Name: ____Solution________________ Instructor: Krousgrill Siegmund Bilal
(Print) (Last) (First) (Circle your instructor)

ME 323 EXAM #1
FALL SEMESTER 2012
8:00 PM – 9:30 PM
Sep. 25, 2010

Instructions

1. Begin each problem in the space provided on the examination sheets. If additional space is required, use the paper provided. Work on one side of each sheet only, with only one problem on a sheet.

2. Each problem is of value as indicated below.

3. To obtain maximum credit for a problem, you must present your solution clearly. Accordingly:
   a. Identify coordinate systems
   b. Sketch free body diagrams
   c. State units explicitly
   d. Clarify your approach to the problem including assumptions

4. **If your solution cannot be followed, it will be assumed that it is in error**.

Prob. 1	(35)	______
Prob. 2	(33)	______
Prob. 3	(32)	______
Total	**(100)**	

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$\sigma = F_n / A \quad \tau_{avg} = V / A \quad F.S. = F_{fail} / F_{allow} \quad F.S. = \sigma_{fail} / \sigma_{allow} \quad F.S. = \tau_{fail} / \tau_{allow}$

$\varepsilon = (L - L_0) / L_0 = \Delta L / L_0 \quad \gamma = (\pi / 2) - \theta' \quad \sigma = E\varepsilon \quad \nu = -\varepsilon_{lat} / \varepsilon_{long} \quad \tau = G\gamma$

$$\Delta L_{th} = \alpha(\Delta T)L \quad \Delta L_{th} = \int_0^L \alpha(\Delta T)\,dx$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T \,,\ \varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T \,,\ \gamma_{xy} = \frac{1}{G}\tau_{xy} \quad ,\gamma_{xz} = \frac{1}{G}\tau_{xz} \quad ,\gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\Delta L = \frac{F_i L_0}{EA} \quad \Delta L = \int_0^L \frac{F_i(x)}{E(x)A(x)}dx \quad u_B = u_A + \Delta L$$

$$\phi\rho = \gamma L \quad \tau = G\rho\frac{\phi}{L} \quad \tau = \frac{T_i\rho}{J} \quad \phi = \frac{T_i L}{GJ}$$

$$\phi = \int_0^L \frac{T_i(x)}{G(x)J(x)}dx \quad \phi_B = \phi_A + \phi_{AB}$$

$$\tau = G\rho\frac{d\phi}{dx}, \gamma = \rho\frac{d\phi}{dx}$$

$$J = I_p = \frac{\pi}{2}r^4 \ldots bar$$

$$J = I_p = \frac{\pi}{2}\left(r_o^4 - r_i^4\right)\ldots tube$$

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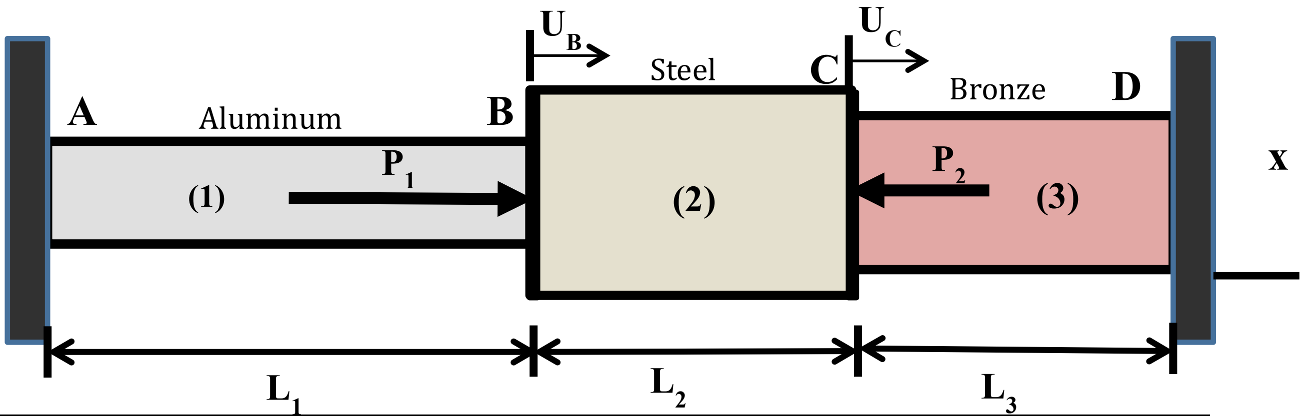
PROBLEM #1 (35 points)

A three-segment rod made of aluminum (element 1), steel (element 2) and bronze (element 3) is initially stress free after it is attached to rigid supports at ends A and D. It is subsequently subjected to load P_1 **=150 kN** and P_2 **= 90 kN** applied at node (joints) B and C, respectively. In addition, the temperature of element (3) is DECREASED by ΔT**=50°C**, while the temperature of element (1) and (2) is NOT changed. Data for the problem are provided in the table below.



Element	Cross-section area A_i (mm^2)	Length L_i (mm)	Modulus E_i (GPa)	Coefficient of Thermal Expansion α_i (°C)$^{-1}$
(1)	900	500	70	
(2)	2000	250	200	
(3)	1200	350	80	10×10^{-6}

(a) Draw appropriate free body diagrams and write down the necessary equilibrium equations for the problem.

(b) Write the force-deformation equations for each element.

(c) Write the compatibility equations.

(d) Calculate the internal forces in each element.

(e) Calculate the stresses in each element.

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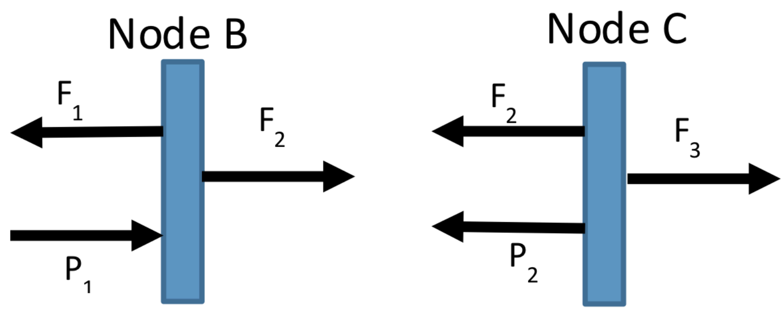
Part (1a): Write the equilibrium equations for node *B* and node *C*.



$$\sum Fx = -F_1 + P_1 + F_2 = 0$$
$$F_1 = P_1 + F_2 \qquad (1a)$$

$$\sum Fx = -P_2 - F_2 + F_3 = 0$$
$$F_3 = P_2 + F_2 \qquad (1b)$$

Part (1b): Write the force-deformation equations for each element

$$e_1 = \frac{F_1 L_1}{A_1 E_1} \qquad (2)$$

$$e_2 = \frac{F_2 L_2}{A_2 E_2} \qquad (3)$$

$$e_3 = \frac{F_3 L_3}{A_3 E_3} + \alpha_3 L_3 \Delta T \qquad (4)$$

Part (1c): Write the compatibility equations

$$e_1 = U_B - \cancelto{0}{U_A} \Rightarrow e_1 = U_B \qquad (5)$$

$$e_2 = U_C - U_B \qquad (6)$$

$$e_3 = \cancelto{0}{U_D} - U_C \Rightarrow e_3 = -U_C \qquad (7)$$

or

$$e_1 = -e_2 - e_3$$

or

$$e_1 + e_2 + e_3 = 0 \qquad (8)$$

or $e_{AD} = 0$

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Part (1d): Calculate the internal forces in each element.

$$\frac{F_1L_1}{A_1E_1}+\frac{F_2L_2}{A_2E_2}+\left(\frac{F_3L_3}{A_3E_3}+\alpha_3L_3\Delta T\right)=0$$

sub for F_1 *and* F_3 *from* (1)

$$\frac{(P_1+F_2)L_1}{A_1E_1}+\frac{F_2L_2}{A_2E_2}+\frac{(F_2+P_2)L_3}{A_3E_3}+\alpha_3L_3\Delta T=0$$

$$F_2=\frac{-\left(\frac{P_1L_1}{A_1E_1}+\frac{P_2L_3}{A_3E_3}+\alpha_3L_3\Delta T\right)}{\frac{L_1}{A_1E_1}+\frac{L_2}{A_2E_2}+\frac{L_3}{A_3E_3}}$$

$$F_2=\frac{-\frac{1}{10^{-6}\times10^{9}}\left(\frac{150\times10^{3}(.5)}{900\times70}+\frac{90\times10^{3}(.35)}{1200\times83}\right)-10\times10^{-6}(.35)(-50)}{\frac{1}{10^{-6}\times10^{9}}\left(\frac{.5}{900\times70}+\frac{.25}{2000\times200}+\frac{.35}{1200\times83}\right)}$$

$$F_2=-110.28\,kN$$

$$F_1=39.71\,kN$$

$$F_3=-20.28\,kN$$

Part (1e): Calculate the stresses in each element

$$\sigma_1=\frac{F_1}{A_1}=44.13\,MPa$$

$$\sigma_2=\frac{F_2}{A_2}=-55.14\,MPa$$

$$\sigma_3=\frac{F_3}{A_3}=-16.9\,MPa$$

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PROBLEM #2 (33 points)

The shaft shown below is made up of segments (1) and (2) with a homogeneous distribution of material having a shear modulus of G. Segment (1) (between B and C) is tubular with an outer diameter d and inner diameter $d/3$. Segment (2) (between C and D) has a solid circular cross section of diameter d. Torques of $2T_0$ and T_0 act a C and D, respectively, as shown in the figure. Determine the maximum shear stress in segment (1) and in segment (2).

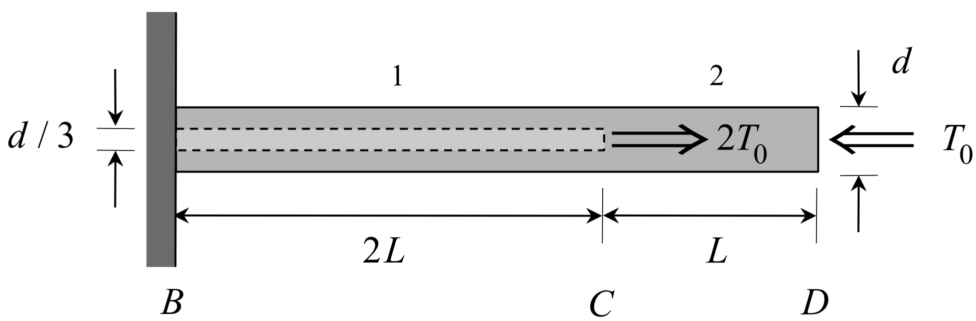
(a) Draw appropriate free body diagrams and write down the necessary equilibrium equations for the problem.

(b) Determine the maximum shear stress in segment (1) and in segment (2).

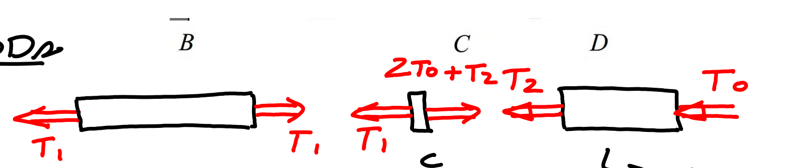
(c) Determine the minimum shear stress in segment (1).

(d) Determine the angles of rotation of the shaft at locations C and D.

Leave your answers in symbolic form in terms of only the parameters defined in the problem and figure.



FBDs



↳ $T_2 = -T_0$

Equilibrium

$C: \sum M_x = -T_1 + 2T_0 + T_2 = 0 \Rightarrow T_1 = 2T_0 + T_2 = T_0$

Polar area moments

$$J_1 = \frac{\pi}{32}\left[d^4 - \left(\frac{d}{3}\right)^4\right] = \frac{5\pi}{162} d^4$$

$$J_2 = \frac{\pi d^4}{32}$$

Shear stress

$$\tau = \frac{T\rho}{J} \Rightarrow \tau_1 = \frac{T_1\rho}{J_1} \ \& \ \tau_2 = \frac{T_2\rho}{J_2}$$

$$\therefore \quad \tau_1 = \frac{162}{5\pi d^4}\rho T_0$$

$$\begin{cases} \tau_{1,max} = \tau_1\left(\rho = \frac{d}{2}\right) = \frac{81}{5\pi d^3}T_0 \\ \tau_{1,min} = \tau_1\left(\rho = \frac{d}{6}\right) = \frac{27}{5\pi d^3}T_0 \end{cases}$$

$$\tau_2 = \frac{T_2 \rho}{J_2} \quad ; \quad J_2 = \frac{\pi}{32}d^4$$

$$= -\frac{32}{\pi d^4}\rho T_0$$

$$\tau_{2,max} = \tau_2\left(\rho = \frac{d}{2}\right) = -\frac{16}{\pi d^3}T_0$$

(c) $$\Delta\phi_c = \Delta\phi_1 = \frac{T_1(2L)}{GJ_1} = \frac{T_0(2L)}{G\,5\pi(d^4/162)}$$

$$= \frac{324 T_0 L}{5G\pi d^4}$$

$$\Delta\phi_D = \Delta\phi_1 + \Delta\phi_2 = \Delta\phi_1 + \frac{T_2 L}{GJ_2}$$

$$= \frac{324 T_0 L}{5G\pi d^4} - \frac{T_0 L}{G(\pi d^4/32)}$$

$$= \left[\frac{324}{5} - 32\right]\frac{T_0 L}{G\pi d^4} = \frac{164}{5}\frac{T_0 L}{G\pi d^4}$$

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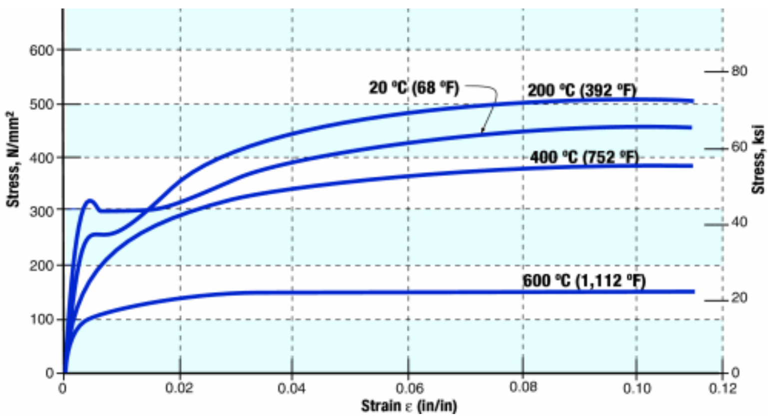
PROBLEM #3 (32 points total)

3.1 (8 points) The diagram shows stress-strain data for a steel at several temperatures. Determine at what temperature the still possesses the highest ultimate stress and at what temperature the highest modulus.

The temperature for the highest ultimate stress is: 200 ºC ______.
The temperature for highest modulus is: 20 ºC ____.

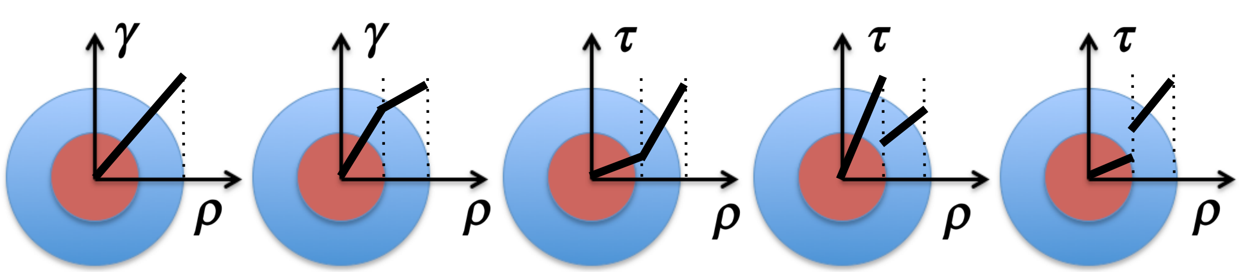


3.2 (8 points) Consider a bar with circular cross section.

- Under axial loading with a force $\boldsymbol{F}$, doubling of the bar diameter reduces the stress in the bar by a factor of: **4.**
- Under torsion loading with a torque $\boldsymbol{T}$, doubling of the bar diameter reduces the maximum stress in the bar by a factor of**: 8**.

3.3 (8 points) A sheet of material position in the $\boldsymbol{x}$-$\boldsymbol{y}$-plane is pulled such the in-plane strains ε_x and ε_y are both 10%. If the material possesses Poisson's ratio $\nu = 0.5$, the thickness of the sheet changes by how many %? Your answer: __10___%

3.4 (8 points) Consider the case of torsion loading of a cylindrical bar consisting of a core of steel and a sleeve of copper. The shear modulus of steel is higher than that of copper. Consider that this composite bar is twisted. Then the following drawings of shear strain and shear stress are either correct or wrong. Mark your answer.



Correct **Incorrect** Incorrect Correct Incorrect