Name (Print)____

(Last) (First)

ME 323 - Mechanics of Materials
Exam # 1
Date: October 5, 2016 Time: 8:00 – 10:00 PM

Instructions:

Circle your lecturer's name and your class meeting time.

Gonzalez	Krousgrill	Zhao	Sadeghi
4:30-5:20PM	11:30-12:20PM	12:30-1:20PM	2:30-3:20PM

Begin each problem in the space provided on the examination sheets. If additional space is required, use the paper provided.

Work on one side of each sheet only, with only one problem on a sheet.

Please remember that for you to obtain maximum credit for a problem, you must present your solution clearly.

Accordingly,

- coordinate systems must be clearly identified,
- free body diagrams must be shown,
- units must be stated,
- write down clarifying remarks,
- state your assumptions, etc.

If your solution cannot be followed, it will be assumed that it is in error.

When handing in the test, make sure that ALL SHEETS are in the correct sequential order.

Remove the staple and restaple, if necessary.

Prob. 1 ____________________

Prob. 2 ____________________

Prob. 3 ____________________

Prob. 4 ____________________

Total ____________________

Useful Equations

$$e_x = \frac{1}{E}\left[s_x - n\left(s_y + s_z\right)\right] + aDT$$

$$e_y = \frac{1}{E}\left[s_y - n\left(s_x + s_z\right)\right] + aDT$$

$$e_z = \frac{1}{E}\left[s_z - n\left(s_x + s_y\right)\right] + aDT$$

$$g_{xy} = \frac{1}{G}t_{xy} \quad g_{xz} = \frac{1}{G}t_{xz} \quad g_{yz} = \frac{1}{G}t_{yz}$$

$$G = \frac{E}{2(1+v)}$$

$$\tau = \frac{Tr}{I_p}$$

$$\phi = \int_0^L \frac{T(x)}{G(x)I_p(x)}\,\mathrm{dx} \qquad \phi = \frac{TL}{GI_p}$$

$I_p = \dfrac{\pi d^4}{32}$ polar area moment for solid circular cross section

$I_p = \dfrac{\pi}{32}(d_o{}^4 - d_i{}^4)$ polar area moment for a circular hollow cross section

$I = \dfrac{1}{12}bh^3$ second area moment for a rectangular cross section

$I = \dfrac{\pi d^4}{64}$ second area moment for a circular cross section

$$e = \int_0^L \frac{F}{EA}dx + \int_0^L \alpha\Delta T dx \qquad e = \frac{FL}{EA} + L\alpha\Delta T$$

$$e = u\cos\theta + v\sin\theta$$

$$Factor\ of\ \ Safety = \frac{Yield\ \ Strength}{Allowable\ Stress}\,or\,\frac{failure\ Stress}{Stress\,member}$$

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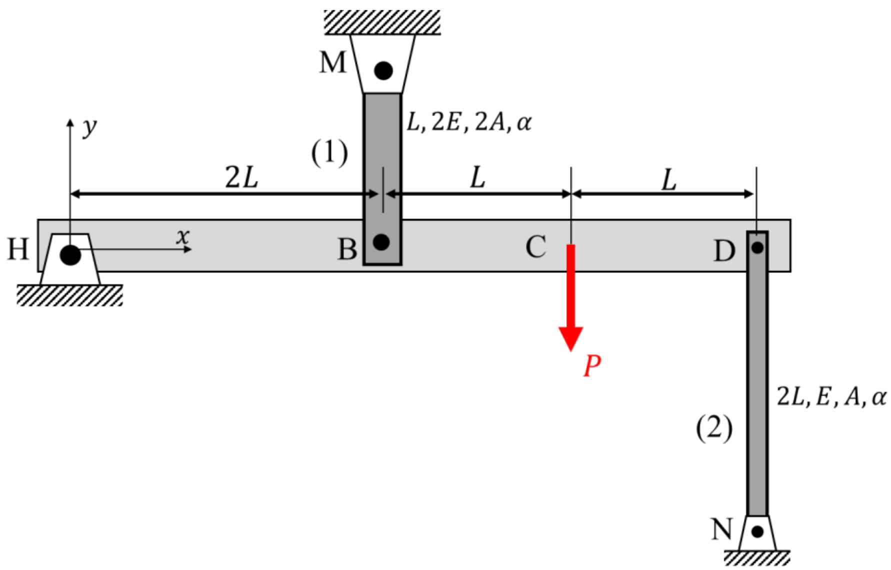
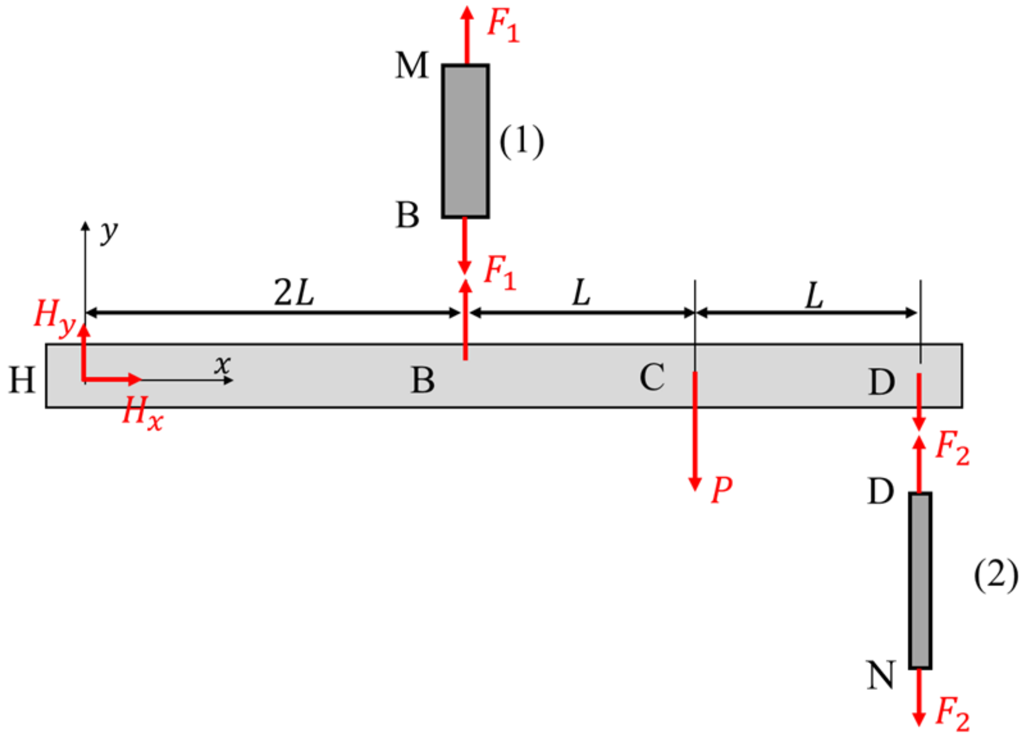
Problem 1 (25 points): A rigid bar HD is pinned to ground at H, and to two elastic rods (1) and (2) at B and D, respectively. Rods (1) and (2) are pinned to a wall at M and N, respectively. The length, Young's modulus, and cross sectional area of each rod is shown in the following figure. The material for each rod possesses a coefficient of thermal expansion of α. A point load P acts at point C on the beam in the negative y direction. The temperature of rod (2) is increased by a temperature ΔT, while the temperature of rod (1) is kept constant. Assuming small rotations:

1. Write down the equilibrium conditions for the rigid bar.
2. Write down the compatibility conditions that relate the elongations of rods (1) and (2).
3. Determine the axial stresses in rods (1) and (2).

Express your answers in terms of P, L, A, E, α, and ΔT.



Solution: 1. Let F_1 and F_2 represent the tensile axial forces carried by rods (1) and (2) respectively.

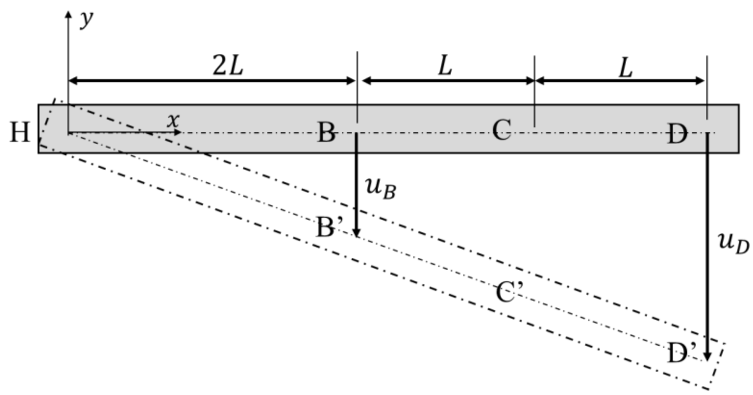


From the FBD of beam HD,

$$\Sigma M_H = F_1(2L) - P(3L) - F_2(4L) = 0$$

$$\Rightarrow F_1 - 2F_2 = \frac{3}{2}P \tag{1.1}$$

2. From the (exaggerated) figure showing the displaced points B', C' and D', where u_B and u_D represent the downward displacements of B and D respectively,



$$\frac{u_B}{2L} = \frac{u_D}{4L} \Rightarrow 2u_B = u_D \tag{1.2}$$

The rotation indicated represents and elongation of rod (1) and a shortening of rod (2), i.e.

$$u_B = e_1; u_D = -e_2 \tag{1.3a}$$

$$\Rightarrow 2e_1 = -e_2 \tag{1.3b}$$

The elongations can be written in terms of the tensile loads carried, and temperature rise as:

$$e_1 = \frac{F_1 L_1}{E_1 A_1} = \frac{F_1 L}{(2E)(2A)} = 0.25\frac{F_1 L}{EA} \tag{1.4a}$$

$$e_2 = \frac{F_2 L_2}{E_2 A_2} + L_2 \alpha \Delta T = \frac{2F_2 L}{EA} + 2L\alpha\Delta T \tag{1.4b}$$

3. From (4.3b), (4.4a), and (4.4b),

$$2 \times 0.25\frac{F_1 L}{EA} = -\frac{2F_2 L}{EA} - 2L\alpha\Delta T$$

$$\Rightarrow F_1 + 4F_2 = -4EA\alpha\Delta T \tag{1.5}$$

Solving (4.1) and (4.5),

$$F_1 = -\frac{4}{3}EA\alpha\Delta T + P \quad ; \quad F_2 = -\frac{2}{3}EA\alpha\Delta T - \frac{1}{4}P \tag{1.6}$$

Therefore, the axial stresses,

$$\sigma_1 = \frac{F_1}{2A} = -\frac{2}{3}E\alpha\Delta T + \frac{P}{2A}$$

$$\sigma_2 = \frac{F_1}{A} = -\frac{2}{3}E\alpha\Delta T - \frac{P}{4A}$$

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<u>Problem 2 (30 points):</u> A shaft is made up of three components: a solid circular shaft (1) of length 2L and diameter d; solid circular shaft (2) of length L and diameter 2d; and circular-cross section tubular shaft (3) of length 3L, outer diameter of 4d and inner diameter of 3d. All three components are made of the same material having a shear modulus of G. Shafts (1) and (2) are connected by a thin, rigid connector C, whereas shafts (2) and (3) are connected by a rigid connector D. Shaft 1 is rigidly attached to ground at B. Torques T and 2T are applied to C and D, respectively, as shown in the figure below. It is desired to know the torque carried by each of the shaft components. To this end:

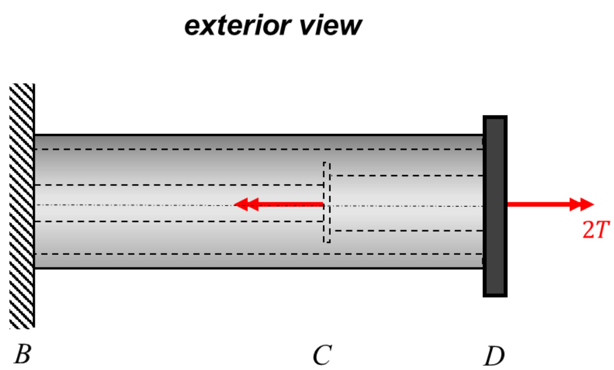
a) Draw free body diagrams (FBDs) of connectors C and D below. Write down the appropriate equilibrium equations from these FBDs.

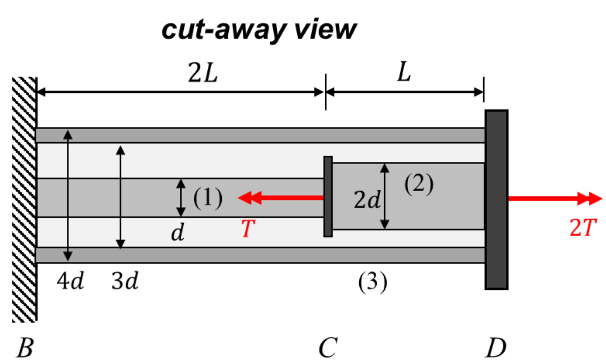
b) Write down the torque/rotation equations for the three shaft components.

c) Write down the appropriate compatibility equations.

d) Solve for the torques in the three shaft components.

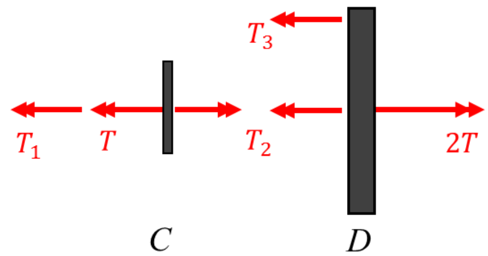
Express your final answers in terms of T, L, d and G.





Solution

Equilibrium



$$\textbf{Connector } \boldsymbol{C}\text{: } \Sigma M_C = -T_1 - T + T_2 = 0 \Rightarrow T_1 = T_2 - T \tag{2.1}$$

$$\textbf{Connector } \boldsymbol{D}\text{: } \Sigma M_D = -T_2 - T_3 + 2T = 0 \Rightarrow T_3 = 2T - T_2 \tag{2.2}$$

Torque/rotation equations

$$I_{P1} = \frac{\pi}{32}d^4; \qquad I_{P2} = \frac{\pi}{32}(2d)^4 = \frac{16\pi}{32}d^4; \qquad I_{P3} = \frac{\pi}{32}\{(4d)^4 - (3d)^4\} = \frac{175\pi}{32}d^4$$

$$\Delta\phi_1 = \left(\frac{TL}{GI_P}\right)_1 = \frac{T_1(2L)(32)}{\pi G d^4} = \frac{64\,T_1 L}{\pi G d^4} \tag{2.3}$$

$$\Delta\phi_2 = \left(\frac{TL}{GI_P}\right)_2 = \frac{T_2(L)(32)}{16\pi G d^4} = \frac{2\,T_2 L}{\pi G d^4} \tag{2.4}$$

$$\Delta\phi_3 = \left(\frac{TL}{GI_P}\right)_3 = \frac{T_3(3L)(32)}{175\pi G d^4} = \frac{96\,T_3 L}{175\,\pi G d^4} \tag{2.5}$$

Compatibility equations

$$\textbf{Shaft 1}\text{: } \phi_C = \phi_B + \Delta\phi_1 = \Delta\phi_1$$

$$\textbf{Shaft 2}\text{: } \phi_D = \phi_C + \Delta\phi_2 = \Delta\phi_1 + \Delta\phi_2 \tag{2.6}$$

$$\textbf{Shaft (3)}\text{: } \phi_D = \phi_B + \Delta\phi_3 = \Delta\phi_3 \tag{2.7}$$

Equating (2.6) and (2.7):

$$\Delta\phi_1 + \Delta\phi_2 = \Delta\phi_3 \tag{2.8}$$

Solve

Using equations (2.3)-(2.5) and (2.8):

$$\frac{64\,T_1 L}{\pi G d^4} + \frac{2\,T_2 L}{\pi G d^4} = \frac{96\,T_3 L}{175\,\pi G d^4} \quad \Rightarrow 64T_1 + 2T_2 = \frac{96}{175}T_3 \tag{2.9}$$

Using equations (2.1), (2.2) and (2.9):

$$64(T_2 - T) + 2T_2 = \frac{96}{175}(2T - T_2) \quad \Rightarrow T_2 = 0.978\,T$$

Using equations (2.1) and (2.2):

$$T_1 = T_2 - T = -0.022\,T$$
$$T_3 = 2T - T_2 = 1.022\,T$$

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<u>Problem 3 (25 Points)</u>: A simply supported beam shown in Fig 3.1, is 10 ft long and is supported at B and M. It is subjected to the following loads as shown. The cross section of the beam is uniform rectangular of height 4 in and width 2 in as shown in Fig 3.2. For the state of loading shown:

1. Construct shear force and bending moment diagrams for the beam AM. Mark numerical values for points A through M along with max/min values on these diagrams for the shear, and moment diagrams respectively. Provide justification for your diagrams in terms of calculations.

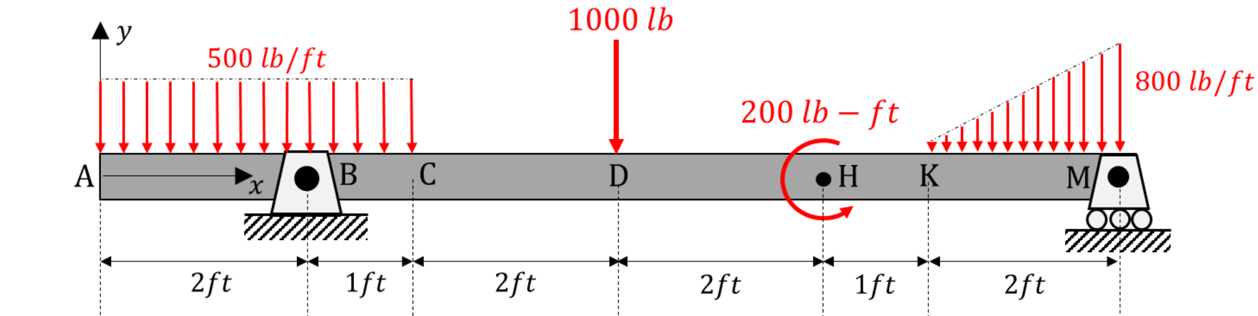
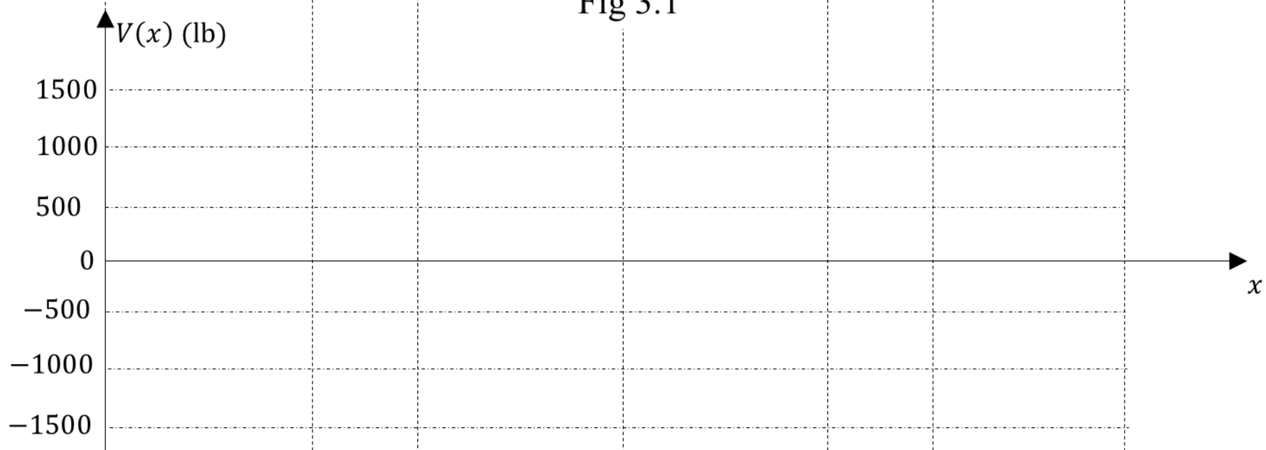
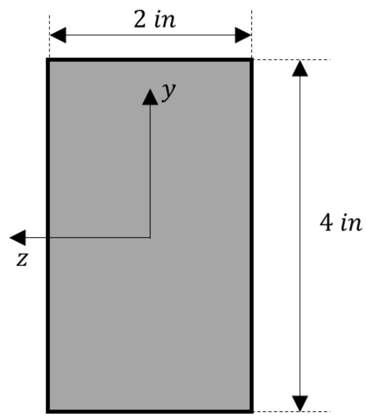


Fig 3.1

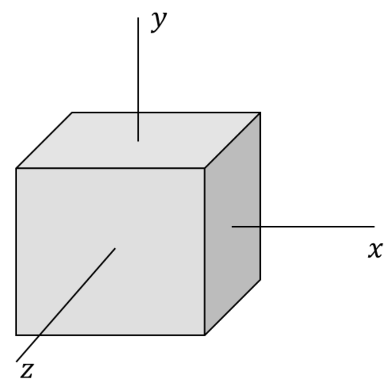


2. Determine the cross section (x), and locations (y) of the max bending normal (flexural) stress in the beam. Also determine the magnitude of the corresponding flexural stress. State whether the stress is compressive, or tensile. Represent the stress element at this location, and label the non-zero stress components in Fig 3.3.



cross section

Fig 3.2

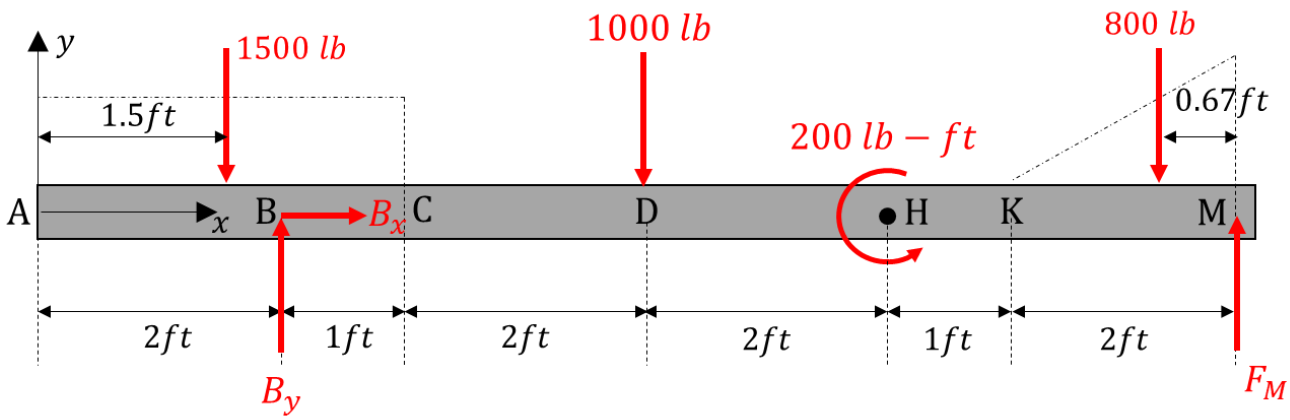


Stress Element for
Max Flexural stress

Fig 3.3

Solution: 1. Net intensity of the uniform load: $500 \times 3 = 1500\ lb$ acting at $x = 1.5\ ft$. Net intensity of the triangular load: $0.5 \times 800 \times 2 = 800\ lb$, acting at $x = 10 - 0.67 = 9.33\ ft$

From the FBD of beam AD,



$$\Sigma F_x = B_x = 0 \tag{3.1}$$

$$\Sigma F_y = -1500 + B_y - 1000 - 800 + F_M = 0 \Rightarrow F_M = 3300 - B_y \tag{3.2a}$$

$$\Sigma M_M = 1500 \times 8.5 - B_y \times 8 + 1000 \times 5 + 200 + 800 \times 0.67 = 0 \tag{3.3a}$$

$$\Rightarrow B_y = 2310.75\ lb \tag{3.3b}$$

Substituting the value of B_y into $(3.2a)$,

$$F_M = 3300 - 2310.75 = 989.25\ lb \tag{3.2b}$$

Inspection approach for the shear force diagram:

Since there are no point loads at A, $V(0) = 0$. Between A and B, shear force drops uniformly with a slope that is equal to the intensity of the uniformly distributed load.

$$V_B^- = V(0) - 500 \times 2 = -1000\ lb$$

Due to the reaction force B_y, the shear force jumps up by an equivalent amount, i.e.

$$V_B^+ = V_B^- + B_y = -1000 + 2310.75 = 1310.75\ lb$$

Between B and C, shear force drops uniformly with a slope that is equal to the intensity of the uniformly distributed load.

$$V_C = V_B^+ - 500 \times 1 = 1310.75 - 500 = 810.75\ lb$$

Since there are no applied loads between C and D, shear force remains constant, i.e.

$$V_D^- = V_{CD} = V_C = 810.75\ lb$$

Due to the applied downward load of 1000 lb at D, shear force drops by an equivalent amount, i.e.

$$V_D^+ = V_D^- - 1000 = -189.25\ lb$$

Since there are no applied forces between D and K, shear force remains constant, i.e.

$$V_K = V_{DK} = V_D = -189.25\ lb$$

Due to the triangular downward load between K and M, shear force drops parabolically by an amount that is equal to the net intensity of the applied load, i.e.

$$V_M = V_k - 800 = -989.25\ lb = -F_M \text{ (check)}$$

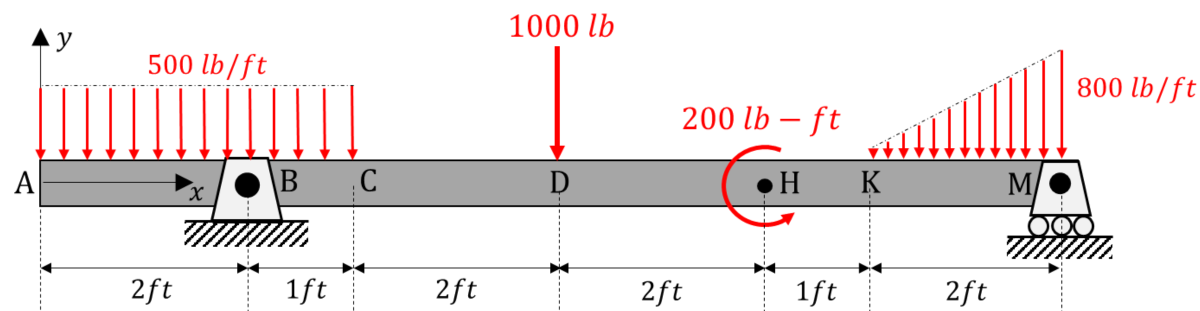
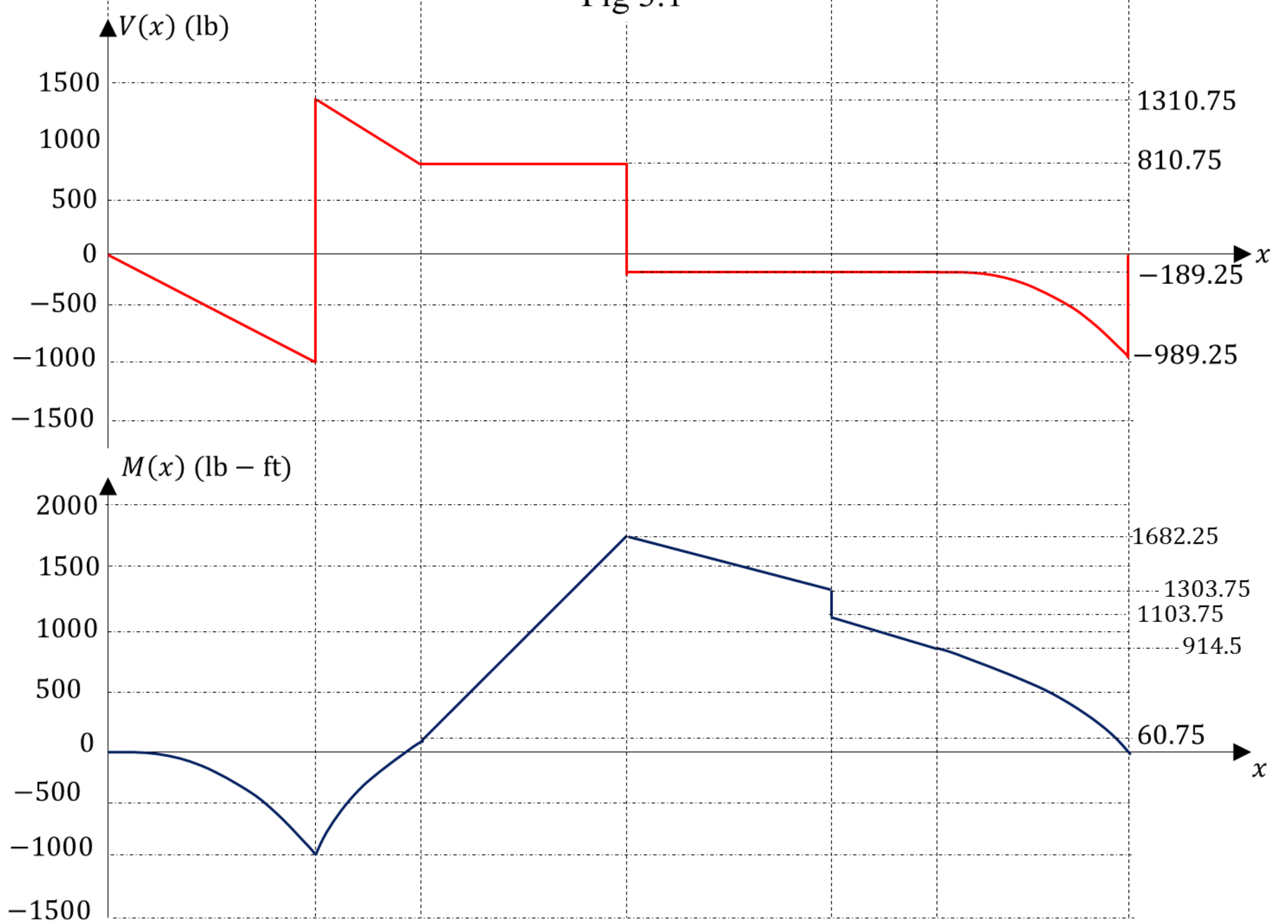


Fig 3.1



Inspection approach for the bending moment diagram:

Since there are no applied moments at A, $M(0) = 0$. Between A and B, bending moment drops parabolically by an amount equal to the triangular area (ΔM_{AB}) in the shear force diagram in this range.

$$M_B = M(0) - 0.5 \times 1000 \times 2 = -1000\ lb - ft$$

Between B and C, bending moment increases parabolically by an amount equal to the area of the trapezium (ΔM_{BC}) in the shear force diagram in this range. However, at B, the bending moment would have a higher positive slope, than at C, due to the corresponding shear forces.

$$\Delta M_{BC} = 0.5(1310.75 + 810.75) \times 1 = 1060.75\ lb - ft$$

$$M_C = M_B + \Delta M_{BC} = -1000 + 1060.75 = 60.75\ lb - ft$$

Between C and D, bending moment increases linearly with a slope that is equal to the intensity of uniform shear force $810.75\ lb$ in this range.

$$M_D = M_C + 810.75 \times 2 = 1682.25\ lb - ft$$

Between D and H, bending moment decreases linearly with a slope that is equal to the intensity of uniform shear force $-189.25\ lb$ in this range.

$$M_H^- = M_D - 189.25 \times 2 = 1303.75\ lb - ft$$

Due to the counter-clockwise moment at H, moment drops by $200\ lb - ft$.

$$M_H^+ = M_H^- - 200 = 1103.75\ lb - ft$$

Between H and K, bending moment decreases linearly with a slope that is equal to the intensity of uniform shear force $-189.25\ lb$ in this range.

$$M_K = M_H^+ - 189.25 \times 1 = 914.5\ lb - ft$$

Between K and M, bending moment drops cubically (order 3) by an amount equal to the parabolic area (ΔM_{KM}) in the shear force diagram in this range. Since end M is pinned, $M_M = 0$.

2. The magnitude of maximum bending moment occurs at D ($x = 5ft$). Therefore, maximum flexural stress also occurs at this cross section. Since the bending moment is positive and the cross section symmtrical about the neutral plane, maximum tensile stress is experienced at the bottom plane ($y = -2\ in$), and maximum compressive stress at the top plane ($y = 2\ in$).

$$M_{max} = 1682.25\ lb - ft = 1682.25 \times 12\ lb - in$$

For a rectangular cross section,

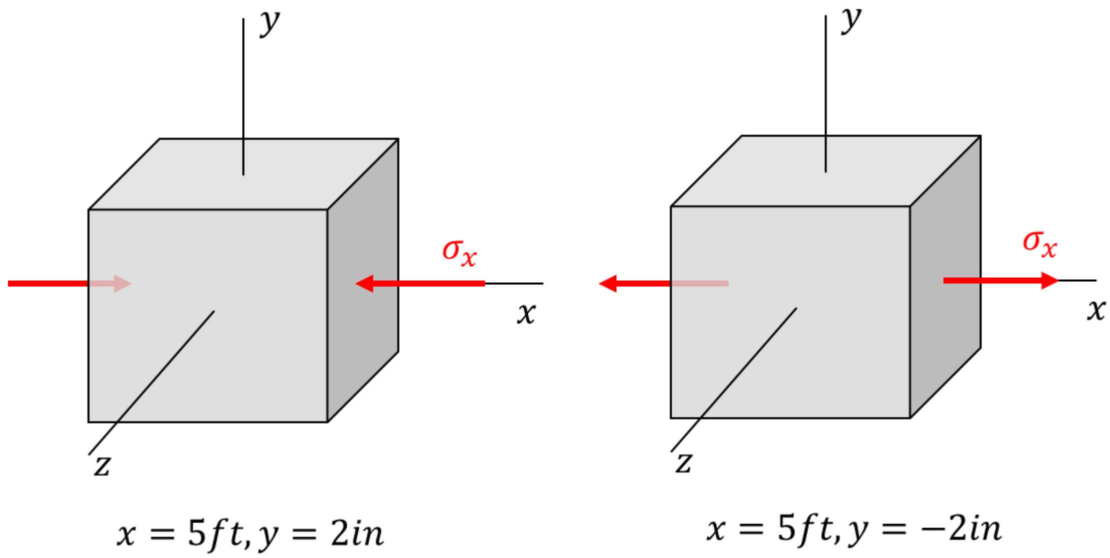
$$I_z = \frac{bh^3}{12} = \frac{(2)(4)^3}{12} = 10.67\ in^4$$

Therefore, at $= 5ft, y = -2\ in$,

$$\sigma_{bottom} = -\frac{M_{max}(-2)}{I_z} = \frac{1682.25 \times 12 \times 2}{10.67} = 3783.88\ psi = 3.78\ ksi$$

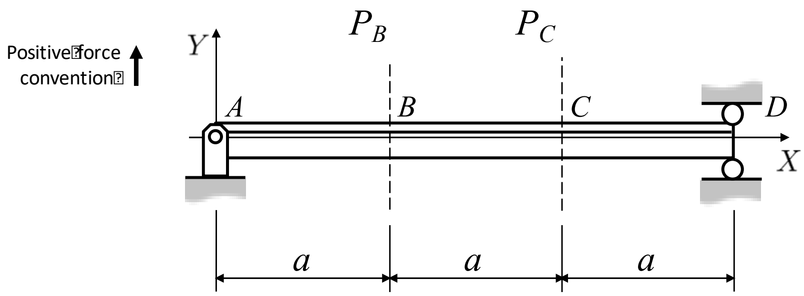
At $= 5ft, y = +2\ in$,

$$\sigma_{bottom} = -\frac{M_{max}(2)}{I_z} = -\frac{1682.25 \times 12 \times 2}{10.67} = -3783.88\ psi = -3.78\ ksi$$

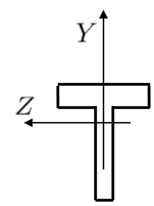


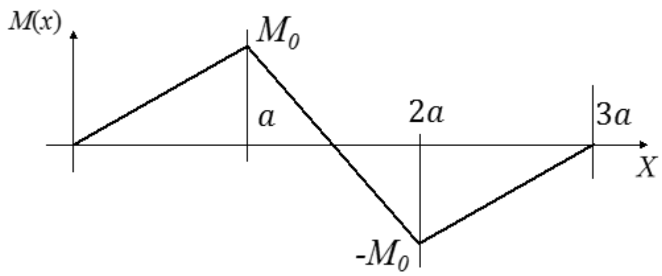
<u>Problem #4 (20 Points)</u>: ***PART A – 8 points***

A T-beam of length $3a$ is supported at the two ends and loaded by forces P_B and P_C. The line of action of the forces is indicated (dashed lines) but the direction is to be determined. The correct moment diagram is properly shown below.



Cross section of the T-beam





(a) Indicate the cross section(s) where the maximum tensile stress is attained:

(i)	$x = 0$	(iii)	$x = 2a$
(ii)	$x = a$	(iv)	$x = 3a$

(b) Indicate the cross section(s) where the maximum compressive stress is attained:

(i)	$x = 0$	(iii)	$x = 2a$
(ii)	$x = a$	(iv)	$x = 3a$

(c) Indicate the value of the reaction at A, that is of A_y:

(i)	$A_y = M_0/a$	(iii)	$A_y = 2M_0/a$	(v)	$A_y = 3M_0/a$
(ii)	$A_y = - M_0/a$	(iv)	$A_y = - 2M_0/a$	(vi)	$A_y = - 3M_0/a$

(d) Indicate the value of the load at B, that is of P_B:

(i)	$P_B = M_0/a$	(iii)	$P_B = 2M_0/a$	(v)	$P_B = 3M_0/a$
(ii)	$P_B = - M_0/a$	(iv)	$P_B = - 2M_0/a$	(vi)	$P_B = - 3M_0/a$

Solution

The distance from the neutral plane is maximum at the bottom of the cross section. Let's assume that the bottom of the cross section is at $(x, -y_b)$.

Flexural stress:

$$\sigma_{max} = -\frac{M(-y_b)}{I_z}$$

(a) Ans: (ii) $x = a$. Since maximum positive bending moment occurs at $x = a$, the maximum tensile stress occurs here, i.e.

$$\sigma_{max,tensile} = -\frac{M(-y_b)}{I_z} = \frac{M_0 y_b}{I_z}$$

(b) Ans: (iii) $x = 2a$. Since maximum negative bending moment occurs at $x = 2a$, the maximum compressive stress occurs here, i.e.

$$\sigma_{max,comp} = -\frac{(-M_0)(-y_b)}{I_z} = -\frac{M_0 y_b}{I_z}$$

(c) Ans: (i) $A_y = M_0/a$.

The slope of the linear bending moment is the shear force $V(0) = A_y$.

(d) Ans: (vi) $P_B = -3M_0/a$.

Since bending moment is linear in $a \le x \le 2a$,

$$M_C = M_B + V_B^+ \times a$$

$$\Rightarrow V_B^+ = \frac{M_C - M_B}{a} = -\frac{2M_0}{a}$$

$$V_B^+ = V_B^- + P_B \ , \quad and \ \ V_B^- = V(0) = A_y = \frac{M_0}{a}$$

$$\Rightarrow P_B = V_B^+ - \frac{M_0}{a} = -\frac{2M_0}{a} - \frac{M_0}{a} = -\frac{3M_0}{a}$$

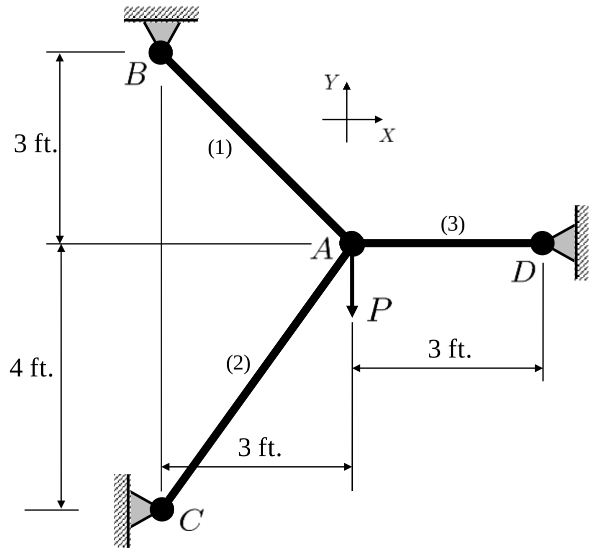
PART B – 6 points

Use only the compatibility condition for the truss structure shown in the figure to find the value of the elongation of member 2 (e_2) in terms of the elongation of member 1 (e_1), and the elongation of member 3 (e_3). Specifically:

(a) Determine an expression for the compatibility condition at A:

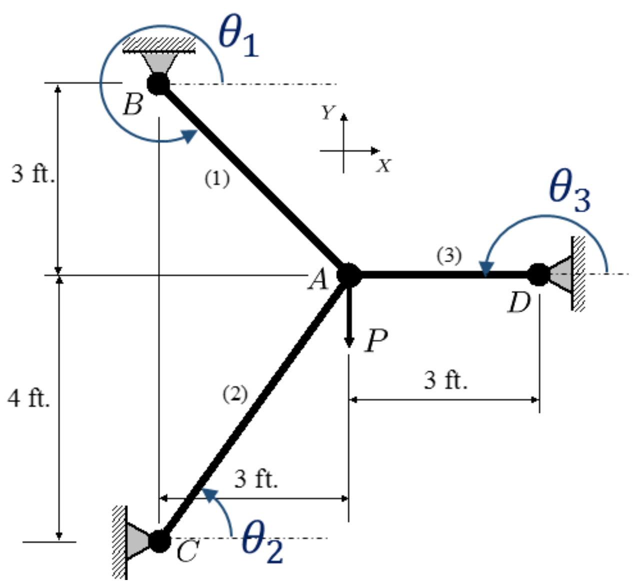
$$e_2 = a\, e_1 + b\, e_3$$

where a and b are numbers.



Please notice that you are not required to solve for the internal axial forces at equilibrium.
Please show your work and thought process.

Solution:
The angular orientation of the three joints are shown below:



$$\theta_1 = 315°, or - 45°$$

$$\theta_2 = \tan^{-1}\frac{3}{4} \Rightarrow \cos\theta_2 = \frac{3}{5}, \sin\theta_2 = \frac{4}{5}$$

$$\theta_3 = 180°$$

Compatibility:

$$e_1 = u_A\cos\theta_1 + v_A\sin\theta_1 = \frac{1}{\sqrt{2}}(u_A - v_A) \qquad (1)$$

$$e_2 = u_A\cos\theta_2 + v_A\sin\theta_2 = \frac{3}{5}\,u_A + \frac{4}{5}\,v_A \qquad (2)$$

$$e_3 = u_A\cos\theta_3 + v_A\sin\theta_3 = -u_A \qquad (3)$$

The following algebraic operation with equations (1) - (3):

$$\begin{array}{rl} 4\sqrt{2}\,e_1 = 4\,u_A - 4\,v_A & [4\sqrt{2}\times(1)] \\ +5e_2 = 3\,u_A + 4\,v_A & [5\times(2)] \\ +7e_3 = -7u_A & [7\times(3)] \\ \hline 4\sqrt{2}\,e_1 + 5e_2 + 7e_3 = 0 & \end{array}$$

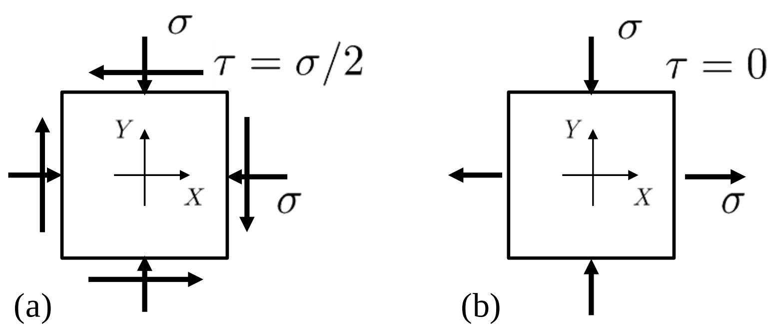
$$\Rightarrow e_2 = -\frac{4\sqrt{2}}{5}\,e_1 - \frac{7}{5}\,e_3$$

PART C – 6 points

For each state of plane stress shown below, i.e., for configurations (a) and (b), indicate whether each component of the state of strain is:

- = 0 (equal to zero)
- > 0 (greater than zero)
- < 0 (less than zero)

The material is linear elastic with Poisson's ratio ν ($0 < \nu < 0.5$), and the deformations are small.



	(a)	(b)
ϵ_x		
ϵ_y		
ϵ_z		
γ_{xy}		
γ_{xz}		
γ_{yz}		

Fill in with '= 0', '> 0', or '< 0'.

Solution:

(a) $\sigma_x = \sigma_y = -\sigma;$ $\sigma_z = 0;$ $\tau_{xy} = -\sigma/2;$ $\tau_{yz} = \tau_{xz} = 0$

(b) $\sigma_x = \sigma;$ $\sigma_y = -\sigma;$ $\sigma_z = 0;$ $\tau_{xy} = \tau_{yz} = \tau_{xz} = 0$

	(a)	(b)
ϵ_x	$\frac{1}{E}(-\sigma + \nu\sigma) = -\frac{\sigma}{E}(1-\nu) \quad (< \mathbf{0})$	$\frac{1}{E}(\sigma + \nu\sigma) = \frac{\sigma}{E}(1+\nu) \quad (> \mathbf{0})$
ϵ_y	$\frac{1}{E}(-\sigma + \nu\sigma) = -\frac{\sigma}{E}(1-\nu) \quad (< \mathbf{0})$	$\frac{1}{E}(-\sigma - \nu\sigma) = -\frac{\sigma}{E}(1+\nu) \quad (< \mathbf{0})$
ϵ_z	$\frac{1}{E}(-\nu(-\sigma - \sigma)) = \frac{2\nu\sigma}{E} \quad (> \mathbf{0})$	$\frac{1}{E}(-\nu(-\sigma + \sigma)) = \mathbf{0}$
γ_{xy}	$\frac{\tau_{xy}}{G} = -\frac{\sigma}{2G} \;\; (< \mathbf{0})$	$\mathbf{0}$
γ_{xz}	$\mathbf{0}$	$\mathbf{0}$
γ_{yz}	$\mathbf{0}$	$\mathbf{0}$