Name (Print)____

(Last) **(First)**

ME 323 - Mechanics of Materials
Exam # 1
Date: October 2, 2013 Time: 8:00 – 10:00 PM
Location: CL50 224 & PHY 114

Instructions:

Circle your lecturer's name and your class meeting time.

Krousgrill	Sadeghi	Bilal
7:30-8:30AM	10:30-11:30AM	3:30-4:30PM

Begin each problem in the space provided on the examination sheets. If additional space is required, use the yellow paper provided.

Work on one side of each sheet only, with only one problem on a sheet.

Please remember that for you to obtain maximum credit for a problem, you must present your solution clearly.

Accordingly,

- coordinate systems must be clearly identified,
- free body diagrams must be shown,
- units must be stated,
- write down clarifying remarks,
- state your assumptions, etc.

If your solution cannot be followed, it will be assumed that it is in error.

When handing in the test, make sure that ALL SHEETS are in the correct sequential order.

Remove the staple and restaple, if necessary.

Prob. 1 _______________________

Prob. 2 _______________________

Prob. 3 _______________________

Prob. 4 _______________________

Total _________________________

Useful Equations

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$G = \frac{E}{2(1+\nu)}$$

$$\tau = \frac{Tr}{I_p}$$

$$\phi = \frac{TL}{GI_p}$$

$$I_p = \frac{\pi d^4}{32}$$ Polar moment of inertia for solid circular cross section

$$I_p = \frac{\pi}{32}(d_o{}^4 - d_i{}^4)$$ Polar moment of inertia for a circular hollow cross section

$$e = \frac{FL}{EA} + L\alpha\Delta T$$

$$e = u\cos\theta + v\sin\theta$$

$$Factor\ of\ Safety = \frac{Yield\ Strength}{Allowable\ Stress}$$

ME 323 Examination # 1 　　　　 **Name** Solution
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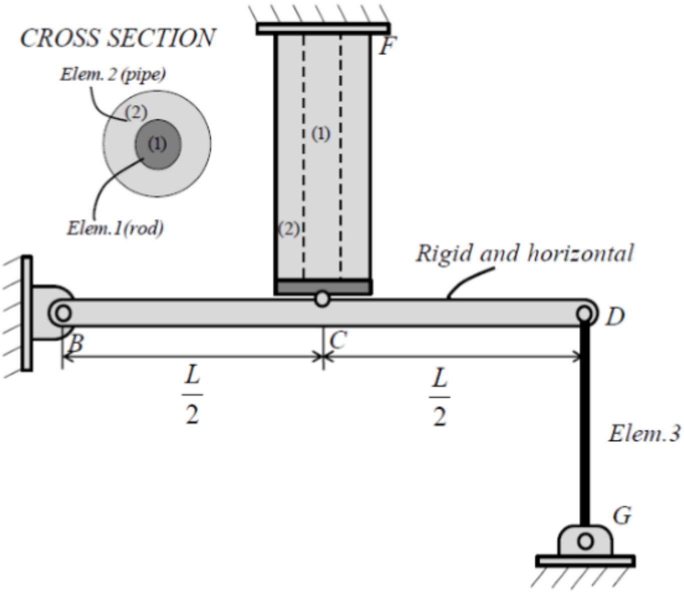
October 2, 2013 　　　　 **Instructor** ____________

PROBLEM #1 (30 points)

A cylindrical rod (element 1) is surrounded by a concentric pipe (element 2), and both are attached at the end to a rigid plate which is connected to bar *BCD* at point *C*. The bar *BCD* is rigid and initially horizontal. Bar *BCD* is also in horizontal plane and NO gravity is acting. The bar *BCD* is also attached to a rod (element 3) at point *D*. The temperature of element (3) is DECREASED by ΔT, while the temperature of element (1) and (2) is NOT changed. Data for the problem are provided in the table below.

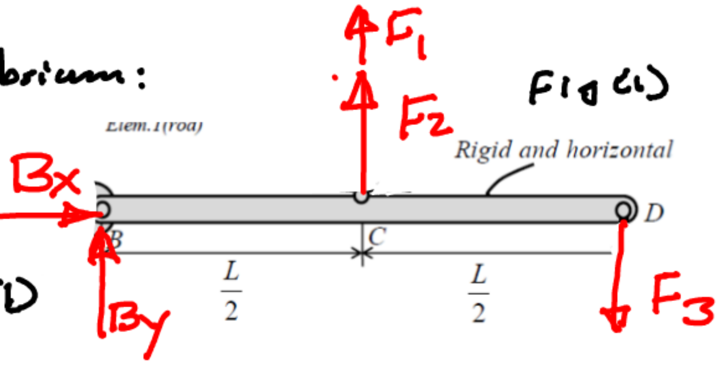


Element	*Area*	*Length*	*Modulus of Elasticity*	*Coefficient of Thermal Expansion (α)*
1	A	L	E	
2	$2A$	L	E/2	
3	A/4	L/2	$2E$	α

1. Draw appropriate free body diagrams and write down the necessary equilibrium equations for the problem.
2. Write the force-deformation equations for each element.
3. Write the compatibility equations.
4. Calculate the internal force in each element.
5. Calculate the change in length of element (3).

Solution:

Step #1: Equilibrium:



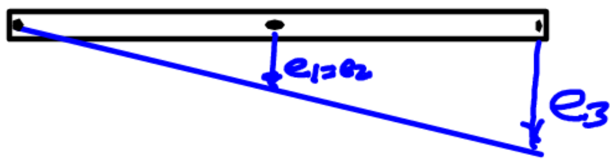
$\circlearrowleft \sum M_B = 0$

$$\Rightarrow \quad -F_3(L) + (F_1+F_2)\frac{L}{2} = 0$$

$$\Rightarrow \quad \boxed{2F_3 = F_1 + F_2} \longrightarrow (1)$$

(2) Step #2:

Force–Deformation Relationships:–



(i) $$e_1 = \frac{F_1 L_1}{A_1 E_1} = \frac{F_1 L}{AE} \longrightarrow (2a)$$

(ii) $$e_2 = \frac{F_2 L_2}{A_2 E_2} = \frac{F_2 L}{2A\,\frac{E}{2}} = \frac{F_2 L}{AE} \longrightarrow (2b)$$

(iii) $$e_3 = \frac{F_3 L_3}{A_3 E_3} + \alpha L_3(-\Delta T) = \frac{F_3 \frac{L}{2}}{\frac{A}{4} \cdot 2E} - \alpha \frac{L}{2}\Delta T$$

$$\boxed{e_3 = \frac{F_3 L}{AE} - \frac{\alpha L \Delta T}{2}} \longrightarrow (2c)$$

Step #3: Compatibility:–

Draw a kinematic diagram that determines a relationship between deformations.

From Figure 2,

$$\frac{e_1}{L/2} = -\frac{e_3}{L}$$

$$\Rightarrow \quad \boxed{\begin{aligned} 2e_1 &= -e_3 \longrightarrow (3a) \\ e_1 &= e_2 \longrightarrow (3b) \end{aligned}}$$

$$(3b) \Rightarrow \frac{F_1 L}{AE} = \frac{F_2 L}{AE}$$

$$\Rightarrow \boxed{F_1 = F_2} \longrightarrow (4).$$

$$(3a) \Rightarrow e_3 = -2e_1$$

$$= -2\frac{F_1 \cancel{L}}{AE} = \frac{F_3 \cancel{L}}{AE} - \frac{\alpha \cancel{L} \Delta T}{2}.$$

$$\boxed{F_1 = -\frac{F_3}{2} + \frac{\alpha \Delta T . AE}{4}} \longrightarrow (5)$$

$$(1) \Rightarrow \quad 2F_3 = F_1 + F_2 = 2F_1 \quad \because \quad F_1 = F_2$$

$$\boxed{F_3 = F_1}$$

$$F_3 + \frac{F_3}{2} = \frac{\alpha \Delta T AE}{4}$$

$$3\frac{F_3}{2} = \frac{\alpha \Delta T AE}{4}.$$

$$\boxed{F_3 = \frac{\alpha \Delta T AE}{6}}$$

$$\therefore \boxed{F_3 = F_1 = F_2}$$

part (b): Change in Length of element 3.

$$e_3 = \frac{F_3 L}{AE} - \alpha . \frac{L}{2} . \Delta T.$$

$$= \frac{\alpha . \Delta T . L \cancel{AE}}{6 \cancel{AE}} - \alpha \frac{L}{2} \Delta T$$

$$\boxed{e_3 = \frac{-1 \alpha L . \Delta T}{3}}$$

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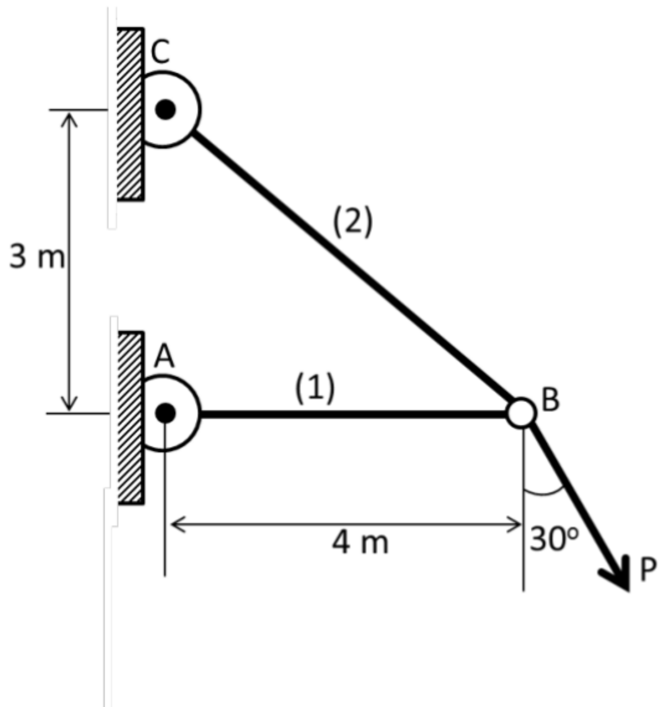
PROBLEM #2 (24 points)

Two bar planar truss has the configuration shown below. If a force P = 40 KN is applied to pin B and at the same time, element (1) is cooled by 50 °C. Determine:

1. the stresses in elements (1) and (2)
2. the horizontal and vertical displacement u_B and v_B respectively.

The following information is also known: $A_1 = A_2 = 0.001\ m^2$, $E_1 = E_2 = 200E9$ Pa, $\alpha_1 = 20E\text{-}6/^oC$



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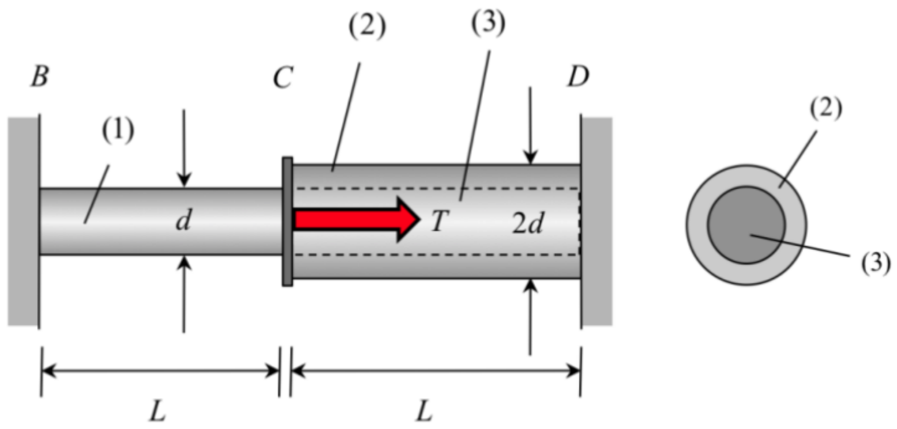
PROBLEM #3 (26 points)

The shaft shown below is made up of three uniform elements, where: element (1) is a solid shaft of diameter d; and, element (2) is a pipe (outer diameter $2d$ and inner diameter d) that surrounds solid element (3), where element (3) is collinear with element (1) and has a diameter of d. The shaft is supported by rigid walls at ends B and D. Elements (1) and (2) are made of the same material, each having a shear modulus of G, whereas element (3) is made up of a material having a shear modulus of 2G. Each of the three elements has a length of L. An axial torque T acts on the rigid connector at joint C.

1. Determine the torque carried in each of the three elements of the shaft. Express your answers in terms of T.
2. Determine the angular rotation of connector C. Express your answer in terms of T, L, d and G.



ME 323 Examination # 1 **Name** ____________________________________
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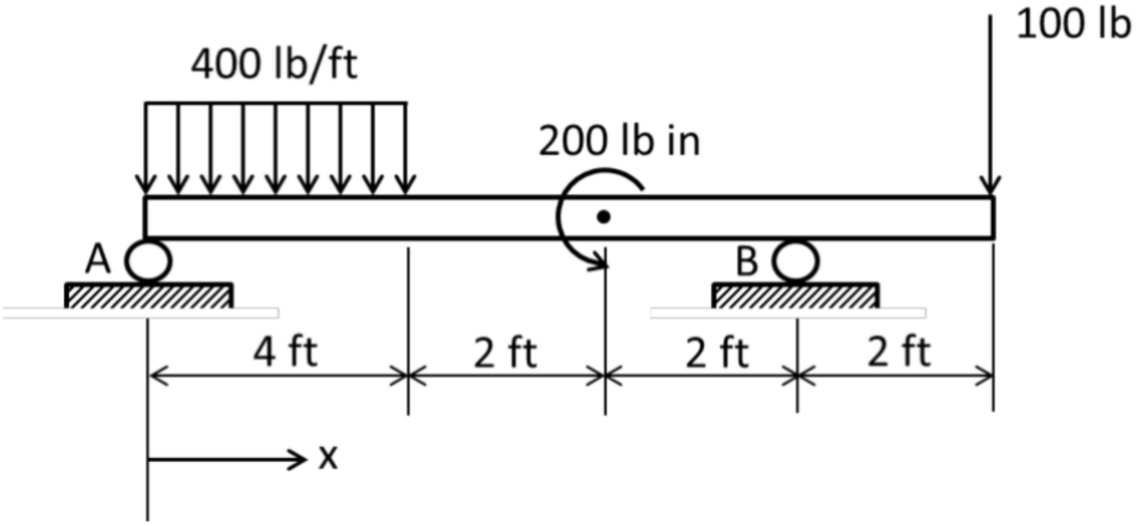
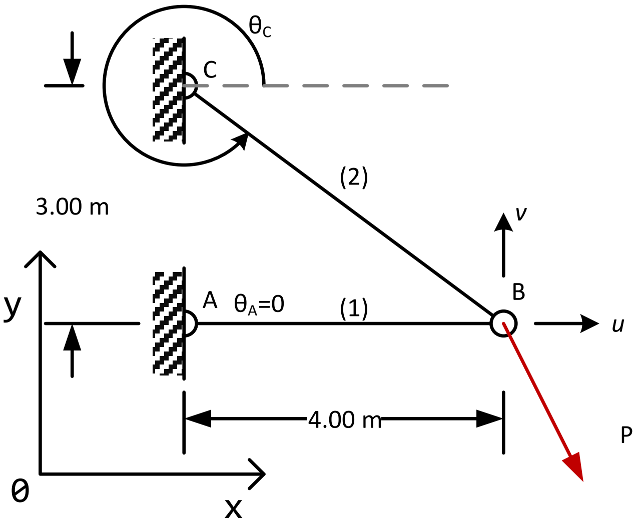
October 2, 2013 **Instructor** ________________________________

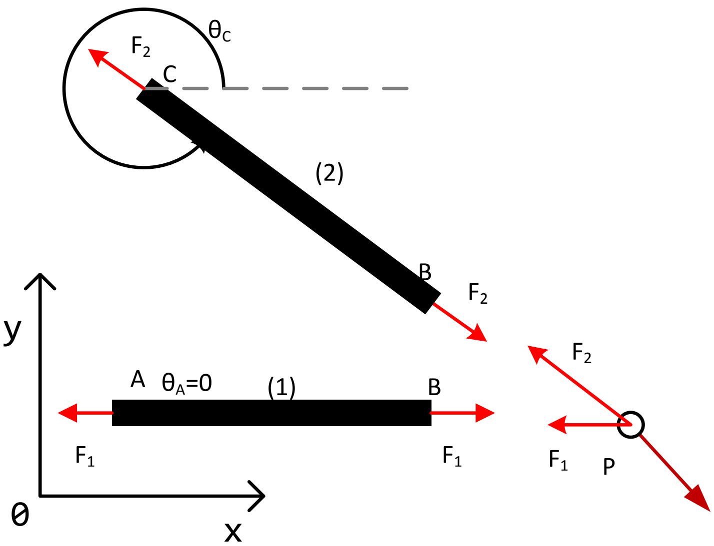
PROBLEM #4 (20 points)

The beam is subject to the loading as shown below.

1. Determine the reactions at the roller supports A and B.
2. Determine the internal shear and moment values at a point 5 ft from the left end.
3. Determine the expressions for shear (V(x)) and moment (M(x)) equations for a 6 < x <8 ft.







Deformation equation

$$e_1 = u\cos\theta_A + v\sin\theta_A = u\cos(0) + v\sin(0) = u \quad (0.1)$$

$$e_2 = u\cos\theta_B + v\sin\theta_B = \frac{4}{5}u - \frac{3}{5}v \quad (0.2)$$

Using equilibrium conditions at B

$$F_2\frac{3}{5} - P\cos(30^\circ) = 0$$
$$\Rightarrow F_2 = P\frac{\sqrt{3}}{2}\times\frac{5}{3} = P\frac{5\sqrt{3}}{6} \quad (0.3)$$

$$F_2 = 40KN\frac{5\sqrt{3}}{6} = 57.735KN \tag{0.4}$$

$$\sigma_2 = \frac{F_2}{A} = \frac{57.735KN}{0.001m^2} = 57.735MPa \tag{0.5}$$

$$P\sin(30^\circ) - F_1 - F_2\frac{4}{5} = 0$$

$$\Rightarrow F_1 = P\sin(30^\circ) - F_2\frac{4}{5} = \frac{P}{2} - P\frac{5\sqrt{3}}{6}\times\frac{4}{5} \text{ From 1.3}$$

$$\Rightarrow F_1 = P\left(\frac{1}{2} - \frac{2\sqrt{3}}{3}\right) \tag{0.6}$$

$$F_1 = 40KN(-0.6547) = -26.188KN$$

$$\sigma_1 = \frac{F_1}{A} = \frac{-26.188KN}{0.001m^2} = -26.188MPa \tag{0.7}$$

From force-elongation equation

$$e_1 = \frac{F_1L_1}{EA} - \alpha\Delta TL_1 \tag{0.8}$$

$$\Rightarrow u = \frac{F_1L_1}{EA} - \alpha\Delta TL_1 = P\left(\frac{1}{2} - \frac{2\sqrt{3}}{3}\right)\frac{L_1}{EA} - \alpha\Delta TL_1 \tag{0.9}$$

$$u = 40,000N\left(\frac{1}{2} - \frac{2\sqrt{3}}{3}\right)\frac{4m}{200\times10^9 Pa\times0.001m^2} - 20\times10^{-6}\times50\times4m$$

$$\Rightarrow u = 5.2376\times10^{-4}m - 4\times10^{-3}m = -4.52376\times10^{-3}m$$

$$\Rightarrow u = -4.52376mm$$

$$e_2 = \frac{F_2L_2}{EA} \tag{0.10}$$

$$\Rightarrow \frac{4}{5}u - \frac{3}{5}v = P\frac{5\sqrt{3}}{6}\frac{L_2}{EA}$$

$$\Rightarrow 0.8\times(-4.52376\times10^{-3}m) - 0.6v = 57,735N\frac{5m}{200\times10^9 Pa\times0.001m^2} = 1.443375\times10^{-3}m$$

$$\Rightarrow 0.6v = -1.443375\times10^{-3}m - 3.619008\times10^{-3}m = -5.062383\times10^{-3}m$$

$$\Rightarrow v = -8.4373\times10^{-3}m = -8.4373mm$$

Alternately,

$$\frac{3}{5}v = \frac{4}{5}u - P\frac{5\sqrt{3}}{6}\frac{L_2}{EA} = 0$$

$$\Rightarrow \frac{3}{5}v = \frac{4}{5}\left(P\left(\frac{1}{2} - \frac{2\sqrt{3}}{3}\right)\frac{L_1}{EA} - \alpha\Delta TL_1\right) - P\frac{5\sqrt{3}}{6}\frac{L_2}{EA}$$

$$\Rightarrow v = \frac{4}{3}\left(P\left(\frac{1}{2} - \frac{2\sqrt{3}}{3}\right)\frac{L_1}{EA} - \alpha\Delta TL_1\right) - P\frac{5\sqrt{3}}{6}\frac{L_2}{EA} \times \frac{5}{3}$$

$$\Rightarrow v = \frac{4}{3}\left(40,000N(-0.6547)\frac{4m}{200\times10^9 Pa\times0.001m^2} - 20\times10^{-6}\times50\times4m\right) - \frac{5\times40,000N\times1.4433757}{200\times10^9 Pa\times0.001m^2}\times\frac{5}{3}$$

$$= \frac{4}{3}\left(-5.2376\times10^{-4}m - 4\times10^{-3}m\right) - 2.405626\times10^{-3}m$$

$$= -\frac{4}{3}\times -4.52376\times10^{-3}m - 2.405626\times10^{-3}m$$

$$= -6.03168\times10^{-3}m - 2.405626\times10^{-3}m = -8.437306\times10^{-3}m$$

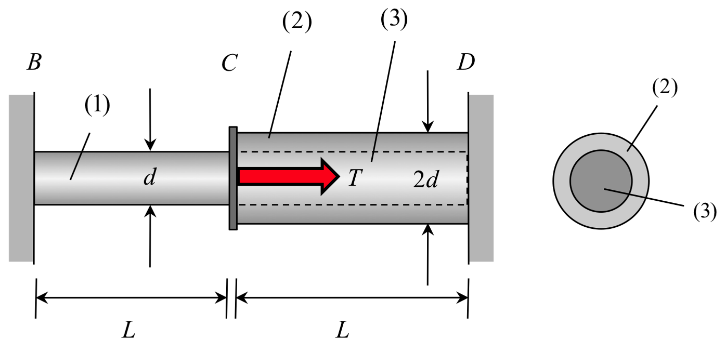
$$\Rightarrow v = -8.437306mm$$

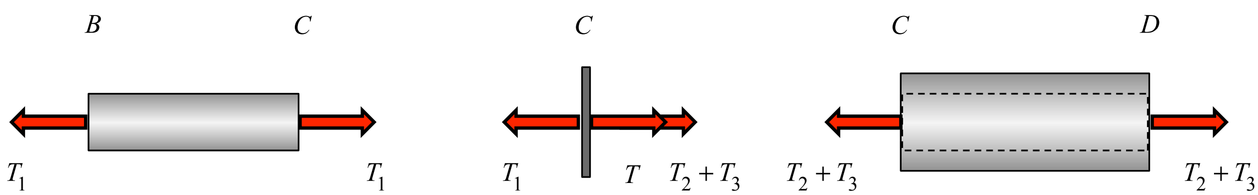
ME 323 Examination # 1 **SOLUTION**
October 2, 2013

PROBLEM #3 (28 points)

The shaft shown below is made up of three uniform elements, where: element (1) is a solid shaft of diameter d; and, element (2) is a pipe (outer diameter $2d$ and inner diameter d) that surrounds solid element (3), where element (3) is collinear with element (1) and has a diameter of d. The shaft is supported by rigid walls at ends B and D. Elements (1) and (2) are made of the same material, each having a shear modulus of G, whereas element (3) is made up of a material having a shear modulus of 2G. Each of the three elements has a length of L. An axial torque T acts on the rigid connector at joint C.

a) Determine the torque carried in each of the three elements of the shaft. Express your answers in terms of T.
b) Determine the angular rotation of connector C. Express your answer in terms of T, L, d and G.





Equilibrium

From FBD of C: $\sum M = T + T_2 + T_3 - T_1 = 0$ (1)

Torque/angle of twist equations:

$$\phi_1 = \frac{T_1 L_1}{G_1 J_1} = \frac{T_1 L}{G\pi\left(d/2\right)^4/2} = 32\frac{T_1 L}{G\pi d^4} \quad (3)$$

$$\phi_2 = \frac{T_2 L_2}{G_2 J_2} = \frac{T_2 L}{G\pi\left[\left(2d/2\right)^4 - \left(d/2\right)^4\right]/2} = \frac{32}{15}\frac{T_2 L}{G\pi d^4} \quad (4)$$

$$\phi_3 = \frac{T_3 L_3}{G_3 J_3} = \frac{T_3 L}{2G\pi\left(d/2\right)^4/2} = 16\frac{T_3 L}{G\pi d^4} \quad (5)$$

Compatibility

$$\phi_C = \phi_1 = 32\frac{T_1 L}{G\pi d^4} \tag{6}$$

$$\phi_D = \phi_C + \phi_2 = 32\frac{T_1 L}{G\pi d^4} + \frac{32}{15}\frac{T_2 L}{G\pi d^4} = \frac{32L}{G\pi d^4}\left[T_1 + \frac{1}{15}T_2\right] = 0 \quad \Rightarrow \quad T_1 = -\frac{1}{15}T_2 \tag{7}$$

$$\phi_2 = \phi_3 \quad \Rightarrow \quad \frac{32}{15}\frac{T_2 L}{G\pi d^4} = 16\frac{T_3 L}{G\pi d^4} \quad \Rightarrow \quad T_3 = \frac{2}{15}T_2 \tag{8}$$

Solve

Substitution of (7) and (8) into (1) gives:

$$T + T_2 + \frac{2}{15}T_2 + \frac{1}{15}T_2 = 0 \quad \Rightarrow \quad T_2 = -\frac{15}{18}T \tag{9}$$

Therefore, using equation (9):

$$T_1 = -\frac{1}{15}T_2 = \frac{1}{18}T$$

$$T_3 = \frac{2}{15}T_2 = -\frac{2}{18}T$$

From equation (6):

$$\phi_C = 32\frac{T_1 L}{G\pi d^4} = \frac{32}{18}\frac{TL}{G\pi d^4}$$

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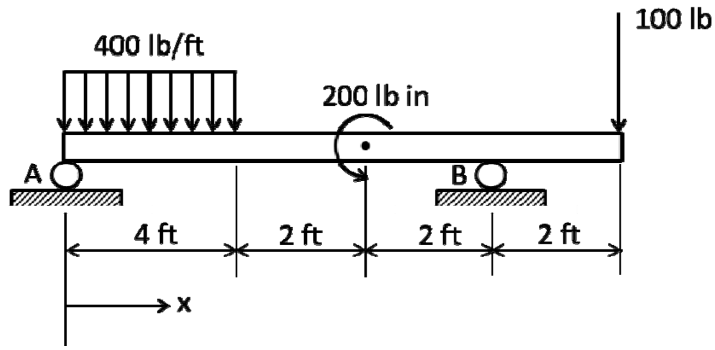
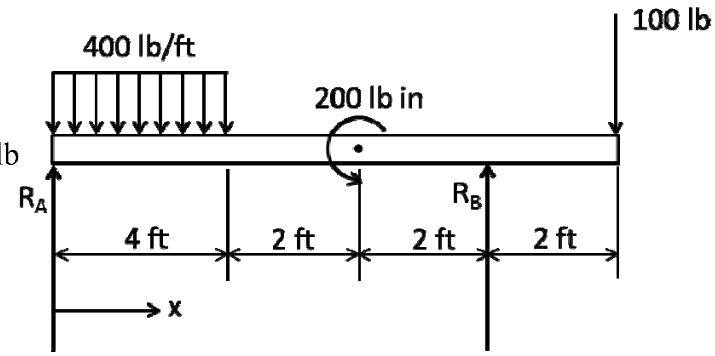
PROBLEM #4 (20 points)

The beam is subject to the loading as shown below.

1. Determine the reactions at the roller supports A and B.
2. Determine the internal shear and moment values at a point 5 ft from the left end.
3. Determine the expressions for shear (V(x)) and moment (M(x)) equations for a $6 < x \leq 8$ ft.

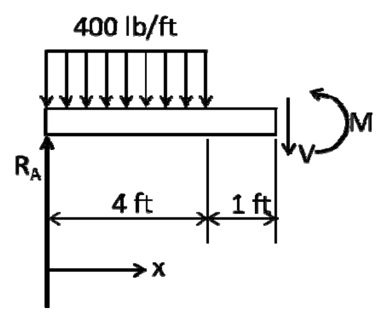




$\sum F_y = R_A + R_B = 1600 + 100 = 1700$ lb

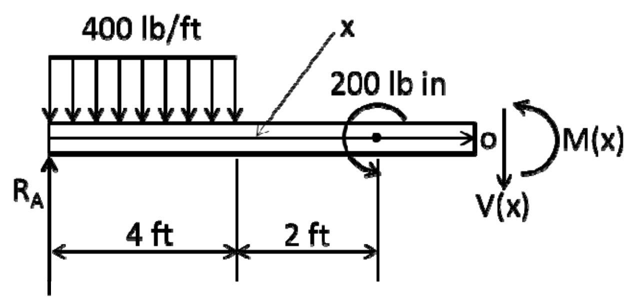
$\sum M_{R_A} = 100*10 - R_B*8 - 200/12 + 1600*2 = 0 \rightarrow R_B = 522.92$ lb

$therefore, \rightarrow R_A = 1177.08$ lb



$\sum F_y = 1177.08 - V - 400*4 = 0 \rightarrow V = -422.92$ lb

$\sum M_{5\,ft\ \text{from the left}} = 1177.08*5 - 1600*3 - M = 0 \rightarrow M = 1085.4$ lb ft



$\sum F_y = 0 = 1177.08 - V(x) - 400*4 = 0 \rightarrow V(x) = -422.92$ lb

$\sum M_o = 0 = M(x) - 1177.08x + 1600(x-2) + 200/12 = 0 \rightarrow M(x) = -422.92x + 3183.33$ lb ft