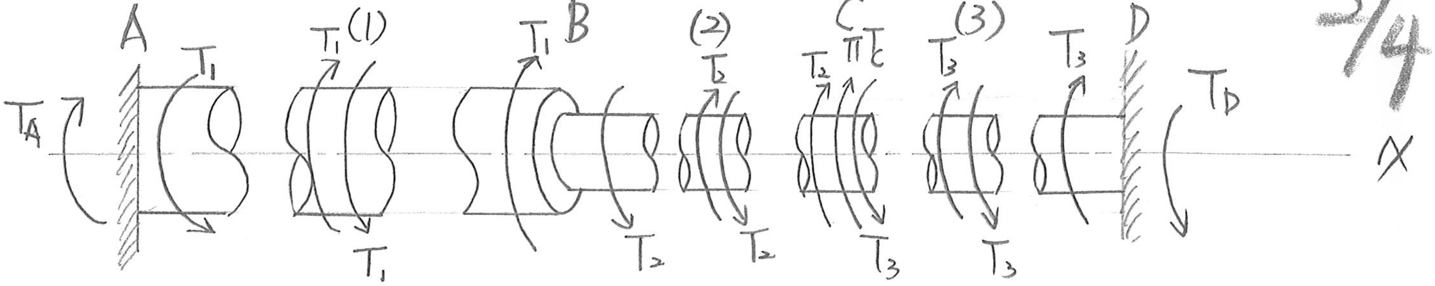
PROBLEM #2 (25 points)

The stepped rod shown below is made of Aluminum and bronze (shear modulus of aluminum G_{al}= 30 GPa and shear modulus of bronze G_{br}= 45 GPa) and is fixed to the walls at A and D. The aluminum section is 20 cm long and has a diameter of 4 cm. The bronze section is 40 cm long and has a diameter of 2 cm. An external torque πT_C is applied as shown. The shear yield stress of the aluminum and bronze are τ_Y = 200 MPa and 128 MPa respectively. Determine the maximum allowable applied torque at C, T_C?

Note: the torque applied is πT_C.



Sol)



$\tau_1 = 2\times10^8$ [Pa] , $G_1 = 30\times10^9$ [Pa]

$\tau_2 = 1.28\times10^8$ [Pa] , $G_2 = 45\times10^9$ [Pa]

$\tau_3 = 1.28\times10^8$ [Pa] , $G_3 = 45\times10^9$ [Pa]

$d_1 = \frac{1}{25}$ [m] , $L_1 = \frac{1}{5}$ [m] , $J_1 = \frac{\pi d_1^4}{32} = \frac{\pi}{125\times10^5}$ [m^4]

$d_2 = \frac{1}{50}$ [m] , $L_2 = \frac{1}{5}$ [m] , $J_2 = \frac{\pi d_2^4}{32} = \frac{\pi}{2\times10^8}$ [m^4]

$d_3 = \frac{1}{50}$ [m] , $L_3 = \frac{1}{5}$ [m] , $J_3 = \frac{\pi d_3^4}{32} = \frac{\pi}{2\times10^8}$ [m^4]

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i) $\Sigma M_A = -T_A + T_1 = 0$ $\quad\therefore T_A = T_1$ — ①

ii) $\Sigma M_B = -T_1 + T_2 = 0$ $\quad\therefore T_1 = T_2$ — ②

iii) $\Sigma M_C = -\pi T_C - T_2 + T_3 = 0$ $\quad\therefore T_3 = \pi T_C + T_2$ — ③

iv) $\Sigma M_D = T_D - T_3 = 0$ $\quad\therefore T_D = T_3$ — ④

v)
$$\left.\begin{aligned}\phi_1 &= \cancel{\phi_A}^{\to 0} - \phi_B\\ \phi_2 &= \phi_B - \phi_C\\ \phi_3 &= \phi_C - \cancel{\phi_D}^{\to 0}\end{aligned}\right) \quad \therefore \phi_1 + \phi_2 + \phi_3 = 0. \text{ — ⑤}$$

In Eq. ⑤,

$$\phi_2 = \left(\frac{T_2 L_2}{G_2 J_2}\right) = -\phi_1 - \phi_3 = -\left[\frac{T_1 L_1}{G_1 J_1} + \frac{T_3 L_3}{G_3 J_3}\right]$$

$\because T_1 = T_2$, $\quad T_1 = T_2 = -\frac{G_2 J_2}{L_2}\left[\frac{T_1 L_1}{G_1 J_1} + \frac{T_3 L_3}{G_3 J_3}\right]$

$\therefore\ T_1 = T_2 = -0.914\,T_3$

Using Eq. ③

$$T_3 = 1.641\,T_c$$

Meanwhile

$$\tau_1 = \frac{\frac{T_1 d_1}{2}}{J_1} = \frac{\frac{-0.914\times1.641}{2}\,T_c d_1}{J_1} = 2\times10^8\,[Pa] \quad \therefore\ T_c = -1675\,[N\cdot m]$$

$$\tau_2 = \frac{\frac{T_2 d_2}{2}}{J_2} = \frac{\frac{-0.914\times1.641}{2}\,T_c d_2}{J_2} = 1.28\times10^8\,[Pa] \quad \therefore\ T_c = -134\,[N\cdot m]$$

$$\tau_3 = \frac{\frac{T_3 d_3}{2}}{J_3} = \frac{\frac{1.641\,T_c}{2}\,d_3}{J_3} = 1.28\times10^8\,[Pa] \quad \therefore\ T_c = 122.5\,[N\cdot m]$$

therefore,

$$T_c = 122.5\ [N\cdot m]$$

Verification)

From the final T_c,

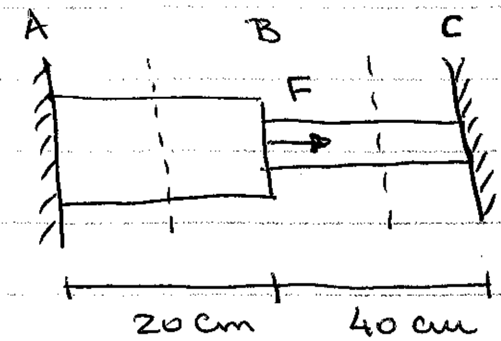
$$T_1 = -183.83\ [N\cdot m]$$
$$T_2 = -183.83\ [N\cdot m]$$
$$T_3 = 201.06\ [N\cdot m]$$

$$\tau_1 = \frac{T_1 d_1}{2J_1} = -1.46\times10^7\ [Pa]$$
$$\tau_2 = \frac{T_2 d_2}{2J_2} = -1.17\times10^8\ [Pa]$$
$$\tau_3 = \frac{T_3 d_3}{2J_3} = 1.28\times10^8\ [Pa]$$

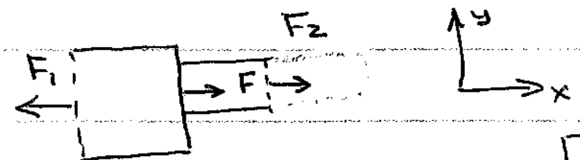
4/4

Therefore, with $T_c = 122.5$ [N.m], the shear stress in each section is safe. Any torque larger than $T_c = 122.5$ [N.m] will be safe in the Aluminium section, but will fail the bronze.

Problem 1



Normal stress in Aluminum and Steel sections.



$$\sum F_x = 0$$

$$\boxed{F + F_2 - F_1 = 0} \quad (1)$$

Compatibility condition

$$\boxed{e_1 + e_2 = 0} \quad (2)$$

Force-elongation

$$\boxed{e_1 = \frac{F_1 L_1}{A_1 E_1}} \quad (3)$$

$$\boxed{e_2 = \frac{F_2 L_2}{A_2 E_2}} \quad (4)$$

Solve for F_1, F_2.

3 and 4 in 2

$$\frac{F_1 \cdot \cancel{20\,10^{-2}\,m}}{\cancel{10\,GPa}\ \tfrac{1}{4}\pi \cdot \cancel{(2\,10^{-2}\,m)^2}}_{4} + \frac{F_2 \cdot \cancel{40\,10^{-2}\,m}^{\,2}}{\cancel{30\,GPa}\ \tfrac{1}{4}\pi \cdot \cancel{(1\,10^{-2}\,m)^2}}_{3}$$

$$\frac{F_1}{4} + \frac{2F_2}{3} = 0$$

$$F_1 = -\frac{8}{3}F_2 \longrightarrow \text{in } 1$$

$$10\ kN + F_2 + \frac{8}{3}F_2 = 0$$

$$\frac{11}{3}F_2 = -10\ kN$$

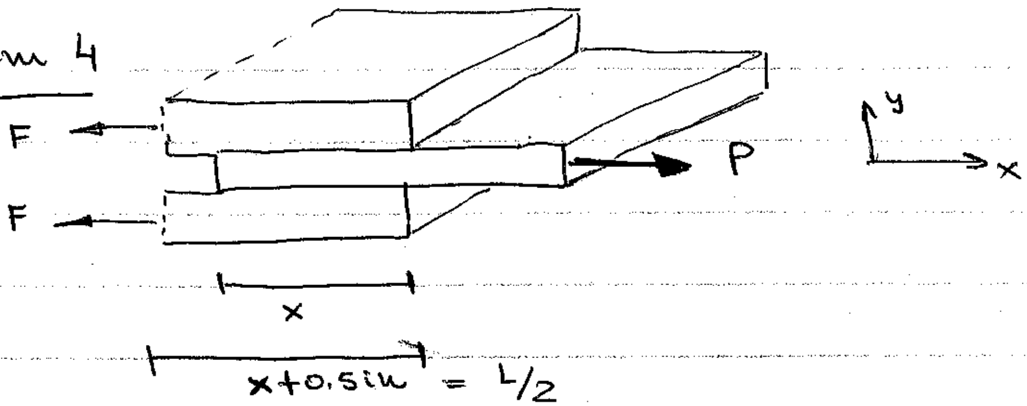
$$F_2 = -\frac{30}{11}\ kN$$

$$F_1 = \frac{80}{11}\ kN$$

$$\sigma_1 = F_1/A_1 = -8.68\ MPa$$

$$\sigma_2 = F_2/A_2 = 5.99\ MPa$$

Problem 4



$\sum F_x = 0 \quad F = P/2$

$$\tau = \frac{P/2}{\text{Area}} = \frac{P/2}{x \cdot 5\,\text{in}}$$

$$\tau < \tau_{allow} = 100\ \text{psi}$$

$$\frac{P/2}{x \cdot 5\,\text{in}} < 100\ \text{psi}$$

$$\Rightarrow x > 10\ \text{in}$$

$$\Rightarrow L = 2x + 1\ \text{in}$$

$$\boxed{L = 21\ \text{in.}}$$

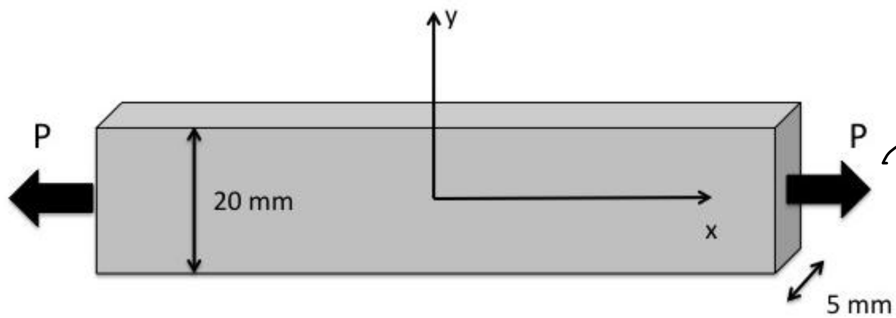
ME 323 Examination # 1 **Name** ______________________
(Print) (Last) (First)

February 15, 2012 **Instructor** Sadeghi Koslowski

PROBLEM #3 (25 points)

A rectangular bar with a base and height of 5 mm and 20 mm respectively is subjected to a tensile load P. The longitudinal (x) and transverse (y) direction strains were measured as $\varepsilon_x = 0.0015$ and $\varepsilon_y = 0.0005$.
Determine:

(a) The Poisson's ratio of the specimen.
(b) If the strains were measured with an axial load of P = 25 kN. What is the Young's modulus of the specimen?



only σ_x

$\sigma_y = \sigma_z = 0$

$$\varepsilon_x = \frac{1}{E}\sigma_x \qquad \varepsilon_y = \frac{1}{E}(0 - \nu\sigma_x) = -\frac{\nu\sigma_x}{E} = -\nu\varepsilon_x$$

(a) $$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = \frac{+0.0005}{0.0015} = \boxed{+\frac{1}{3}}$$

(b) $$E = \frac{\sigma_x}{\varepsilon_x} = \frac{P}{A\,\varepsilon_x} = \frac{P}{w\cdot h\cdot\varepsilon_x} = \frac{25\,kN}{(5\cdot 20)\,mm^2\cdot 0.0015}$$

$$\boxed{E = 166.7\ GPa}$$