ME 315 Final Exam
BASIC EQUATION SHEET

Conservation Laws

Control Volume Energy Balance: $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$; $\dot{E}_{st} = mC_p \, dT/dt$; $\dot{E}_{gen} = \dot{q}V$

Surface Energy Balance: $\dot{E}_{in} - \dot{E}_{out} = 0$

Conduction

Fourier's Law: $q''_{cond,x} = -k \frac{\partial T}{\partial x}$; $q''_{cond,n} = -k \frac{\partial T}{\partial n}$; $q_{cond} = q''_{cond} A$

Heat Diffusion Equation:

Cartesian Coordinates: $\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$

Cylindrical Coordinates: $\frac{1}{r}\frac{\partial}{\partial r}\left(kr \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k \frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$

Spherical Coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(k \frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(k \sin\theta \frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$$

Thermal Resistance:

Conduction Resistance: Plane wall: $R_{t,cond} = \frac{L}{kA}$; Cylindrical shell: $R_{t,cond} = \frac{\ln(r_o/r_i)}{2\pi l k}$;

Spherical shell: $R_{t,cond} = \frac{(1/r_i) - (1/r_o)}{4\pi k}$

Convection Resistance: $R_{t,conv} = \frac{1}{h_{conv} A}$

Radiation Resistance: $R_{t,rad} = \frac{1}{h_{rad} A}$

Thermal Energy Generation:

Plane wall with uniform energy generation, given temperature T_s at both surfaces, width = $2L$, and x=0 at the center of the wall:

$$T(x) - T_s = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right)$$

General solution for plane wall with uniform energy generation:

$$T(x) = -\frac{\dot{q}x^2}{2k} + C_1 x + C_2$$

Cylinder with uniform energy generation, surface temperature T_s:

$$T(r) - T_s = \frac{\dot{q} r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right)$$

General solution for a cylindrical shell, with uniform heat generation:

$$T(r) = -\frac{\dot{q}}{4k} r^2 + C_1 \ln r + C_2$$

Extended Surfaces/Fins:

Convective Tip: $\dfrac{\theta(x)}{\theta_b} = \dfrac{\cosh\left[m(L-x)\right] + (h/mk)\sinh\left[m(L-x)\right]}{\cosh(mL) + (h/mk)\sinh(mL)}$

$$q_{fin} = (hPkA_c)^{1/2}\,\theta_b\,\frac{\sinh(mL) + (h/mk)\cosh(mL)}{\cosh(mL) + (h/mk)\sinh(mL)}$$

Adiabatic Tip: $\dfrac{\theta(x)}{\theta_b} = \dfrac{\cosh\left[m(L-x)\right]}{\cosh(mL)}$; $q_{fin} = (hPkA_c)^{1/2}\,\theta_b \tanh(mL)$

Prescribed Tip Temperature: $\dfrac{\theta(x)}{\theta_b} = \dfrac{(\theta_L/\theta_b)\sinh(mx) + \sinh\left[m(L-x)\right]}{\sinh(mL)}$

$$q_{fin} = (hPkA_c)^{1/2}\,\theta_b\,\frac{\cosh(mL) - (\theta_L/\theta_b)}{\sinh(mL)}$$

Infinitely Long Fin: $\dfrac{\theta(x)}{\theta_b} = e^{-mx}$; $q_{fin} = (hPkA_c)^{1/2}\,\theta_b$

where $m^2 = \dfrac{hP}{kA_c}$; $\theta_b = T_b - T_\infty$; $q_{fin} = q_{conv,\,finsurface} + q_{conv,tip}$; $q_{conv,tip} = hA_c\theta_L$

$$\sinh x = \frac{e^x - e^{-x}}{2};\ \cosh x = \frac{e^x + e^{-x}}{2};\ \tanh x = \frac{\sinh x}{\cosh x}$$

Fin Effectiveness: $\varepsilon_{fin} = \dfrac{q_{fin}}{hA_{c,b}\theta_b}$; $\varepsilon_{fin} = \dfrac{R_{t,conv-base}}{R_{t,cond-fin}}$

Fin Efficiency: $\eta_{fin} = \dfrac{q_{fin}}{hA_{fin}\theta_b}$; for adiabatic tip, $\eta_{fin} = \dfrac{\tanh(mL)}{mL}$; an extended length of a fin using adiabatic tip $L_c = L + \dfrac{A_c}{P}$; $\eta_{fin} = \dfrac{\tanh(mL_c)}{mL_c}$

Fin array:

$$\eta_o = \frac{q_{total}}{hA_{total}\theta_b} = 1 - \frac{NA_{fin}}{A_{total}}(1 - \eta_{fin});\ R_{t,cond-fin} = \frac{1}{\eta_{fin} hA_{fin}};\ R_{t,cond-fin\,array} = \frac{1}{\eta_o hA_{total}}$$

Two Dimensional Steady Conduction:

Conduction Shape Factor: $R_{t,cond} = \frac{1}{Sk}$

Finite Volume Method: For uniform mesh, interior point, no energy generation, steady state,

$$T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} = 4T_{i,j}$$

Transient Conduction:

Lumped Capacitance Analysis: $Bi = \frac{R_{t-cond}}{R_{t-conv}} = \frac{h_{conv}L_c}{k_{solid}}$;

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{t}{\tau_t}\right); \ \tau_t = \frac{\rho V C_p}{h_{conv}A_s} = C_{t,solid}R_{t,conv}$$

$$Fo = \frac{\alpha t}{L_c^2} \ ; \ \frac{\theta}{\theta_i} = \exp\left[-\left(\frac{h_{conv}L_c}{k_{solid}}\right)\left(\frac{\alpha t}{L_c^2}\right)\right] = \exp\left[-(Bi)(Fo)\right]$$

Thermal diffusivity $\alpha = \frac{k}{\rho C_p}$

To use Bi to estimate the approximation of the Lumped Capacitance Method, for plane slab, $L_c = L$; for cylinder, $L_c = r_o/2$; for sphere, $L_c = r_o/3$.

Analytical Solutions for Transient Conduction with Spatial Effect:

$$\theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty}; \ x^* = \frac{x}{L}; \ r^* = \frac{r}{r_o}; \ t^* = \frac{\alpha t}{L^2}; \ Q_o = \rho C_p V (T_i - T_\infty)$$

Plane Wall: $\theta^* \cong C_1 \exp\left(-\zeta_1^2 Fo\right)\cos\left(\zeta_1 x^*\right)$; $Fo = t^* = \frac{\alpha t}{L^2}$; $Bi = \frac{hL}{k}$

Center temperature at x=0: $\theta_o^* = C_1 \exp\left(-\zeta_1^2 Fo\right)$; $\frac{Q}{Q_o} = 1 - \theta_o^* \frac{\sin(\zeta_1)}{\zeta_1}$

Long Cylinder: $\theta^* \cong C_1 \exp\left(-\zeta_1^2 Fo\right) J_0\left(\zeta_1 r^*\right)$; $Fo = t^* = \frac{\alpha t}{r_o^2}$; $Bi = \frac{hr_o}{k}$

Center temperature at r =0: $\theta_o^* = C_1 \exp\left(-\zeta_1^2 Fo\right)$; $\frac{Q}{Q_o} = 1 - 2\theta_o^* \frac{J_1(\zeta_1)}{\zeta_1}$

Sphere: $\theta^* \cong C_1 \exp\left(-\zeta_1^2 Fo\right)\frac{\sin\left(\zeta_1 r^*\right)}{\zeta_1 r^*}$; $Fo = t^* = \frac{\alpha t}{r_o^2}$, $Bi = \frac{hr_o}{k}$

Center temperature at r =0: $\theta_o^* = C_1 \exp\left(-\zeta_1^2 Fo\right)$; $\frac{Q}{Q_o} = 1 - 3\theta_o^* \frac{\left[\sin(\zeta_1) - \zeta_1\cos(\zeta_1)\right]}{\zeta_1^3}$

$J_0(x)$ and $J_1(x)$ are the 0th and 1st order Bessel functions of the first kind and their values will be provided, if needed.

Semi-Infinite Solid: Similarity variable $\eta = \frac{x}{\sqrt{4\alpha t}}$; $\frac{d^2T}{d\eta^2} = -2\eta \frac{dT}{d\eta}$

Constant Surface Temperature (T_s): $\frac{T(\mathrm{x,t}) - T_s}{T_i - T_s} = erf\left(\frac{x}{\sqrt{4\alpha t}}\right)$; $q_s^{"}(t) = \frac{k(T_s - T_i)}{\sqrt{\pi\alpha t}}$

Constant Surface Heat Flux ($q_o^{"}$): $T(\mathrm{x,t}) - T_i = \frac{2q_o^{"}}{k}\sqrt{\frac{\alpha t}{\pi}}\exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{q_o^{"}x}{k} erfc\left(\frac{x}{\sqrt{4\alpha t}}\right)$;

Surface Convection ($-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = h\left[T_\infty - T(0,t)\right]$):

$$\frac{T(\mathrm{x,t}) - T_i}{T_\infty - T_i} = erfc\left(\frac{x}{\sqrt{4\alpha t}}\right) - \left[\exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k}\right)\right]\left[erfc\left(\frac{x}{\sqrt{4\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

Convection

Newton's Law of Cooling: $q_{conv}^{"} = h_{conv}(T_s - T_\infty)$; $q_{conv} = q_{conv}^{"}A$

Mass Transfer: $n_A^{"} = h_m(\rho_{A,s} - \rho_{A,\infty})$; $q_{evap} = n_A^{"}Ah_{fg}$

Average Heat Transfer Coefficient: $\overline{h_{conv}} = \frac{1}{A_s}\int_{A_S} h_{conv} dA_s$

Average Mass Transfer Coefficient: $\overline{h_m} = \frac{1}{A_s}\int_{A_S} h_m dA_s$

Dimensionless Parameters:

Reynolds Number: $Re_{L_c} = \frac{\rho u L_c}{\mu} = \frac{uL_c}{\nu}$; Prandtl Number: $Pr = \frac{\nu}{\alpha}$;

Schmidt Number: $Sc = \frac{\nu}{D_{AB}}$; Lewis Number: $Le = \frac{\alpha}{D_{AB}}$

Nusselt Number: $Nu = \frac{h_{conv}L_c}{k_{fluid}}$; Sherwood Number: $Sh = \frac{h_m L_c}{D_{AB}}$

Boundary Layer Thickness: $\frac{\delta}{\delta_t} \approx Pr^n$; $\frac{\delta}{\delta_c} \approx Sc^n$; $\frac{\delta_t}{\delta_c} \approx Le^n$

Heat-Mass Analogy: $\frac{Nu}{Sh} = \frac{Pr^n}{Sc^n}$; $\frac{h}{h_m} = \frac{k}{D_{AB}Le^n} = \rho C_p Le^{1-n}$

External Flow:

Flat Plate

Flat Plate (Laminar Local): $\delta = \frac{5x}{Re_x^{1/2}}$; $C_{f,x} = 0.664 Re_x^{-1/2}$; $Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}$

Flat Plate (Laminar Average): $\overline{C_{f,L}} = 1.328 Re_L^{-1/2}$; $\overline{Nu_L} = 0.664 Re_L^{1/2} Pr^{1/3}$;

$\overline{Sh_L} = 0.664 Re_L^{1/2} Sc^{1/3}$

Flat Plate (Turbulent Local): $\delta = \frac{0.37x}{Re_x^{1/5}}$; $C_{f,x} = 0.0592 Re_x^{-1/5}$; $Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3}$;

Flat Plate (Turbulent Average): $\overline{C_{f,L}} = 0.074 Re_L^{-1/5}$; $\overline{Nu_L} = 0.037 Re_L^{4/5} Pr^{1/3}$

Flat Plate (Mixed Average): $\overline{C_{f,L}} = 0.074 Re_L^{-1/5} - 1742 Re_L^{-1}$; $\overline{Nu_L} = \left(0.037 Re_L^{4/5} - 871\right) Pr^{1/3}$

Cylinder in cross flow: $\overline{Nu_D} = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{\left[1 + \left(0.4/Pr\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282000}\right)^{5/8}\right]^{4/5}$ for $Re_D Pr > 0.2$;

Sphere: $\bar{N}u_D = 2 + \left(0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3}\right) \mathrm{Pr}^{0.4} \left(\frac{\mu}{\mu_s}\right)^{1/4}$

Internal Flow:

Mean Velocity: $u_m = \frac{\int_{A_c} \rho u(r,x)\, dA_c}{\rho A_c}$; $u_m \overset{circular}{=} \frac{2}{r_o^2} \int_0^{r_o} u(r,x)\, r\, dr$

Reynolds Number: $Re_{D_h} = \frac{u_m D_h}{\nu}$; $D_h = \frac{4A_c}{P}$; $Re_D \overset{circular}{=} \frac{u_m D}{\nu}$, $Re_D = \frac{4\dot{m}}{\pi D \mu}$

Turbulent: $Re_D \geq 2{,}300$

Hydrodynamic Entrance Lengths: $\left(\frac{x_{fd,hydrodynamic}}{D}\right)^{laminar} = 0.05 Re_D$; $60 > \left(\frac{x_{fd,hydrodynamic}}{D}\right)^{turbulent} > 10$

Thermal Entrance Lengths: Laminar flow: $\left(\frac{x_{fd,thermal}}{D}\right) \approx 0.05 Re_D Pr$;

Turbulent flow: $60 > \left(\frac{x_{fd,thermal}}{D}\right) > 10$

Mean (Bulk) Temperature: $T_m = \frac{\int_{A_c} \rho u C_p T\, dA_c}{\dot{m} C_p}$; $T_m \overset{circular}{=} \frac{2}{u_m r_o^2} \int_0^{r_o} u T(r,x)\, r\, dr$

Constant Heat Flux: $T_m(x) = T_{m,i} + \frac{q''_{conv} P}{\dot{m} C_p} x = T_{m,i} + \frac{q''_s P}{\dot{m} C_p} x$; $q_{conv} = q''_s A_s = q''_s (PL)$

Constant Surface Temperature: $\frac{T_s - T_{m,x}}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m} C_p} \overline{h_{conv}}\right)$; $\Delta T_{LMTD} = \frac{\Delta T_o - \Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m} C_p} \overline{h_{conv}}\right); \; q_{conv} = \overline{h_{conv}} A_s \Delta T_{LMTD} = \dot{m} C_p \left(T_{m,o} - T_{m,i}\right)$$

Constant Outside Free Stream Temperature:

$$\frac{\Delta T_o}{\Delta T_i}=\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}}=\exp\left(-\frac{PL}{\dot{m}C_p}\bar{U}\right)=e\,\mathrm{xp}\left(-\frac{1}{\dot{m}C_p R_{tot}}\right);\; q_{conv}=\bar{U}A_s\Delta T_{LMTD};$$

$\bar{U}$ = averaged overall heat transfer coefficient.

Circular Pipe

Fanning Friction Factor :

$$f^{laminar}=\frac{64}{Re_D};\; f\overset{turbulent}{=}\frac{0.316}{Re_D^{1/4}} \text{ for } Re_D < 2\times10^4;\; f\overset{turbulent}{=}\frac{0.184}{Re_D^{1/5}} \text{ for } Re_D > 2\times10^4$$

Laminar Fully-developed Region: $Nu_D \underset{q''=constant}{=} 4.36;\; Nu_D \underset{T_s=constant}{=} 3.66$

Laminar Entrance Region:

$$\overline{Nu_D}\overset{T_s=constant}{=}3.66+\frac{0.0668(D/L)Re_D Pr}{1+0.04\left[(D/L)Re_D Pr\right]^{2/3}}$$

Turbulent Fully-Developed Region: $Nu_D = 0.023 Re_D^{4/5} Pr^n;\; n \overset{fluid\ cooling}{=} 0.3$

$n \overset{fluid\ heating}{=} 0.4$ (Applicable to both q_s" = constant and T_s = constant)

Free Convection:

Boundary Layer Parameters: $\eta=\frac{y}{x}\left(\frac{Gr_x}{4}\right)^{1/4};\; u=\frac{2\nu}{x}\left(Gr_x\right)^{1/2} f'(\eta)$

$$Gr_x=\frac{g\beta(T_s-T_\infty)x^3}{\nu^2};\; Ra_x = Gr_x Pr=\frac{g\beta(T_s-T_\infty)x^3}{\nu\alpha}$$

Vertical Flat Plate: $\overline{Nu_L}=\left\{0.825+\frac{0.387Ra_L^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2$

Horizontal Flat Plate: $L_c = A_s/P;\; \overline{Nu_{L_c}} \underset{upper\,hot/lower\,cold}{=} 0.54 Ra_{L_c}^{1/4},\; for\ 10^4 < Ra_L < 10^7;$

$$\overline{Nu_{L_c}} \underset{upper\,hot/lower\,cold}{=} 0.15 Ra_{L_c}^{1/3},\ \text{for } 10^7 < Ra_L < 10^{11};\; \overline{Nu_{L_c}} \underset{lower\,hot/upper\,cold}{=} 0.27 Ra_{L_c}^{1/4}$$

Horizontal Cylinder: $\overline{Nu_D} = C Ra_D^n$, C and n depends on Ra.

Sphere: $\overline{Nu_D}=2+\frac{0.589Ra_D^{1/4}}{\left[1+(0.469/Pr)^{9/16}\right]^{4/9}}$

Boiling:

Basic Equation: $q_{boiling}=h_{boiling}A\Delta T_e = h_{boiling}A(T_s - T_{sat})$

Nucleate Boiling: $q_s'' = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left[\frac{c_{p,l}\Delta T_e}{C_{sf}h_{fg}Pr_l^n}\right]^3;$

Critical heat flux: $q_{max}''=\frac{\pi}{24}h_{fg}\rho_v\left[\frac{\sigma g(\rho_l-\rho_v)}{\rho_v^2}\right]^{1/4}$

Minimum Heat Flux: $q''_{min} = 0.09 h_{fg} \rho_v \left[\frac{\sigma g \left(\rho_l - \rho_v\right)}{\left(\rho_l + \rho_v\right)^2} \right]^{1/4}$

Film Boiling: $\overline{Nu_D} = \frac{\overline{h_D} D}{k_v} = C \left[\frac{g \left(\rho_l - \rho_v\right) h'_{fg} D^3}{\nu_v k_v \left(T_s - T_{sat}\right)} \right]^{1/4}$;

$h'_{fg} = h_{fg} + 0.8 C_{p,v} \left(T_s - T_{sat}\right)$; $C \overset{cylinder}{=} 0.62$; $C \overset{sphere}{=} 0.67$

Condensation:

Basic Equation: $q_{condensation} = h_{condensation} A \Delta T_d = h_{condensation} A \left(T_{sat} - T_s\right)$; $\dot{m}_{condensation} = \frac{q_{condensation}}{h'_{fg}}$

Film Condensation, Vertical Flat Plate, laminar: $\overline{Nu_L} = \frac{\overline{h_L} L}{k_l} = 0.943 \left[\frac{g \left(\rho_l - \rho_v\right) h'_{fg} L^3}{\nu_l k_l \left(T_{sat} - T_s\right)} \right]^{1/4}$;

$$h'_{fg} = h_{fg} + 0.68 C_{p,l} \left(T_{sat} - T_s\right)$$

<u>Radiation</u>

Radiation leaving one surface and intercepted by a second surface:

$q_{1-2} = I_e \left(A_1 cos\theta_1\right) d\omega_{2-1}; d\omega_{2-1} = \frac{dA_{2,normal}}{r^2} = \frac{A_2 cos\theta_2}{r^2}$

Emissive Power: Spectral: $E_\lambda \left(\lambda\right) = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,e} \left(\lambda, \theta, \phi\right) cos\theta sin\theta d\theta$;

Total: $E = \int_0^\infty E_\lambda \left(\lambda\right) d\lambda$

$E_\lambda \left(\lambda\right) \overset{diffuse\ emitter}{=} \pi I_{\lambda,e} \left(\lambda\right)$; $E \overset{diffuse\ emitter}{=} \pi I_e$

Irradiation: $G_\lambda \left(\lambda\right) = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i} \left(\lambda, \theta, \phi\right) cos\theta sin\theta d\theta$; Total: $G = \int_0^\infty G_\lambda \left(\lambda\right) d\lambda$

$G_\lambda \left(\lambda\right) \overset{diffuse\ irradiation}{=} \pi I_{\lambda,i} \left(\lambda\right)$; $G \overset{diffuse\ irradiation}{=} \pi I_i$

Radiosity: $J_\lambda \left(\lambda\right) = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,e+r} \left(\lambda, \theta, \phi\right) cos\theta sin\theta d\theta$; $J = \int_0^\infty J_\lambda \left(\lambda\right) d\lambda$

$J_\lambda \left(\lambda\right) \underset{diffuse\ reflector}{\overset{diffuse\ emitter}{=}} \pi I_{\lambda,e+r} \left(\lambda\right)$; $J \underset{diffuse\ reflector}{\overset{diffuse\ emitter}{=}} \pi I_{e+r}$ (opaque surface)

Black Body Emission: $E_{\lambda,b} \left(\lambda, T\right) = \frac{C_1}{\lambda^5 \left[\exp\left(\frac{C_2}{\lambda T}\right) - 1 \right]}$; $E_b \left(T\right) = \int_0^\infty E_{\lambda,b} \left(\lambda, T\right) d\lambda = \sigma T^4$; $E_b = \pi I_b$;

$\lambda_{max} T = 2898 \mu m K$ Wein's displacement law

Radiative Properties:

Emissivity: spectral directional: $\varepsilon_{\lambda,\theta} = \frac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{\lambda,eb}(\lambda,T)}$;

Spectral hemispherical: $\varepsilon_{\lambda} = \frac{E_{\lambda}}{E_{\lambda,b}} = \frac{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi,T) cos\theta sin\theta d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,eb}(\lambda,T) cos\theta sin\theta d\theta}$;

Total hemispherical: $\varepsilon = \frac{\int_0^{\infty} E_{\lambda}(\lambda,T) d\lambda}{\int_0^{\infty} E_{\lambda,b}(\lambda,T) d\lambda} = \frac{\int_0^{\infty} \varepsilon_{\lambda} E_{\lambda,b}(\lambda,T) d\lambda}{\sigma T^4}$

Absorptivity: spectral directional: $\alpha_{\lambda,\theta} = \frac{I_{\lambda,i\ absorbed}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$;

Spectral hemispherical: $\alpha_{\lambda} = \frac{G_{\lambda,absorbed}(\lambda)}{G_{\lambda}(\lambda)} = \frac{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i\ absorbed}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}$;

Total hemispherical: $\alpha = \frac{G_{absorbed}}{G}$; $\alpha = \frac{\int_0^{\infty} \alpha_{\lambda} G_{\lambda}(\lambda) d\lambda}{\int_0^{\infty} G_{\lambda}(\lambda) d\lambda}$

Reflectivity: Spectral directional: $\rho_{\lambda,\theta} = \frac{I_{\lambda,i\ reflected}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$;

Spectral hemispherical: $\rho_{\lambda} = \frac{G_{\lambda,reflected}(\lambda)}{G_{\lambda}(\lambda)} = \frac{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i\ reflected}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}$;

Total hemispherical: $\rho = \frac{G_{reflected}}{G}$; $\rho = \frac{\int_0^{\infty} \rho_{\lambda} G_{\lambda}(\lambda) d\lambda}{\int_0^{\infty} G_{\lambda}(\lambda) d\lambda}$

Transmissivity: Spectral directional: $\tau_{\lambda,\theta} = \frac{I_{\lambda,i\ transmitted}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$;

Spectral hemispherical: $\tau_{\lambda} = \frac{G_{\lambda,transmitted}(\lambda)}{G_{\lambda}(\lambda)} = \frac{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i\ transmitted}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}$;

Total hemispherical: $\tau = \frac{G_{transmitted}}{G}$; $\tau = \frac{\int_0^{\infty} \tau_{\lambda} G_{\lambda}(\lambda) d\lambda}{\int_0^{\infty} G_{\lambda}(\lambda) d\lambda}$

Semi-transparent Surface: $\alpha_{\lambda} + \rho_{\lambda} + \tau_{\lambda} = 1$; $\alpha + \rho + \tau = 1$

Opaque Surface: $\alpha_\lambda + \rho_\lambda = 1$; $\alpha + \rho = 1$

Kirchhoff's Law: $\varepsilon_{\lambda,\theta} = \alpha_{\lambda,\theta}$

Diffuse surface: $\varepsilon_\lambda = \alpha_\lambda$

Gray Surface: $\varepsilon_\lambda \neq f(\lambda)$; $\alpha_\lambda \neq f(\lambda)$;

Diffuse-Gray Surface: $\varepsilon = \alpha$

View Factor: $F_{ij} = \frac{q_{i-j}}{A_i J_i} = \int_{A_i}\int_{A_j} \frac{cos\theta_i cos\theta_j}{\pi r^2} dA_i dA_j$; $F_{ji} = \frac{q_{j-i}}{A_j J_j} = \int_{A_i}\int_{A_j} \frac{cos\theta_i cos\theta_j}{\pi r^2} dA_i dA_j$

Reciprocity: $A_i F_{ij} = A_j F_{ji}$

Summation: $\sum_{j=1}^{N} F_{ij} = 1$; $F_{ii} \underset{\text{plane surface}}{\overset{\text{convex surface}}{=}} 0$; $F_{ii} \overset{\text{concave surface}}{\neq} 0$

Surface with multiple sub-surfaces: $F_{(j)i} = \frac{\sum_{k=1}^{n} A_k F_{ki}}{\sum_{k=1}^{n} A_k}$

Radiative Exchange:

Black enclosure: $q_{ij} = A_i F_{ij} \sigma \left(T_i^4 - T_j^4\right)$;

Diffuse-gray enclosure: $q_i = \frac{E_{bi} - J_i}{(1-\varepsilon_i)/A_i\varepsilon_i}$; $q_i = \sum_{j=1}^{N} \frac{J_i - J_j}{1/A_i F_{ij}}$

Radiation between two parallel, infinitely large surfaces: $q_{12} = \frac{E_{b,1} - E_{b,2}}{\frac{(1-\varepsilon_1)}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{(1-\varepsilon_2)}{A_2\varepsilon_2}}$

Radiation Shields: $q_{12} \overset{\text{Single Shield}}{=} \frac{E_{b,1} - E_{b,2}}{\frac{(1-\varepsilon_1)}{A_1\varepsilon_1} + \frac{1}{A_1F_{13}} + \frac{(1-\varepsilon_{31})}{A_3\varepsilon_{31}} + \frac{(1-\varepsilon_{32})}{A_3\varepsilon_{32}} + \frac{1}{A_3F_{32}} + \frac{(1-\varepsilon_2)}{A_2\varepsilon_2}}$

Net Radiation Leaving a Surface: $q''_{rad} = J - G = \varepsilon E - \alpha G$

Large isothermal surroundings: $G = J = \sigma T_{surr}^4$

$$q''_{rad} \overset{\text{large isothermal}}{=} \varepsilon\sigma\left(T_s^4 - T_{surr}^4\right) = h_{rad}\left(T_s - T_{surr}\right)$$

$$h_{rad} = \varepsilon\sigma\left(T_s + T_{surr}\right)\left(T_s^2 + T_{surr}^2\right)$$

Useful Constants

σ = Stefan-Boltzmann's Constant = 5.67×10^{-8} W/m²K⁴

R_u = Universal Gas Constant = 8.314 kJ/kmolK

Geometry

Cylinder: $A = 2\pi r l$; $V = \pi r^2 l$; Sphere: $A = 4\pi r^2$; $V = \frac{4}{3}\pi r^3$

Triangle: $A = bh/2$ *b*: base *h*: height