ME 323 Examination # 1 **Name** SOLUTION

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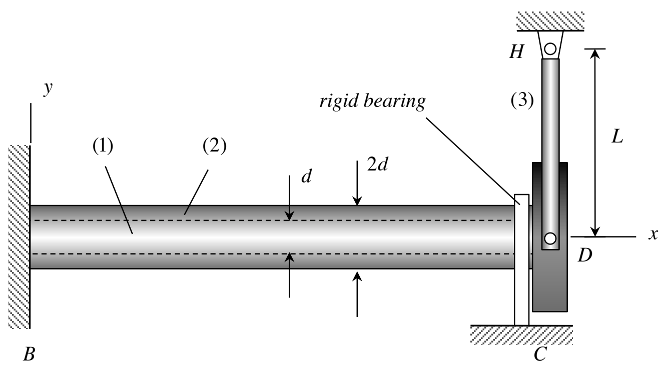
PROBLEM NO. 1 – 25 points max.

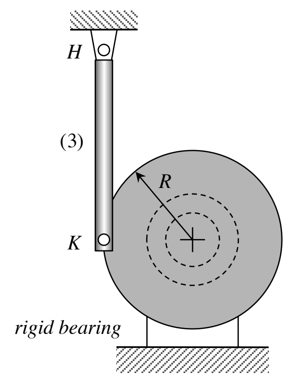
Shaft BC is made up of elements (1) and (2), with (1) being a core concentrically placed within the tubular element (2). Elements (1) and (2) have shear moduli of G and 2G, respectively. The shaft is rigidly connected to a fixed wall at B and to a rigid connecter at D. A rigid bearing supports the shaft just inside the connector D. Rod (3) (of length L, Young's modulus E, thermal expansion coefficient α and cross-sectional area A) connects point K on connector D to a fixed support at H. The temperature of (3) is increased by an amount of ΔT, with the rest of the system being held fixed in temperature.

a) Determine the torques carried by elements (1) and (2) of the shaft.

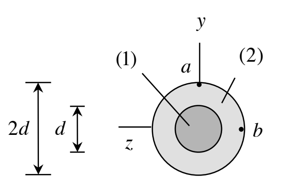
b) Consider the points "a" and "b" on the outer radius of segment (2). Draw the stress elements to represent the stress states at "a" and "b".

HINTS: The bearing near the end of the shaft restricts transverse deflection of the shaft, but allows free rotation of that end of the shaft. Assume small rotations of connector D as the temperature of (3) is increased. Note that connector D provides a rigid connection between the centerline of the shaft and the connection point K.

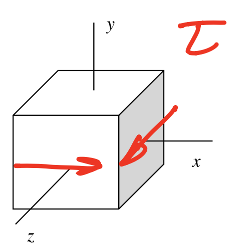




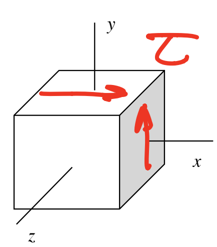
end view of connector D



cross section of shaft



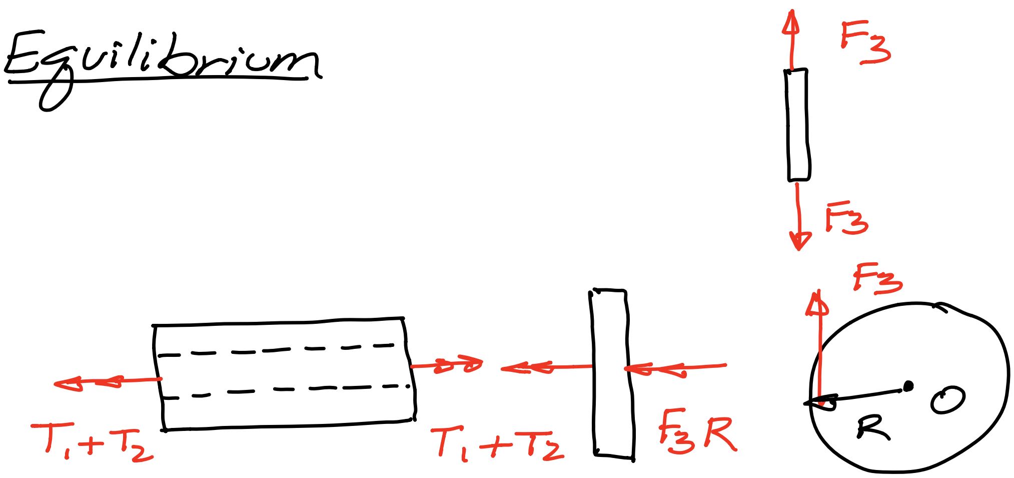
stress element at "a"



stress element at "b"

***Please** start your work on the following page.*

1. Equilibrium



(1) $\underline{D}$: $\sum M = -F_3R - T_1 - T_2 = 0$

(INDETERMINATE: 1 eqn/3 unknowns)

2. Load/deformation

(2) $$\Delta\phi_1 = \frac{T_1(3L)}{GI_{p1}} \quad ; \quad I_{p1} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32}d^4$$

(3) $$\Delta\phi_2 = \frac{T_2(3L)}{2GI_{p2}} \quad ; \quad I_{p2} = \frac{\pi}{2}\left[\left(\frac{2d}{2}\right)^4 - \left(\frac{d}{2}\right)^4\right]$$
$$= \frac{15}{32}\pi d^4$$

(4) $$e_3 = \frac{F_3L}{EA} + \alpha\Delta TL$$

3. Compatibility

(5) $$\phi_D = \Delta\phi_1 = \Delta\phi_2$$

(6) $$e_3 = R\phi_D = R\Delta\phi_1$$

4. <u>Solve</u>

(2), (3), (5): $\dfrac{3T_1 \cancel{L}}{\cancel{G} I_{p1}} = \dfrac{3T_2 \cancel{L}}{2\cancel{G} I_{p2}} \Rightarrow T_2 = 2\dfrac{I_{p2}}{I_{p1}} T_1$ (7)

(2), (4), (6): $\dfrac{F_3 L}{EA} + \alpha \Delta T L = R\,\dfrac{3T_1 L}{G I_{p1}} \Rightarrow$

$$F_3 = \frac{3REA}{GI_{p1}} T_1 - EA\Delta T \alpha L \quad (8)$$

(1), (7), (8):

$$-\frac{3R^2EA}{GI_{p1}} T_1 + EA\Delta TLR\alpha - T_1 - 2\frac{I_{p2}}{I_{p1}} T_1 = 0$$

$$\hookrightarrow T_1 = \frac{EA\Delta TRL\alpha}{1 + \dfrac{3R^2EA}{G(\pi d^4/32)} + 2\left(\dfrac{15\cancel{\pi d^4}/\cancel{32}}{\cancel{\pi d^4}/\cancel{32}}\right)}$$

$$\begin{cases} T_1 = \dfrac{EA\Delta TRL\alpha}{31 + \dfrac{96R^2EA}{\pi G d^4}} \\[2em] T_2 = 2\left(\dfrac{\frac{15}{32}\pi d^4}{\frac{1}{32}\pi d^4}\right) T_1 = 30\left(\dfrac{EA\Delta TRL\alpha}{31 + \dfrac{96R^2EA}{\pi G d^4}}\right) \end{cases}$$

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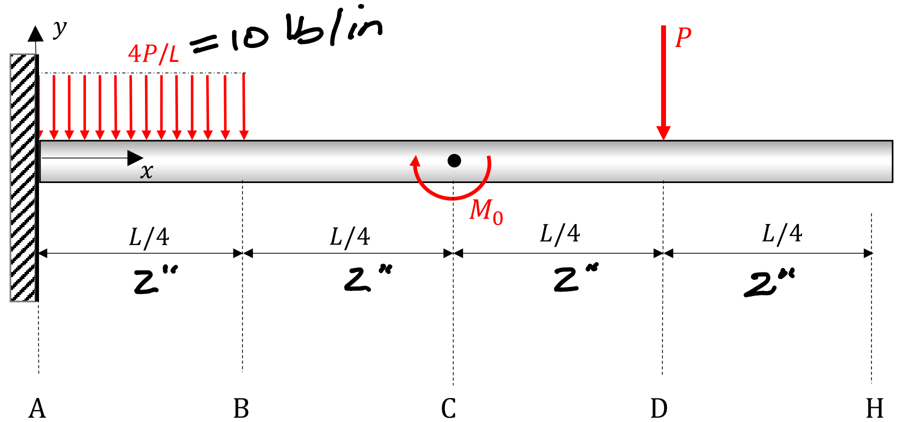
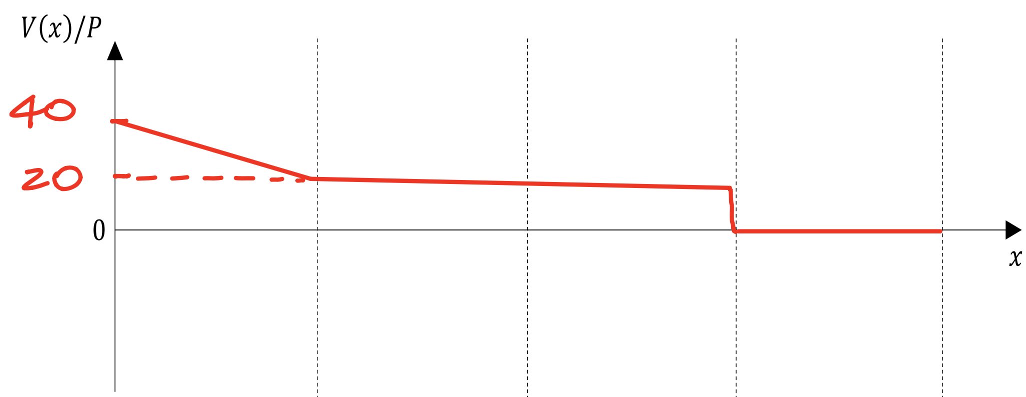
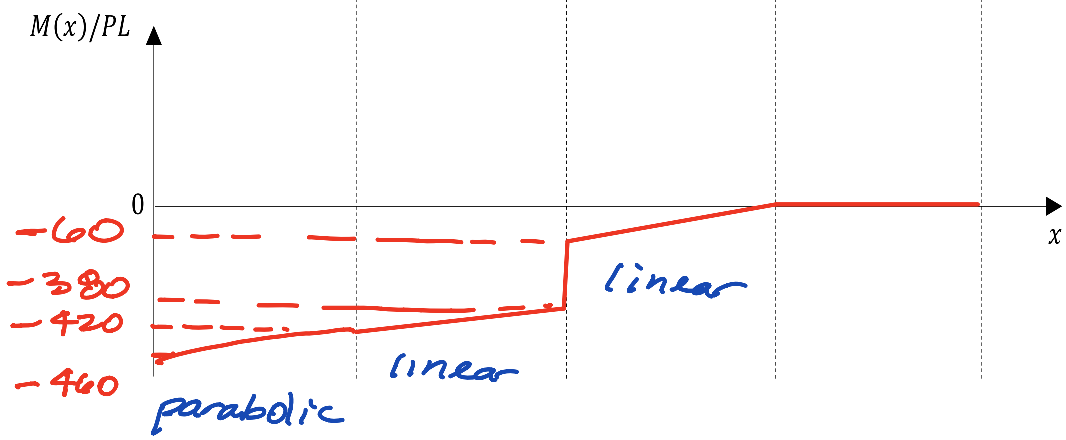
PROBLEM NO. 2 – 25 points max.

The cantilever beam AH is acted upon by a uniformly distributed downward load of intensity $4P/L$ (force/length) between A and B, a concentrated moment $M_0 = 2PL$ at C, and a vertical force P at D. Using $P = 20\ lb$ and $L = 8\ in$, construct the shear force and bending moment diagrams for the beam. Mark numerical values for points A through H, along with max/min values on these diagrams.

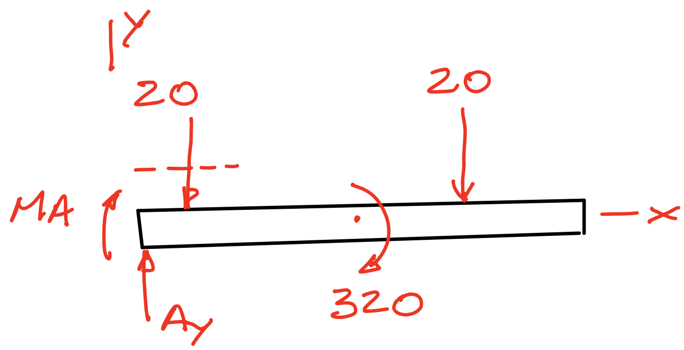
NOTE: You are *not* required to show your calculations needed for constructing these diagrams; that is, if your diagrams are correct, you will receive full credit without calculations. However, if you have numerical errors in your answers without supporting calculations, you will *not receive partial credit*. It is suggested that you show calculations.







External reactions



$$\sum M = -320 - (20)(1) - (20)(6) - M_A = 0$$

$$\hookrightarrow M_A = -320 - 20 - 120 = -460 \text{ lb.in}$$

$$\sum F_y = A_y - 20 - 20 = 0 \Rightarrow A_y = 40 \text{ lb}$$

- AB

$V(0) = A_y = 40$

$V(2) = V(0) - (2)(10) = 20$

$M(0) = M_A = -460$

$M(2) = M(0) + (20)(2) + \frac{1}{2}(20)(2) = -420$

- BC

$V(4) = V(2) + 0 = 20$

$M(4^-) = M(2) + (20)(2) = -380$

- CD

$V(6^-) = V(4) + 0 = 20$

$M(4^+) = M(4^-) - (-320) = -60$

$M(6) = M(4^+) + (20)(2) = 0$ ($\checkmark$ checks)

- DH

$V(6^+) = V(6^-) - 20 = 0$ ($\checkmark$ checks)

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PROBLEM NO. 3 – 25 points max.

Each of the three truss members in the figure has a modulus of elasticity E. The cross-sectional areas of the members are $A_1 = A_3 = A$ and $A_2 = 2A$. Member (2), whose coefficient of thermal expansion is α, is heated by an amount $\Delta T_2 = \Delta T$, and a vertical load P is applied to the truss at joint D. The temperature of the other members is held constant.

a) Draw the free-body diagram for joint D.

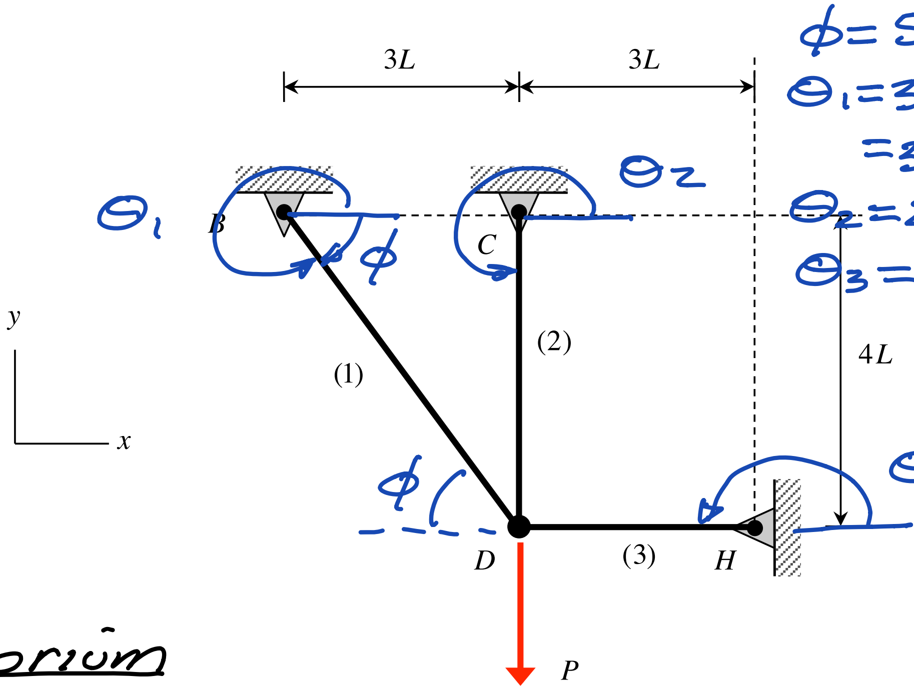
b) Determine expressions for the member axial forces F_1, F_2, and F_3 in terms of parameters defined here in the problem statement (E, A, P, α and ΔT).

$\phi = 53.13°$

$\theta_1 = 360 - 53.13°$
$= 306.87°$

$\theta_2 = 270°$

$\theta_3 = 180°$

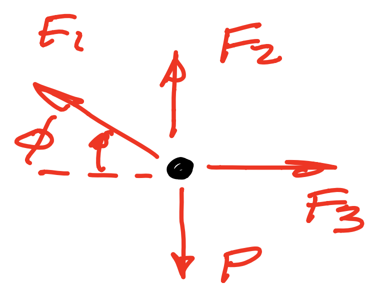


1. Equilibrium

(1) $\Sigma F_x = F_3 - F_1 \cos\phi = 0 \quad \Rightarrow F_3 = 0.6 F_1$

(2) $\Sigma F_y = F_2 + F_1 \sin\phi - P = 0 \Rightarrow F_2 = -0.8 F_1 + P$

2. Force/elongation



(3) $e_1 = \dfrac{F_1 (5L)}{EA}$

(4) $e_2 = \dfrac{F_2 (4L)}{E(2A)} + \alpha \Delta T (4L)$

(5) $e_3 = \dfrac{F_3 (3L)}{EA}$

3. <u>Compatibility</u>

(6) $e_1 = u_D\cos\theta_1 + v_D\sin\theta_1 = 0.6u_D - 0.8v_D$

(7) $e_2 = u_D\cancel{\cos\theta_2}^{\,0} + v_D\cancel{\sin\theta_2}^{\,-1} = -v_D$

(8) $e_3 = u_D\cancel{\cos\theta_3}^{\,-1} + v_D\cancel{\sin\theta_3}^{\,0} = -u_D$

4. <u>Solve</u>

(3) – (8): $e_1 = 0.8e_2 - 0.6e_3$

$$\frac{5F_1\cancel{L}}{EA} = 0.8\left[\frac{2F_2\cancel{L}}{EA} + 4\alpha\Delta T\cancel{L}\right] - 0.6\left(\frac{3F_3\cancel{L}}{EA}\right)$$

$$5F_1 = 1.6F_2 - 1.8F_3 + 3.2\alpha\Delta TEA$$

$$= 1.6(-0.8F_1 + P) - 1.8(0.6F_1) + 3.2\alpha\Delta TEA$$

$\hookrightarrow [5 + 1.28 + 1.08]F_1 = 3.2\alpha\Delta TEA + 1.6P$

$$\hookrightarrow \begin{cases} F_1 = \dfrac{1}{7.36}\left[3.2\alpha\Delta TEA + 1.6P\right] \\ F_2 = -0.8\left(\dfrac{1}{7.36}\right)\left[3.2\alpha\Delta TEA + 1.6P\right] + P \\ F_3 = \dfrac{0.6}{7.36}\left[3.2\alpha\Delta TEA + 1.6P\right] \end{cases}$$

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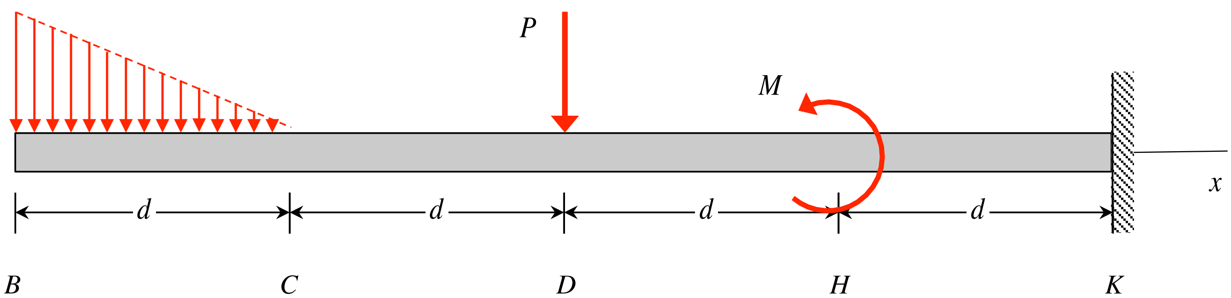
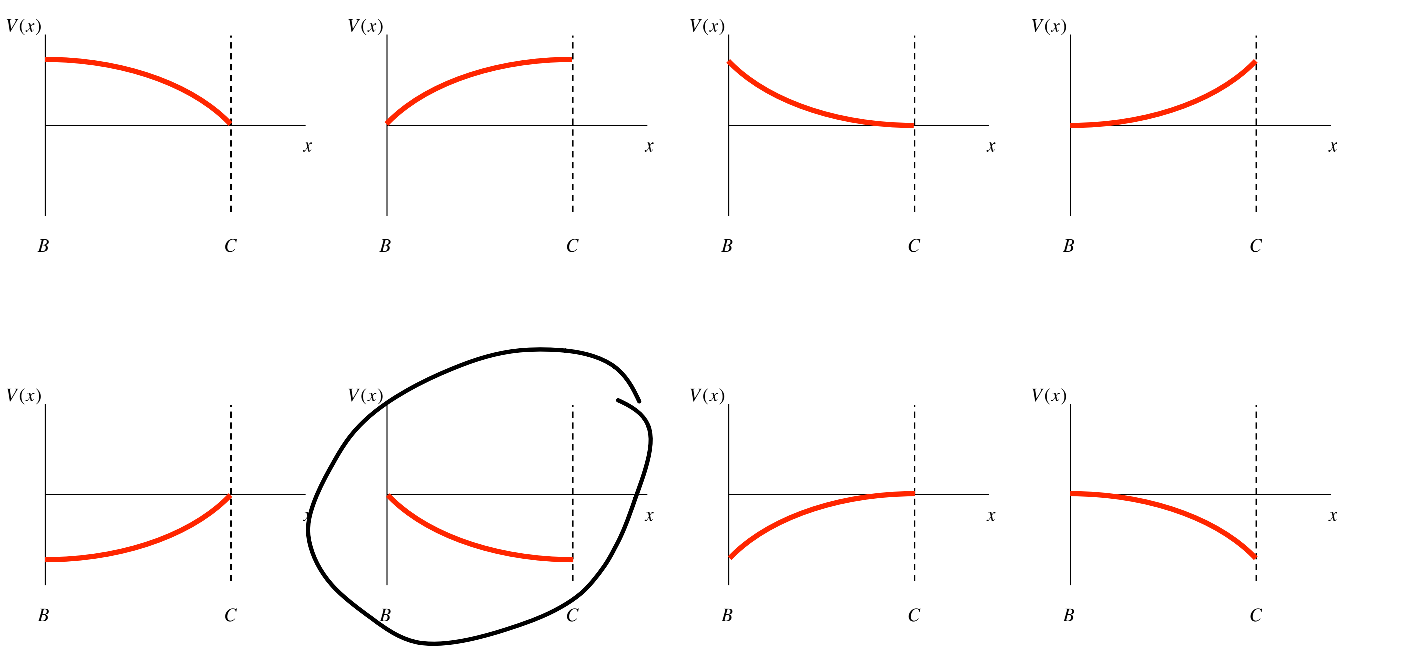
You are NOT required to show your work for any part of Problem No. 4.

PROBLEM NO. 4 - PART A – 3 points max.



The above loading is applied to a cantilevered beam. *Circle* the figure below which most accurately describes the internal shear force resultant in the beam between locations B and C.



$$p = \frac{dV}{dx} \Rightarrow \frac{dV}{dx}(0) = p(0) < 0 \quad \text{(negative slope)}$$

$$\frac{dV}{dx}(d) = p(d) = 0 \quad \text{(zero slope)}$$

$$V(0) = 0 \quad \text{(free end)}$$

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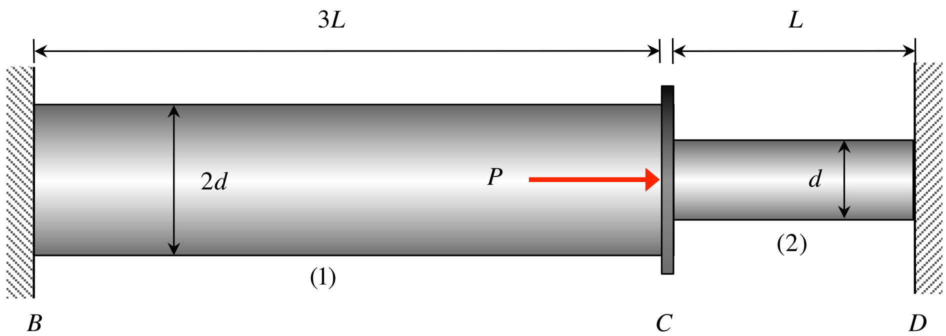
PROBLEM NO. 4 - <u>PART B</u> – 3 points max.



A rod is made up of solid elements (1) and (2) joined by a rigid connector C, with the material of (1) and (2) having the same modulus of elasticity E. An axial load P is applied to C with no thermal loads being present. Let F_1 and F_2 represent the axial loads in elements (1) and (2), respectively. Circle the response below which most accurately describes the relative sizes of $|F_1|$ and $|F_2|$:

a) $|F_1| > |F_2|$

b) $|F_1| = |F_2|$

c) $|F_1| < |F_2|$

d) More information is needed to answer this question.

$$e_1 + e_2 = 0$$

$$\hookrightarrow \frac{F_1(3L)}{\cancel{E}\pi\left(\frac{2d}{2}\right)^2} + \frac{F_2 L}{\cancel{E}\pi\left(\frac{d}{2}\right)^2} = 0$$

$$3F_1 = -4F_2 \Rightarrow |F_1| = \frac{4}{3}|F_2|$$

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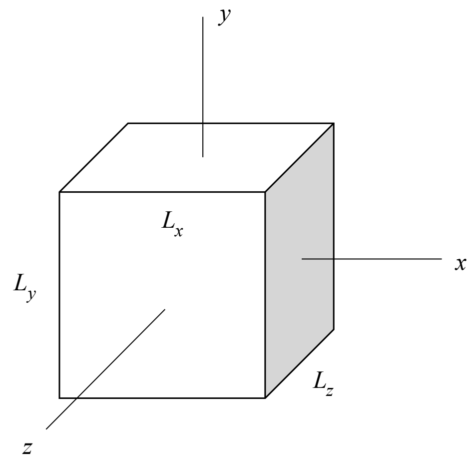
PROBLEM NO. 4 - PART C – 4 points max.



A cube of dimensions $\left(L_x, L_y, L_z\right)$ experiences a state of stress with uniform components of stress though out the cube. The material of the cube has a Young's modulus of E and a Poisson's ratio of $\nu = 0.4$. As a result of the loading on the cube, it is known that $\sigma_y = \sigma_z = \sigma_x / 2 > 0$. As a result of this loading (circle the correct answer):

a) The dimension L_z is *increased*.

b) The dimension L_z remains the *same*.

c) The dimension L_z is *decreased*.

d) More information is needed to answer this question.

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$
$$= \frac{1}{E}\left[\frac{\sigma_x}{2} - \nu\left(\sigma_x + \frac{\sigma_x}{2}\right)\right]$$
$$= \frac{\sigma_x}{2E}\underbrace{(1 - 3\nu)}_{-0.2} = -0.1\frac{\sigma_x}{E} < 0$$

$\hookrightarrow$ L_z is decreased

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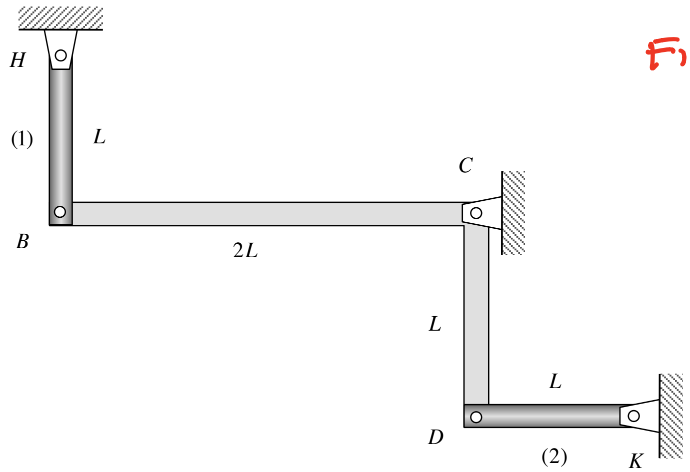

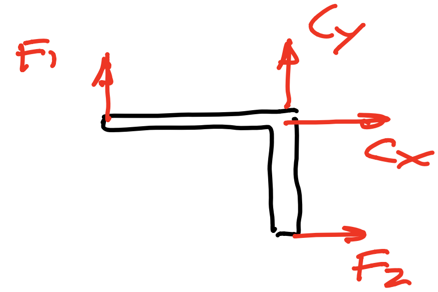

Identical elements (1) and (2) (each having a Young's modulus E, coefficient of thermal expansion α and cross-sectional area A) are connected between ends B and D, respectively, of a rigid, L-shaped bar BCD. The temperature of (1) is *increased* by an amount of $\Delta T > 0$, with the temperature of element (2) being held constant.

PROBLEM NO. 4 - PART D – 3 points max.

Consider the *load* (force) carried by element (1):

a) The load in (1) is *compressive*.

b) The load in (1) is *zero*.

c) The load in (1) is *tensile*.

- $\sum M_C = -F_1(2L) + F_2(L) = 0$

$\hookrightarrow F_2 = -2F_1$

- $\frac{e_1}{2L} = -\frac{e_2}{L} \Rightarrow$

$e_1 = -2e_2$

$\hookrightarrow \frac{F_1 L}{EA} + \alpha \Delta T L = -2\frac{F_2 L}{EA}$

$\hookrightarrow F_1 = - \frac{\alpha \Delta T E A}{5}$

compression

PROBLEM NO. 4 - PART E – 3 points max.

Consider the *strain* in element (1):

a) The strain in (1) is *compressive*.

b) The strain in (1) is *zero*.

c) The strain in (1) is *tensile*.

$$\varepsilon_1 = \frac{F_1}{EA} + \alpha \Delta T$$

$$= \frac{1}{\cancel{EA}}\left[-\frac{\alpha \Delta T \cancel{EA}}{5}\right] + \alpha \Delta T$$

$$= + \frac{4}{5}\alpha \Delta T$$

tensile

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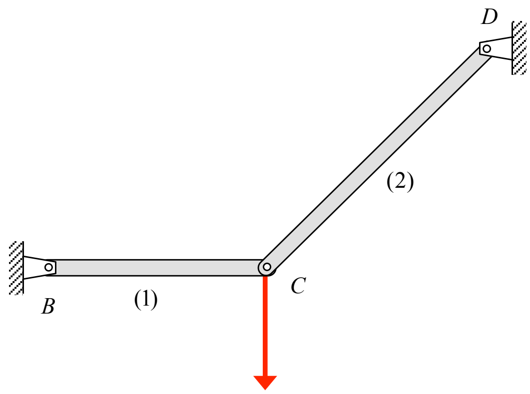
PROBLEM NO. 4 - PART F – 3 points max.

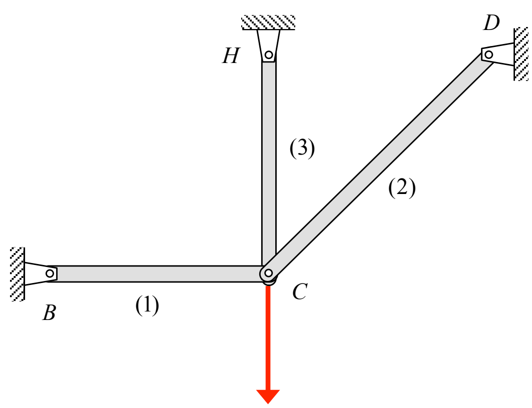


Consider the truss above that is made up of elements (1) and (2).

TRUE or *FALSE*: The stress in element (1) depends on the material makeup of element (2).

DETERMINATE. Stresses found straight from equilibrium

PROBLEM NO. 4 - PART G – 3 points max.



Consider the truss above that is made up of elements (1), (2) and (3).

TRUE or *FALSE*: The stress in element (1) depends on the material makeup of elements (2) and (3).

INDETERMINATE. Need to include material properties in analysis for stress

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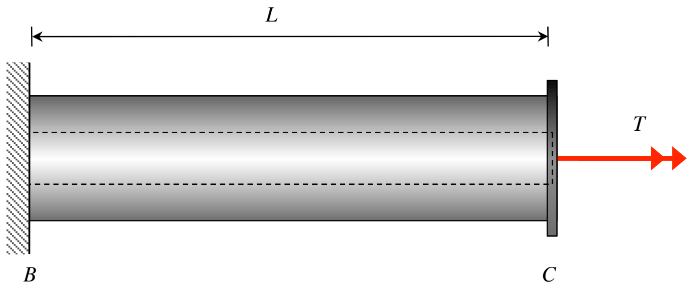
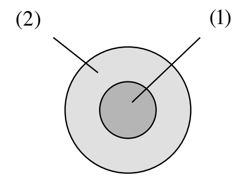
PROBLEM NO. 4 - PART H – 3 points max.





A shaft is made up of a tubular element (2) and core element (1), with the elements have shear moduli of G_2 and G_1, respectively, with $G_2 > G_1$. These elements at attached to a rigid support at B and are attached to a rigid connector C. A torque T is applied to connector C. The resulting shear strain distribution γ on the shaft cross section is shown in the figure below left. In the figure below right, make sketch of the shear stress distribution τ in the shaft cross section. Clearly indicate the slopes of the shear stress curves in terms of the shear strain slope *m*.