Name: Solution (Print) (Last) (First)

Instructor: Siegmund Sarkar Susilo (Circle one)

Sarkar

ME 323 EXAM #1
FALL SEMESTER 2010
8:00 PM – 9:30 PM
Sep. 29, 2010

Instructions

1. Begin each problem in the space provided on the examination sheets. If additional space is required, use the paper provided. Work on one side of each sheet only, with only one problem on a sheet.

2. Each problem is of value as indicated below.

3. To obtain maximum credit for a problem, you must present your solution clearly. Accordingly:
   a. Identify coordinate systems
   b. Sketch free body diagrams
   c. State units explicitly
   d. Clarify your approach to the problem including assumptions

4. **If your solution cannot be followed, it will be assumed that it is in error.**

Prob. 1 _(33)	
Prob. 2 _(34)	
Prob. 3 _(33)	
Total (100)	

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$$\sigma = F_n / A \quad \tau_{avg} = V / A \quad F.S. = F_{fail} / F_{allow} \quad F.S. = \sigma_{fail} / \sigma_{allow} \quad F.S. = \tau_{fail} / \tau_{allow}$$

$$\varepsilon_{avg} = (\Delta s' - \Delta s) / \Delta s = \delta / L_0 \quad \gamma = (\pi / 2) - \theta' \quad \sigma = E\varepsilon \quad \nu = -\varepsilon_{lat} / \varepsilon_{long} \quad \tau = G\gamma$$

$$\delta_{th} = \alpha(\Delta T)L \quad \delta_{th} = \int_0^L \alpha(\Delta T)dx$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\overset{\delta}{\Delta L} = \frac{F_i L_0}{EA} \quad \overset{\delta}{\Delta L} = \int_0^L \frac{F_i(x)}{E(x)A(x)}dx \quad u_B = u_A + \overset{\delta_{AB}}{\cancel{\Delta L_{AB}}}$$

$$\phi\rho = \gamma L \quad \tau = Gc\frac{\phi}{L} \quad \tau = \frac{T_i\rho}{J} \quad \phi = \frac{T_i L}{GJ} \quad \phi = \int_0^L \frac{T_i(x)}{G(x)J(x)}dx \quad \phi_B = \phi_A + \phi_{AB}$$

$$J = \frac{\pi c^4}{2} \ldots bar \quad J = \frac{\pi(c_o^4 - c_i^4)}{2} \quad \cdots tube$$

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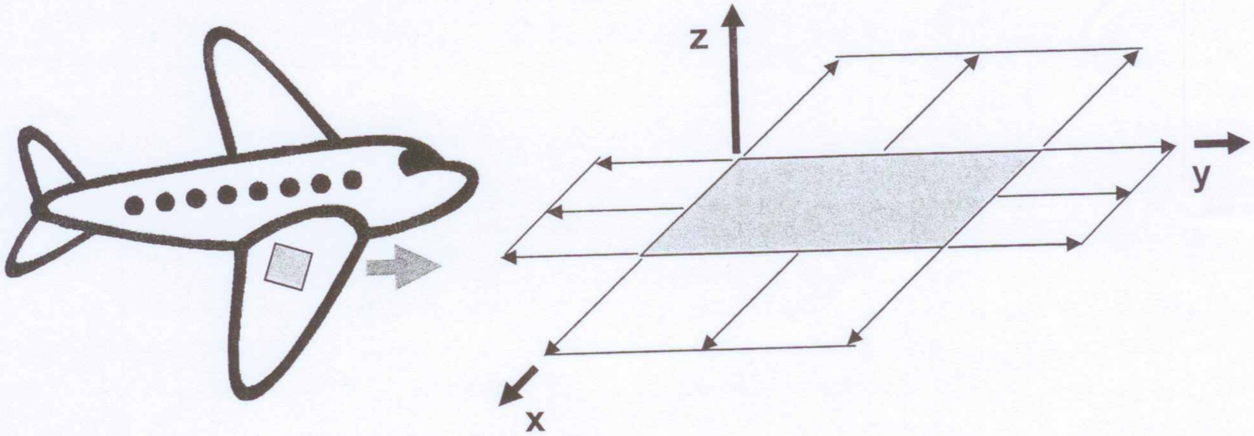
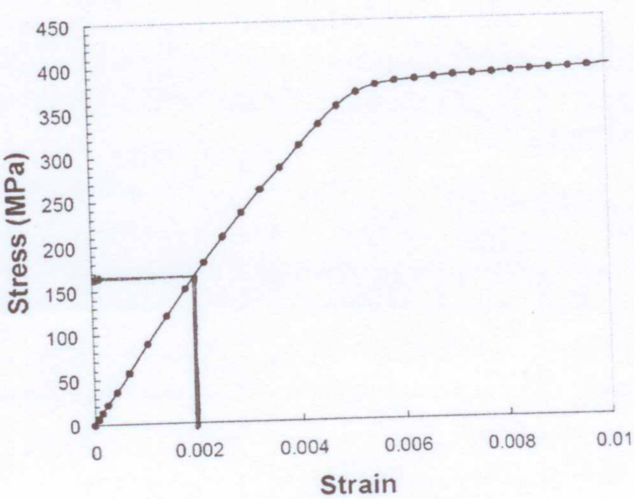
PROBLEM #1 (33 points)

When thin sheets of material, like the outer skin of an airplane are subjected to stress, they are said to be in a state of plane stress ($\sigma_z = \tau_{xz} = \tau_{yz} = 0$). On an airplane (as shown below) it was determined that a biaxial state of stress exists such that $\tau_{xy} = 0$, i.e. no shear stresses occur. The strains in the x and y direction were measured as $\varepsilon_x = 200\times10^{-6}$ and $\varepsilon_y = 140\times10^{-6}$..

Determine the stresses σ_x and σ_y.

For the analysis consider that the aircraft is made of an aluminum alloy. The relevant stress strain curve is given below. In addition, it is known that Poisson's ratio is $\nu = 0.3$.





$E = \frac{\sigma}{\varepsilon} = \frac{170}{0.002} =$

85,000 MPa

$\sigma_z = 0$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\sigma_y\right] \Rightarrow \quad \varepsilon_x E = \sigma_x - \nu\sigma_y$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\cdot\sigma_x\right] \Rightarrow \quad \varepsilon_y E = \sigma_y - \nu\cdot\sigma_x$$

$$\begin{array}{r}
\varepsilon_x E = \sigma_x - \nu\sigma_y \\
\nu\,\varepsilon_y E = \nu\sigma_y - \nu^2\sigma_x \\
\hline
\varepsilon_x E + \nu\varepsilon_y E = \sigma_x - \nu^2\sigma_x
\end{array}$$

$$\sigma_x = \frac{E}{(1-\nu^2)}\left[\varepsilon_x + \nu\varepsilon_y\right] = \boxed{22 \text{ MPa}}$$

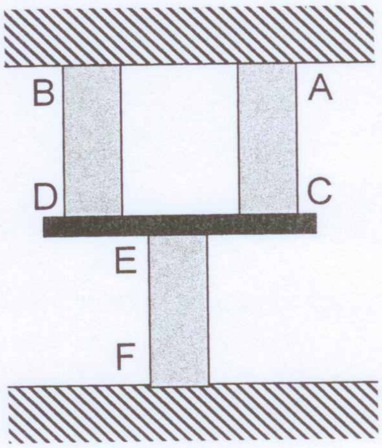
or $$\sigma_y = \frac{E}{(1-\nu^2)}\left[\varepsilon_y + \nu\varepsilon_x\right] = \boxed{18 \text{ MPa}}$$

PROBLEM #2 (34 points)

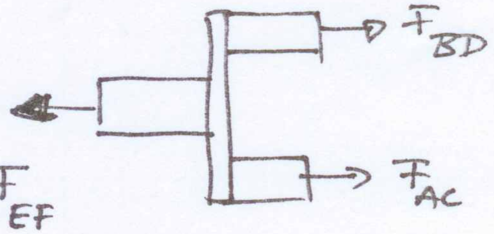
An assembly consists of three identical bars ($L = 1.0\ m$, $A = 5.0\ mm^2$). The bars are made of steel ($E = 200\ GPa, \nu = 0.3, \alpha = 16 \times 10^{-6}\ /^\circ C$). The assembly is fixed at walls on its two ends. The bar EF is heated from room temperature ($T = 20^\circ C$) to an elevated temperature ($T = 200^\circ C$).

(a) **Calculate the stresses in the bars after heating to** $T = 200^\circ C$.

(b) **If the yield strength of steel is** $\sigma_Y = 750\ MPa$ **(independent of temperature), determine to which temperature the structure can be heated such that the bars do not yield.**



FBD

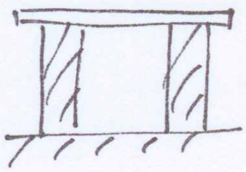


$$-F_{EF} + F_{BD} + F_{AC} = 0$$

$$-F_{EF} + 2F_{AC} = 0$$

$$F_{EF} = 2F_{AC}$$

First determin relationships between F_{BD} and F_{AC}:



Since $\cancel{F_{BD}}$ & DC remains parallel

$$\Delta L_{BD} = \Delta L_{AC} \Rightarrow F_{BD} = F_{AC}$$

$$\Delta L_{EF} = \frac{F_{EF} L}{EA} + \alpha \Delta T L$$

$$\Delta L_{AC} = \frac{F_{AC} L}{EA}$$

$\cancel{\Delta l_n}$ $\quad u_c = u_A + \Delta L_{AC}$

$$u_F = u_E + \Delta L_{EF}$$

BC

$u_A = 0 \quad , u_F = 0$

$u_c = u_E$

$$\Delta L_{AC} + \Delta L_{EF} = 0$$

$$\frac{F_{EF} L}{EA} + \alpha \Delta T L + \frac{F_{AC} L}{EA} = 0$$

$$\frac{2\overset{\sigma_{AC}}{\frac{F_{AC}}{A}} L}{E} + \alpha \Delta T L + \frac{\overset{\sigma_{AC}}{\frac{F_{AC}}{A}} L}{E} = 0$$

$$\boxed{\sigma_{AC} = -\frac{1}{3} E \alpha \Delta T} = -192\ \text{MPa}$$

$$F_{EF} = 2F_{AC} = 2\left[-\frac{1}{3}E\alpha\Delta T A\right] = -\frac{2}{3}E\alpha\Delta T A$$

$$\boxed{\sigma_{EF} = -\frac{2}{3}E\alpha\Delta T} \; \boxed{= -384\ MPa}$$

$$\sigma_{EF} > \sigma_{AC}$$

$$-\sigma_y = -\frac{2}{3}E\alpha\Delta T^* = -\frac{2}{3}E\alpha(T^* - 20^\circ C)$$

$$\boxed{\frac{\frac{3}{2}\sigma_y}{E\alpha} + 20^\circ C = T^*} \; \boxed{= 371.5\ ^\circ C}$$

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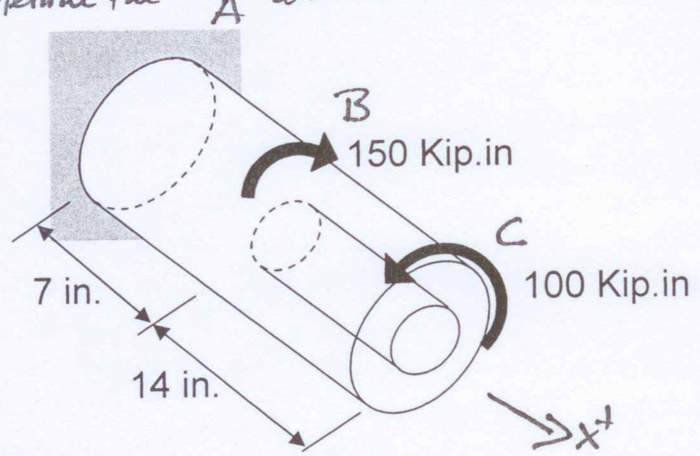
PROBLEM #3 (33 points)

The bar shown below as a solid circular cross-section over a segment of length $L_1 = 7.0\,in.$ and a hollow circular cross section over a segment of length $L_2 = 14.0\,in.$. The shear modulus of the material of the bar is $G = 3.8 \times 10^6\ psi$. The outer radius is $c_o = \overset{2}{\cancel{4}}.0\,in.$ and the inner radius is $c_i = \overset{1}{\cancel{2}}.0\,in.$ The bar is loaded by two torques. A torque of magnitude 150 Kip.in. acts at the location where the solid section transitions to the hollow section, and a torque of magnitude 100 kip.in acts at the free end. The bar is attached to a rigid wall.

(a) Determine the rotation of the free end of the bar. Provide magnitude and direction of rotation.

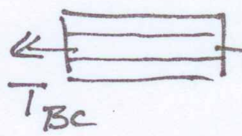
(b) Determine the stress state in AB and BC.



$$J_{Solid} = \frac{\pi}{2}(2\,in)^4 = 25.13\ in^4$$

$$J_{Hollow} = \frac{\pi}{2}\left[(2\,in)^4 - (1\,in)^4\right] = 23.56\ in^4$$

FBD

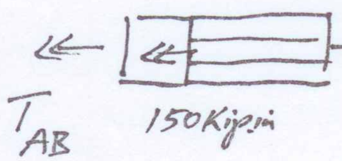


$$\Sigma M_x = 0 \quad -T_{BC} + 100\ Kip.in = 0$$

$$\boxed{T_{BC} = 100\ Kip.in}$$

$$\Sigma M_x = 0 \quad -T_{AB} - 150\,Kip.in + 100\ Kip.in = 0$$

$$\boxed{T_{AB} = -50\ Kip.in}$$



$$\phi_{AB} = \frac{T_{AB}\,L_{AB}}{G\,J_S} = \frac{-50\times10^3 \cdot 7}{25.13 \cdot 3.8\times10^6}\left[\frac{\cancel{lb}.in\ \ in}{\cancel{psi}\ lb/in^2 \cdot in^4}\right] = -0.00367\ rad$$

$$\phi_{BC} = \frac{T_{BC}\,L_{BC}}{G\,J_H} = \frac{100\times10^3 \cdot 14}{23.56 \cdot 3.8\times10^6}\left[\frac{lb\ in\ \ in}{in^4\ \ lb/in^2}\right] = 0.0159\ rad$$

$$\phi_B = \phi_A + \phi_{AB}$$

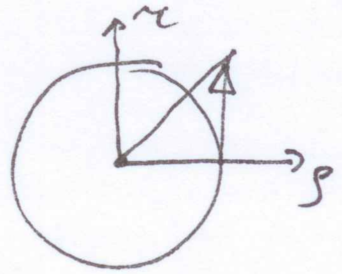
$$\phi_C = \phi_B + \phi_{BC}$$

BC: $\phi_A = 0 \Rightarrow \phi_C = \phi_{AB} + \phi_{BC} =$

$$\phi_C = -0.00367 + 0.0159 = \underline{\underline{0.0122 \text{ rad}}}$$

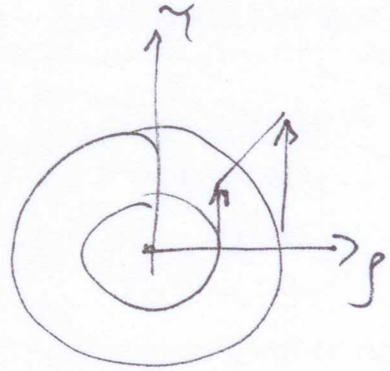
Rotation along the curl of right hand with thumb in x^+

Shear State in AB



$$\tau_{max}(AB) = \frac{T_{AB}\, c_o}{J_s} = 3{,}980 \text{ lb/in}^2$$

Shear State in BC



$$\tau(c_i) = \frac{T_{BC}\, c_i}{J_H} = \frac{1}{2}\tau(c_o) = 4325 \text{ lb/in}^2$$

$$\tau(c_o) = \frac{T_{BC} \cdot c_o}{J_H} = 8{,}650 \text{ lb/in}^2$$