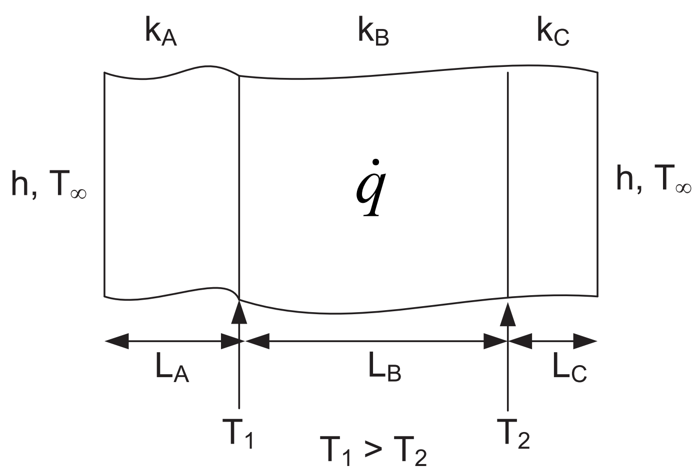
ME 315 Example Final Exam

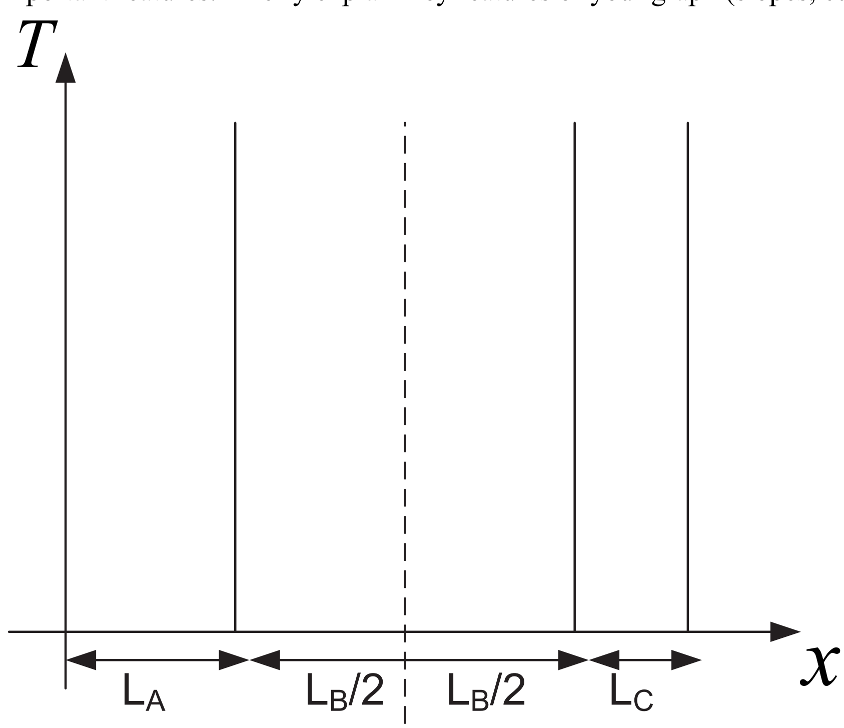
Problem 1 (25 pts)

(a) (**7 pts**) A plane composite wall consists of three layers as shown below. The central layer dissipates heat uniformly at a volumetric rate of $\dot{q}$. Thermal conductivity of material C is the highest while that of material B is the lowest. Temperature at the interface between A and B (T_1) is higher than the temperature (T_2) at the interface between B and C. The convective conditions on both sides are identical. Assume steady-state conditions and no contact resistance between layers.

$k_C > k_A > k_B$

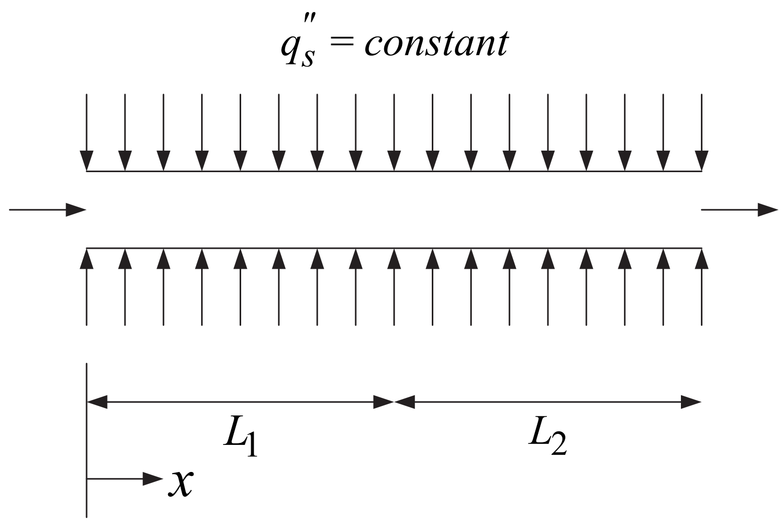


On the axes provided, sketch the qualitative temperature distribution through the three layer composite wall showing all the important features. Briefly explain key features of your graph (slopes, curve shapes, etc.).

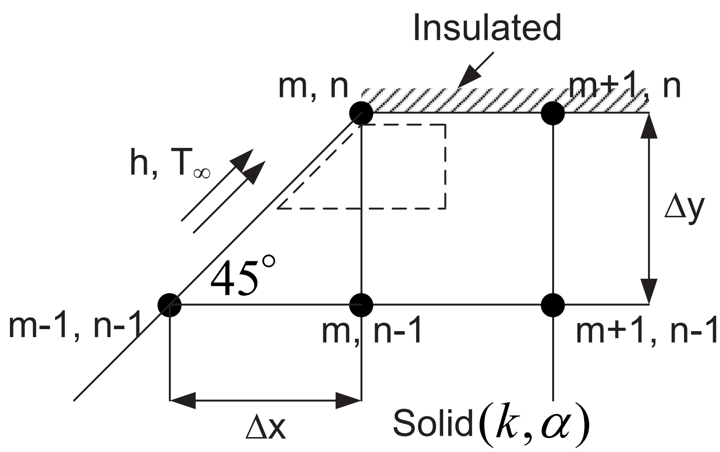


(b) (**8 pts**) Consider a liquid flowing inside a tube. The entire tube surface is supplied with constant heat flux. Flow becomes fully developed at a distance L_1 from the entrance of the tube, and thus is fully-developed in the second portion of the tube of length L_2.

On the axes provided, qualitatively sketch the variation of mean temperature of liquid and surface temperature of tube in the axial direction. Briefly explain key features of your graph



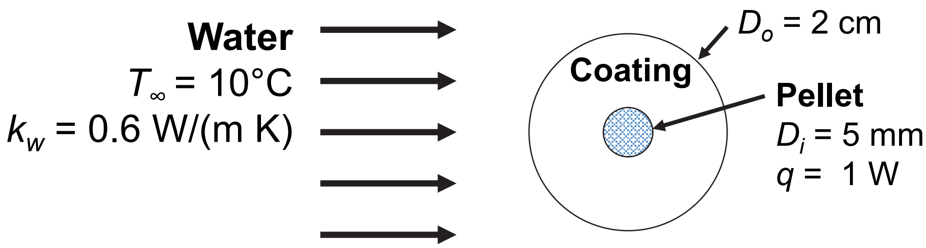
(c) (**10 pts**) Consider the corner node (m, n) in a two-dimensional uniform mesh with cell width $\Delta x = \Delta y$ shown below. The solid has thermal conductivity k and thermal diffusivity α. The top boundary is perfectly insulated, while the inclined boundary is exposed to an ambient fluid at T_∞ with a convective heat transfer coefficient h.



Using the explicit time stepping method and applying an energy balance about the corner node (m, n), derive an equation for $T_{m,n}^{P+1}$ in terms of $T_{m,n}^{P}$ and temperatures of relevant surrounding nodes as well as ambient fluid, the finite difference Biot number (Bi), the finite difference Fourier number (Fo), and known parameters.

Problem 2 (25 pts)

A 5 mm diameter spherical pellet generates 1 W of energy. The outside of the pellet is coated with a layer of non-reacting material ($\dot{q} = 0$) with identical thermophysical properties. The pellet is submerged in cool water at 10°C that flows over the coated pellet. For such flow, the Nu_D is approximately a constant value of 2.



Pellet and Coating Properties: $k = 15$ W/m/K $(\rho\, c_p) = 2.3\text{x}10^6$ J/(m^{3}K)

Recall for a sphere, $V = \dfrac{\pi}{6} D^3$ $A_s = \pi D^2$ $L_c = \dfrac{V}{A_s} = \dfrac{D}{6}$

(a) (**6 pts**) The outer diameter of the coated sphere is D_o = 2 cm = 0.02 m, calculate the outer surface temperature of the sphere at steady state.

(b) (**5 pts**) Is the outer surface temperature of the coating a good indicator of the hottest temperature in the spherical pellet? In other words, will the hottest portion of the sphere be significantly hotter than the surface temperature? Briefly justify your answer.

(c) (**7 pts**) Derive a differential equation that could be solved to find the time required to heat the sphere from an initial temperature of T_i = 10°C to a final temperature of T_f = 20°C. Then calculate the rate of change in temperature of the sphere $\left(\dfrac{\mathrm{dT}}{\mathrm{dt}}\right)$ when it is at T_f = 20°C.

(d) (7 **pts**) To prevent catastrophic damage, the outer surface of the pellet must not exceed 30°C at steady state. But the coating material is expensive. Is it possible to reduce the thickness of the coating and still meet this thermal requirement? If so, calculate the minimum sphere diameter required to meet this requirement. If not, briefly justify your answer.

Problem 3 (25 pts)

Consider a cylindrical tungsten filament with a diameter D = 0.8 mm and length L = 15 mm. The surface is diffuse with $\varepsilon_\lambda = 0.5$ for $\lambda < 2$ μm and $\varepsilon_\lambda = 0.2$ for $\lambda > 2$ μm. The filament is enclosed in an evacuated bulb and is heated by an electrical current to a steady-state temperature of T_s = 3000 K. For simplicity, the surrounding of the filament can be assumed large with a temperature T_{sur} = 300 K.

(a) (5 pts) On the following figure, qualitatively sketch the blackbody spectral emissive power $E_{\lambda,b}$ at T_s and the actual spectral emissive power E_λ for the filament surface at T_s.

(b) (5 pts) Compute the total absorptivity of the filament.

(c) (5 pts) Compute the total emissivity of the filament.

(d) (5 pts) Compute the power consumption of the filament (in [W]) to maintain its steady-state temperature. Neglect all conduction and convection heat transfer.

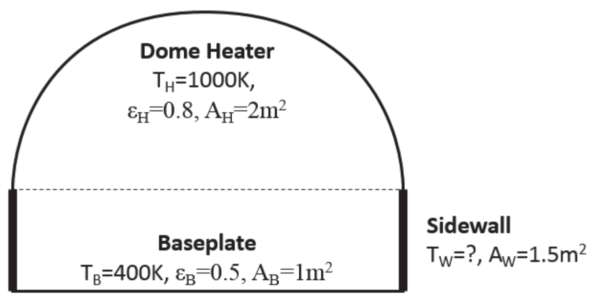
(e) (5 pts) Compute the power fraction (in percentage) emitted by the filament in the visible range 0.4 – 0.7 μm.

Problem 4 (25 pts)

A large radiation oven is designed to cure small parts in vacuum environment as shown in the figure below. All surfaces are gray and diffuse with uniform radiosities.

The internal surface of oven is consist of *i*) a dome-shaped electrical heater thermally insulated at its backside, *ii*) a cylindrical sidewall thermally insulated at its backside, and *iii*) a metallic baseplate at the bottom cooled at its backside. During its steady-state operation, the heater and baseplate temperatures are measured to be $\mathbf{T_H=1{,}000K}$ and $\mathbf{T_B=400K}$ respectively, while the sidewall temperature $\mathbf{T_W}$ is unknown.

The view factor from the bottom baseplate to the sidewall is given as $\mathbf{F_{BW}=0.5}$. All dimensions, surface and material properties are provided in the figure below.



(a) **(8 points)** Draw the radiation network of this curing oven and symbolically label all resistances and nodes.

(b) **(9 points)** Using the radiation network from (a), calculate the electrical heating power ($\mathbf{q_e}$) of the dome and the temperature of the oven sidewall ($\mathbf{T_W}$) at steady state.

(c) **(8 points)** A small and opaque polymer plate (thickness of **d=5cm**, surface emissivity of $\mathbf{\varepsilon_s=0.5}$, and unknown thermal conductivity) is placed at the center of the bottom metallic baseplate (as shown to the right) in order to cure the top surface of the polymer. The view factor from the exposed top surface of polymer plate to the sidewall of the oven is $\mathbf{F_{SW}=0.2}$. The bottom of the polymer plate is maintained at the same temperate of baseplate (T_B = 400 K). The polymer plate is small enough that it does not affect the radiosity of other surfaces in the oven. Neglect the heat transfer through the side edges of the polymer plate and assume its surface is gray and diffuse. Find the **thermal conductivity of the polymer plate** if the upper surface temperature of the polymer plate is $\mathbf{T_S= 900\ K}$ at steady state.