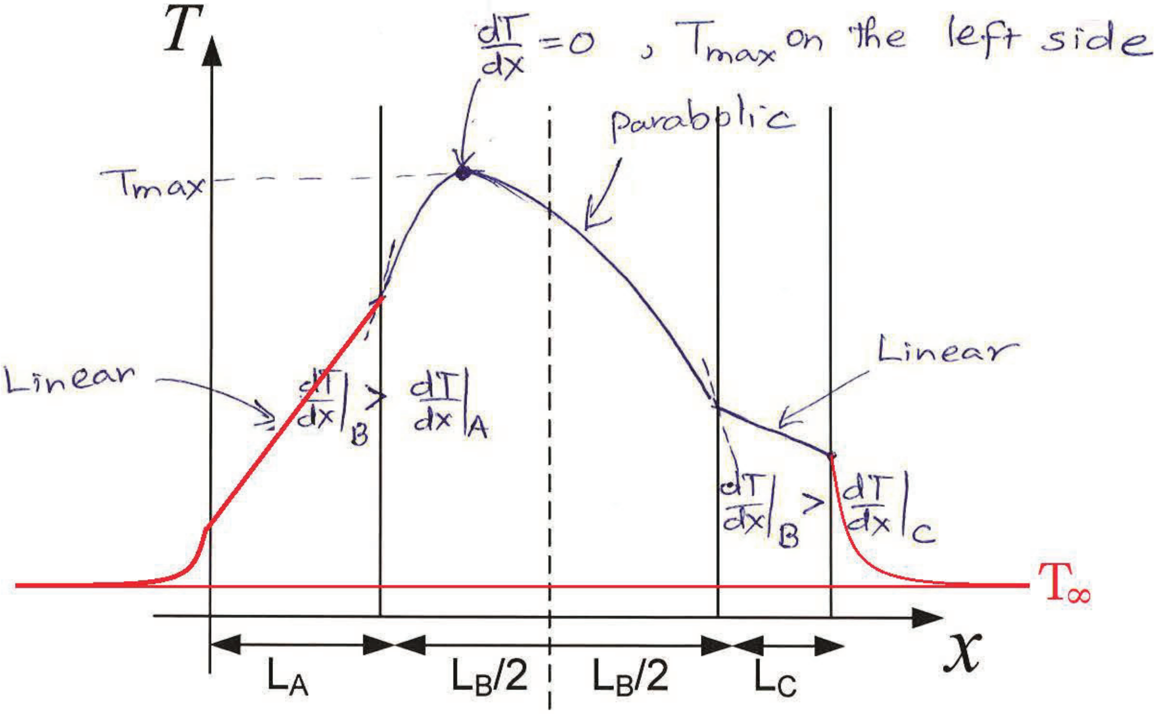
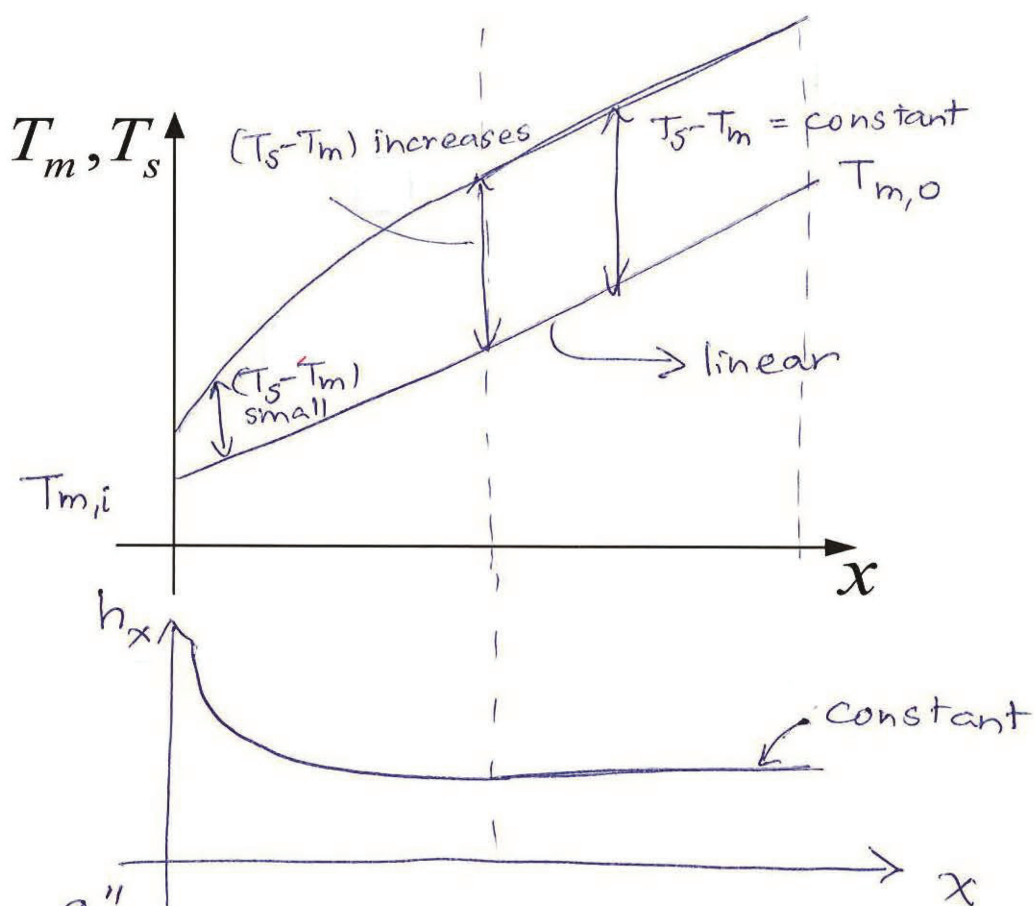
**Problem 1**

(a)




(b)




$$T_s = T_m + \frac{q_s''}{h}$$

(c)

Considering energy balance for the control volume, we have:

$$\dot{E}_{in} - \cancel{\dot{E}_{out}} + \cancel{\dot{E}_{gen}} = \dot{E}_{st} \Rightarrow \dot{E}_{in} = q_e + q_s + q_b$$

$$q_e = k\left(\frac{\Delta y}{2}\times 1\right)\left(\frac{T^P_{m+1,n} - T^P_{m,n}}{\Delta x}\right) = \frac{k}{2}\left(T^P_{m+1,n} - T^P_{m,n}\right)$$

$$q_s = k\left(\Delta x \times 1\right)\left(\frac{T^P_{m,n-1} - T^P_{m,n}}{\Delta y}\right) = k\left(T^P_{m,n-1} - T^P_{m,n}\right)$$

$$q_b = h\left(\frac{\Delta x}{\sqrt{2}}\times 1\right)\left(T^P_{\infty} - T^P_{m,n}\right)$$

$$\dot{E}_{st} = mC_p\frac{dT}{dt} = \rho VC_p\frac{dT}{dt} = \rho\left[\frac{3}{8}(\Delta x)^2\right]C_p\left(\frac{T^{P+1}_{m,n} - T^P_{m,n}}{\Delta t}\right)$$

Substituting in the energy balance, we have:

$$\frac{k}{2}\left(T^P_{m+1,n} - T^P_{m,n}\right) + k\left(T^P_{m,n-1} - T^P_{m,n}\right) + \frac{h\Delta x}{\sqrt{2}}\left(T^P_{\infty} - T^P_{m,n}\right) = \rho\left[\frac{3}{8}(\Delta x)^2\right]C_p\left(\frac{T^{P+1}_{m,n} - T^P_{m,n}}{\Delta t}\right)$$

Multiplying throughout by 2/k, we get:

$$\left(T^P_{m+1,n} - T^P_{m,n}\right) + 2\left(T^P_{m,n-1} - T^P_{m,n}\right) + \sqrt{2}\frac{h\Delta x}{k}\left(T^P_{\infty} - T^P_{m,n}\right) = \frac{3}{4}\frac{\rho C_p}{k}\frac{(\Delta x)^2}{\Delta t}\left(T^{P+1}_{m,n} - T^P_{m,n}\right)$$

$$Bi = \frac{h\Delta x}{k},\ \alpha = \frac{k}{\rho C_p},\ \text{and } Fo = \frac{\alpha\Delta t}{(\Delta x)^2}$$

$$\left(T^P_{m+1,n} - T^P_{m,n}\right) + 2\left(T^P_{m,n-1} - T^P_{m,n}\right) + \sqrt{2}Bi\left(T^P_{\infty} - T^P_{m,n}\right) = \frac{3}{4Fo}\left(T^{P+1}_{m,n} - T^P_{m,n}\right)$$

Re-arrangement of the equation gives:

$$\boldsymbol{T^{P+1}_{m,n} = \frac{4}{3}FoT^P_{m+1,n} + \frac{8}{3}FoT^P_{m,n-1} + \frac{4\sqrt{2}}{3}BiFoT^P_{\infty} + \left(1 - \frac{4\sqrt{2}}{3}BiFo - 4Fo\right)T^P_{m,n}}$$

## Problem 2

(a)

| |
|---|
| $\cancel{\dot{E}_{in}} - \dot{E}_{out} + \dot{E}_{gen} = \cancel{\dot{E}_{st}}$ [energy balance around sphere, no $E_{in}$, steady state] |
| $-q_{conv} + q_{gen} = 0$ |
| Newton's law of cooling: $q_{conv} = \bar{h}A_s(T_s - T_\infty) \rightarrow -\bar{h}A_s(T_s - T_\infty) + q_{gen} = 0$ |
| Rearrange energy balance to find $T_s = \frac{q_{gen}}{\bar{h}A_s} + T_\infty$ |
| $\overline{Nu_{D_o}} = \frac{\bar{h}D_o}{k_w} = 2 \rightarrow \bar{h} = \frac{2k_w}{D_o} = \frac{2(0.6W/(m\ K))}{0.02m} = 60\frac{W}{m^2K}$ |
| $T_s = \frac{q_{gen}}{\bar{h}A_s} + T_\infty = \frac{1W}{\left(60\frac{W}{m^2K}\right)\left(\pi(0.02m)^2\right)} + 10°C$ <br> $= \frac{1W}{\left(60\frac{W}{m^2K}\right)\left(1.26x10^{-3}m^2\right)} + 10°C = 13.2K + 10°C$ |
| $T_s = 23.2°C$ |

(b) Yes, the outer surface temperature of the coating a good indicator of the hottest temperature in the spherical pellet.

explanation. Several options including:

1. Calculate $Bi = \frac{\bar{h}L_c}{k_p} = \frac{\bar{h}(D/6)}{k_p} = \frac{\left(60\frac{W}{m^2K}\right)\left(\frac{0.02m}{6}\right)}{15\frac{W}{m\ K}} = 0.0133 << 0.1$. This indicates that lumped capacitance would apply which means that the temperature gradients inside the object are negligible.

2. Explain that Conduction Resistances << Convection Resistances and that q*R = ΔT, so internal temperature gradients are negligible.

$$R_{cond} \approx \frac{L_c}{kA} = \frac{\cancel{D}/6}{k\pi D^{\cancel{2}}} = \frac{1}{6k\pi D} = 0.177\frac{K}{W}$$

$$R_{cond} = \frac{1}{\bar{h}A} = 13.3\frac{K}{W}$$

3. Calculate temperature at the center of the sphere.

Resistance of non-heated spherical shell

$$R_{cond} = \frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{1}{4\pi\left(15\frac{W}{m\ K}\right)}\left[\frac{1}{0.0025m} - \frac{1}{0.01m}\right] = 1.59\frac{K}{W}$$

T at interface between heated and non-heated region

$$T_i - T_s = qR_{cond} \rightarrow T_i - T_s = 1W\ 1.5\frac{K}{W} = 1.5K$$

T at center of sphere

$$T_c - T_i = \frac{\dot{q}r_i^2}{6k}\left[1 - \frac{0^2}{r_i^2}\right] = \frac{\cancel{6}q}{\pi D_i^{\cancel{3}}}\frac{D_i^{\cancel{2}}}{\cancel{6}4k} = \frac{q}{4\pi kD_i} = \frac{1W}{4\pi\left(15\frac{W}{m\ K}\right)(0.005m)}$$

$$= 1.06K$$

Thus, the center of the sphere is 2.56°C hotter than the surface of the sphere at steady state, which is a small temperature rise compared to ($T_s - T_\infty$) = 13.2°C

(c)

Verify/Assume lumped capacitance applies.

$$Bi = \frac{\bar{h}L_c}{k_p} = \frac{\bar{h}(D/6)}{k_p} = \frac{\left(60\frac{W}{m^2K}\right)\left(\frac{0.02m}{6}\right)}{15\frac{W}{m\ K}} = 0.0133 << 0.1$$

[Bi may have been calculated in part (b) and students must list assumption or reference somehow that lumped capacitance is valid]

$$\cancel{\dot{E}_{in}} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$ [energy balance about sphere]

$$-q_{conv} + q_{gen} = Mc_p\frac{dT}{dt}$$

$$-\bar{h}A_s(T - T_\infty) + q_{gen} = \rho c_p V\frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{-\bar{h}A_s}{\rho c_p V}(T - T_\infty) + \frac{q_{gen}}{\rho V c_p}$$

$$\left.\frac{dT}{dt}\right|_{T=20°C} = \frac{-\left(60\frac{W}{m^2K}\right)\left(\cancel{\pi}\cancel{(0.02m)^2}\right)}{\left(2.3x10^6\frac{J}{m^3K}\right)\left(\frac{\cancel{\pi}}{6}(0.02m)^{\cancel{3}}\right)}(20°C - 10°C) + \frac{1W}{\left(2.3x10^6\frac{J}{m^3K}\right)\left(\frac{\pi}{6}(0.02m)^3\right)}$$

$$\frac{dT}{dt} = -0.078\frac{K}{s} + 0.103\frac{K}{s} = 0.0255\frac{K}{s}$$

(d) **Yes**, the thickness of the coating can be reduced and the requirements can still be met.

Should recognize that the steady state temperature of the sphere from (a) is less than the maximum allowed temperature. The surface area for convection is reduced if the sphere diameter is reduced, but the system can tolerate a reduction in surface area and meet the requirements.

Calculate sphere diameter including that *h* is a function of diameter:

$$-\bar{h}A_s(T_s - T_\infty) + q_{gen} = 0$$ [From (a) or from an energy balance on the sphere]

$$\overline{Nu}_{D_o} = \frac{\bar{h}D_o}{k_w} = 2 \rightarrow \bar{h} = \frac{2k_w}{D_o}$$

$$-\frac{2k_w}{\cancel{D_o}}\pi D_o^{\cancel{2}}(T_s - T_\infty) + q_{gen} = 0$$

$$-2k_w\pi D_o(T_s - T_\infty) + q_{gen} = 0 \rightarrow D_o = \frac{q_{gen}}{2k_w\pi(T_s - T_\infty)}$$

$$D_o = \frac{1W}{2\left(0.6\frac{W}{m\ K}\right)\pi(30°C - 10°C)} = 0.013m = 1.3cm$$

or if the changes in *h* with diameter are neglected

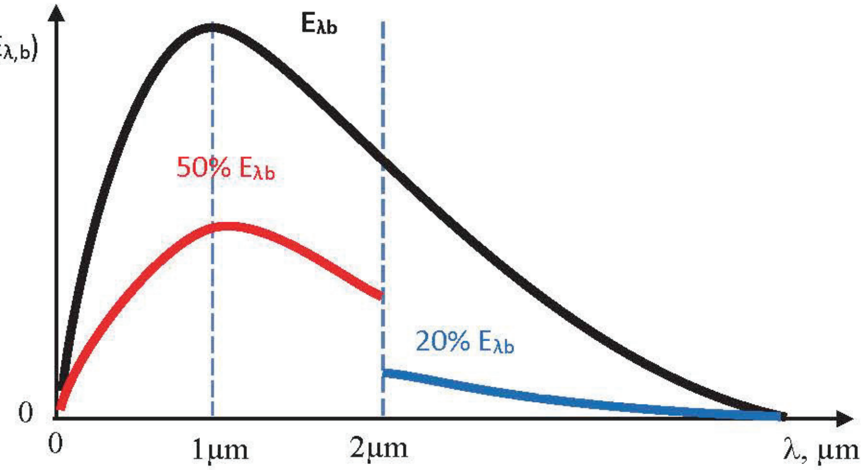
$$-\bar{h}A_s(T_s - T_\infty) + q_{gen} = 0$$ [From (a) or from an energy balance on the sphere]

$$-\bar{h}\pi D_o^2(T_s - T_\infty) + q_{gen} = 0 \rightarrow D_o = \sqrt{\frac{q_{gen}}{\bar{h}\pi(T_s - T_\infty)}}$$

$$D_o = \sqrt{\frac{1W}{\left(60\frac{W}{m^2K}\right)\pi(30°C - 10°C)}} = 0.016m = 1.6cm$$

**Problem 3**

(a)




(b) For diffuse surface, $\varepsilon = \alpha$

$$\alpha_{Total} = \frac{\left[0.5 \times F(2\mu m \times 300K) + 0.2 \times (1 - F(2\mu m \times 300K)\right] E_{b,300K}}{E_{b,300K}} = 0.2$$

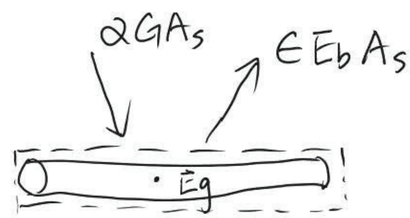
Or state the background irradiation spectrum is mainly in the region of longer than 2um where the total absorptivity is only affected by the spectrum absorptivity at >2um range. Therefore, we can know $\alpha_{Total}$=0.2 without calculations.

(c)

$$E = \left[0.5 \times F(2\mu m \times 3000K) + 0.2 \times (1 - F(2\mu m \times 3000K)\right] E_b$$

$$\varepsilon_{Total} = \frac{E}{E_b} = 0.42$$

(d) Control volume and energy balance.




Alternatively, one can state G<<E from $T_S$ >> $T_{Sur}$, and ignore irradiation.

$$\dot{E}_g = \varepsilon_{Total} E_b A_s - \alpha G A_s = \varepsilon_{Total} \sigma T_s^4 - \alpha \sigma T_{sur}^4$$

$$\dot{E}_g = 72[W]$$

(e) Power fraction in the visible range 0.4 – 0.7 μm for a blackbody

$$F_b(0.4 - 0.7\mu m) = F(0.7\mu m \times 3000K) - F(0.4\mu m \times 3000K)$$
$$= 0.0838 - 0.0021 = 0.0817$$

For the real filament:

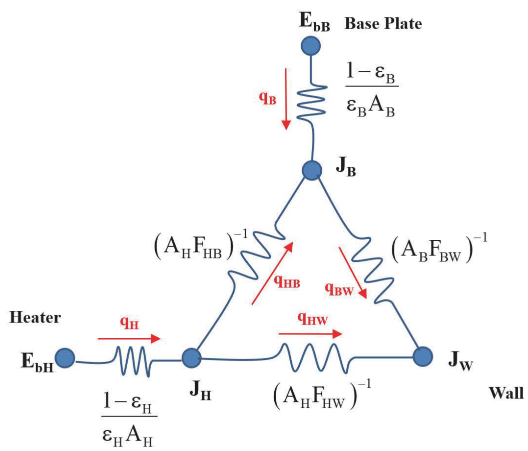
$$E_{visible} = 0.5 \times F_b(0.4 - 0.7\mu m) \times E_b$$

$$\eta = \frac{E_{visible}}{E} = \frac{E_{visible}}{\varepsilon_{Total} E_b} = 9.7\%$$

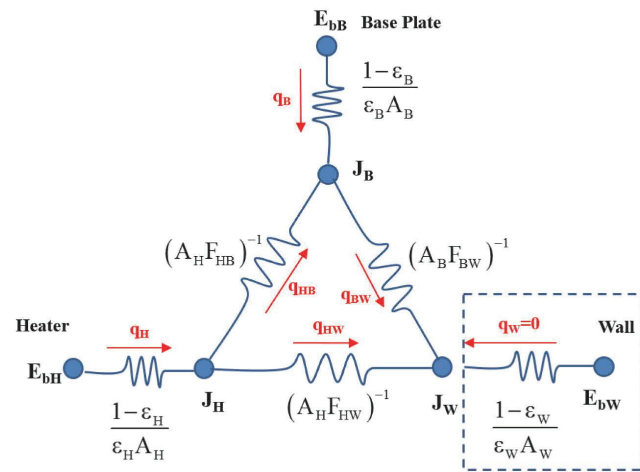
## Problem 4

**Part (a)** There are two possible radiation networks for this 3-surface enclosure.

Network 1:




Network 2:




**Part (b)**

Find the two of the needed view factors:

$F_{BW}=0.5 \rightarrow F_{BH}=1-F_{BW}-F_{BB}=1-0.5-0=0.5$

$F_{HW}=(A_{H'}/A_H)F_{H'\to W}=1/2*0.5=0.25$ or $F_{WH}=F_{W\to H'}=F_{H'\to W}(A_{H'}/A_W)=0.333$

Find $q_e$ and $T_W$:

*Method 1: Using effective 1D network*

Calculate total resistance:

$$R_{total}=\frac{1-\varepsilon_H}{\varepsilon_H A_H}+\left[\left(\frac{1}{A_H F_{HB}}\right)\Bigg|\Bigg|\left(\frac{1}{A_H F_{HW}}+\frac{1}{A_W F_{WB}}\right)\right]+\frac{1-\varepsilon_B}{\varepsilon_B A_B}=2.46 \quad [m^{-2}]$$

Find the two blackbody potentials:

$$E_{bH} = \sigma T_H^4 = 56,700$$
$$E_{bB} = \sigma T_B^4 = 1,450$$

The $q_e$ can be calculated as:

$$q_e = q_H = \frac{E_{bH} - E_{bB}}{R_{total}} = 22,500 \quad [W]$$

Using $q_e$, we can find $J_B$ and $J_H$ using potential drop:

$$J_H = 53,900$$
$$J_B = 23,900$$

Using $J_B$ and $J_H$ and resistances, we can find $J_W$ and $T_W$:

$$J_W = 38,900$$
$$T_W = \left(\frac{J_W}{\sigma}\right)^{1/4} = 910 \quad [K]$$

*Method 2: Using energy balance at each J nodes*

Write one equation at each of the 3 J nodes with 5 currents using 5 nodal potentials:

$$\begin{cases} -q_H + q_{HB} + q_{HW} = 0 \\ q_{HW} + q_{BW} = 0 \\ -q_B - q_{HB} + q_{BW} = 0 \end{cases}$$

$$E_{bH} = \ldots \qquad E_{bB} = \ldots$$
$$q_H = \ldots \qquad q_B = \ldots$$
$$q_{HB} = \ldots \qquad q_{HW} = \ldots \qquad q_{BW} = \ldots$$

Reduce these equations and solve for all J nodes:

$$J_H = 53,900$$
$$J_B = 23,900$$
$$J_W = 38,900$$

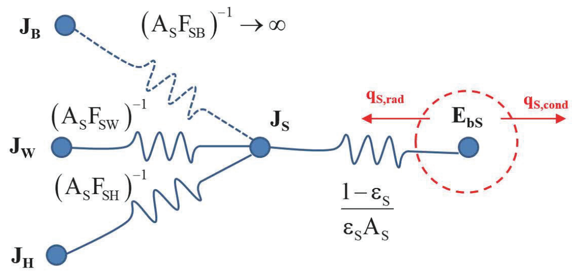
From J nodal values, we can find $q_H$ and $T_W$:

$$q_e = q_H = 22,500 \quad [W]$$

$$T_W = \left(\frac{J_W}{\sigma}\right)^{1/4} = 910 \quad [K]$$

**Part (c)**

Draw a portion of radiation network around surface S:




Find the view factors: $F_{SB}=0$; $F_{SW}=0.2$; $F_{SH}=0.8$

Find $J_S$ using nodal energy balance:

$$E_{bS} = \sigma T_S^{\ 4} = 37,200$$

$$\frac{J_W - J_S}{\dfrac{1}{A_S F_{SW}}} + \frac{J_H - J_S}{\dfrac{1}{A_S F_{SH}}} + \frac{E_{bS} - J_S}{\dfrac{1-\varepsilon_S}{\varepsilon_S A_S}} = 0 \quad \Rightarrow \quad J_S = 44,050$$

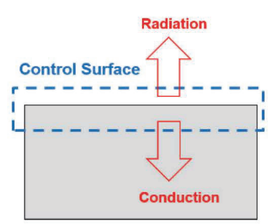
Find $q_S$ or $q_{S,rad}$:

$$q_S = q_{S,rad} = \frac{E_{bS} - J_S}{\dfrac{1-\varepsilon_S}{\varepsilon_S A_S}} \approx -6,850 \times A_S \quad [W] \Rightarrow q''_S = q''_{S,rad} \approx -6,850 \quad [\frac{W}{m^2}]$$

From the surface energy balance of the hot plate's top surface:

$$q_{i,ext} = q_{i,cond} + q_{i,rad} + q_{i,conv} \Rightarrow 0 = q_{S,cond} + q_{S,rad}$$

$$-q''_{S,rad} = q''_{S,cond} = k_S \frac{T_S - T_B}{d} = k_S \frac{900-400}{0.05} = 10,000 \times k_s \quad [\frac{W}{m^2}]$$




Find $k_S$

$k_S \approx 0.7 \quad [\frac{W}{mK}]$ (Only one significant digit is OK at this step)