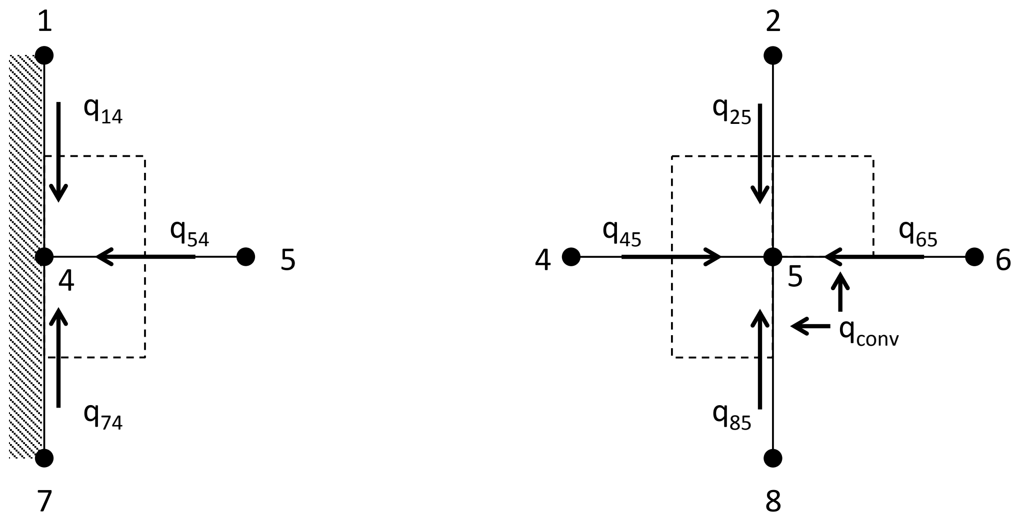
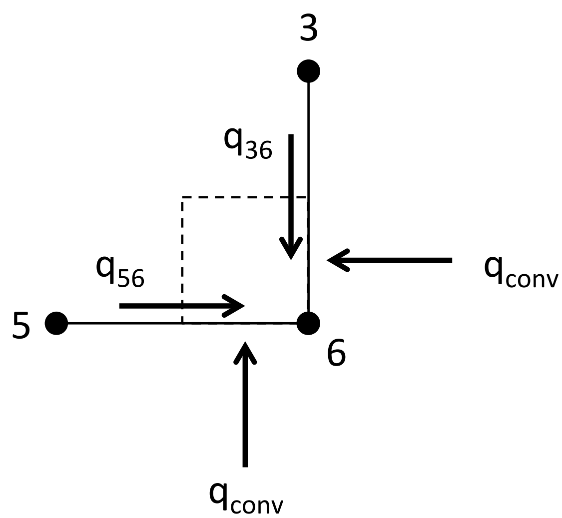
Problem 1

(a)


1
q14
q54
5
4
q74
7
2
q25
4
q45
q65
6
5
qconv
q85
8


3
q36
qconv
q56
5
6
qconv

Node 4

$$\Delta x = \Delta y$$

$$q_{14} = -k\frac{T_4^p - T_1^p}{\Delta y}\cdot\frac{\Delta x}{2}\cdot d$$

$$q_{54} = -k\frac{T_4^p - T_5^p}{\Delta x}\cdot\Delta y\cdot d$$

$$q_{74} = -k\frac{T_4^p - T_7^p}{\Delta y}\cdot\frac{\Delta x}{2}\cdot d$$

$$\dot{E}_{gen} = \dot{q}\cdot V = \dot{q}\cdot\frac{\Delta x}{2}\cdot\Delta y\cdot d$$

$$\dot{E}_{store} = mC_p\frac{\partial T}{\partial t} = \rho\left(\frac{\Delta x}{2}\cdot\Delta y\cdot d\right)C_p\frac{T_4^{p+1} - T_4^p}{\Delta t}$$

$$\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} \quad \rightarrow \quad \dot{E}_{out} = 0 \quad and \quad \dot{E}_{in} = q_{14} + q_{54} + q_{74}$$

$$\rho\left(\frac{\Delta x}{2}\cdot\Delta y\cdot 1\right)c_p\left(\frac{T_4^{P+1} - T_4^P}{\Delta t}\right) = k\left(\frac{\Delta x}{2}\cdot 1\right)\left(\frac{T_1^P - T_4^P}{\Delta y}\right) + k\left(\frac{\Delta x}{2}\cdot 1\right)\left(\frac{T_7^P - T_4^P}{\Delta y}\right) + k\left(\Delta y\cdot 1\right)\left(\frac{T_5^P - T_4^P}{\Delta y}\right) + \dot{q}\left(\frac{\Delta x}{2}\cdot\Delta y\cdot 1\right)$$

$$Fo = \frac{\alpha\Delta t}{\Delta x^2} = \frac{k\Delta t}{\rho c_p\Delta x^2} \quad \rightarrow \quad T_4^{P+1} = T_4^P + Fo\left\{T_1^P + T_7^P + 2T_5^P - 4T_4^P\right\} + \frac{\dot{q}\Delta t}{\rho c_p}$$

$$\boxed{T_4^{P+1} = T_4^P + Fo\left\{T_1^P + T_7^P + 2T_5^P - 4T_4^P\right\} + \frac{\dot{q}\Delta t}{\rho c_p}}$$

Node 5

$$q_{25} = -k\frac{T_5^p - T_2^p}{\Delta y}\cdot\Delta x\cdot d$$

$$q_{45} = -k\frac{T_5^p - T_4^p}{\Delta x}\cdot\Delta y\cdot d$$

$$q_{65} = -k\frac{T_5^p - T_6^p}{\Delta x}\cdot\frac{\Delta y}{2}\cdot d$$

$$q_{85} = -k\frac{T_5^p - T_8^p}{\Delta y}\cdot\frac{\Delta x}{2}\cdot d$$

$$q_{conv} = h\left(T_\infty - T_5^p\right)\cdot\frac{\Delta x}{2}\cdot d + h\left(T_\infty - T_5^p\right)\cdot\frac{\Delta y}{2}\cdot d$$

$$\dot{E}_{gen} = \dot{q}\cdot V = \dot{q}\cdot\frac{3\Delta x\Delta y}{4}\cdot d$$

$$\dot{E}_{store} = mC_p\frac{\partial T}{\partial t} = \rho\left(\frac{3\Delta x\Delta y}{4}\cdot d\right)C_p\frac{T_5^{p+1} - T_5^p}{\Delta t}$$

$$\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} \quad\rightarrow\quad \dot{E}_{out} = 0 \quad and \quad \dot{E}_{in} = q_{25} + q_{45} + q_{65} + q_{85} + q_{conv}$$

$$\rho\left(\frac{3}{4}\Delta x\cdot\Delta y\cdot 1\right)c_p\left(\frac{T_5^{P+1} - T_5^P}{\Delta t}\right) = k\Delta y\left(\frac{T_4^P - T_5^P}{\Delta x}\right) + k\Delta x\left(\frac{T_2^P - T_5^P}{\Delta y}\right) + k\frac{\Delta x}{2}\left(\frac{T_8^P - T_5^P}{\Delta y}\right) + k\frac{\Delta y}{2}\left(\frac{T_6^P - T_5^P}{\Delta x}\right)$$
$$+ h\left(\frac{\Delta x}{2} + \frac{\Delta y}{2}\right)\left(T_\infty - T_5^P\right) + \dot{q}\left(\frac{3}{4}\Delta x\cdot\Delta y\cdot 1\right)$$

$$Fo = \frac{\alpha\Delta t}{\Delta x^2} = \frac{k\Delta t}{\rho c_p\Delta x^2} \quad\rightarrow\quad T_5^{P+1} = T_5^P + \frac{2}{3}Fo\left\{T_8^P + T_6^P + 2T_4^P + 2T_2^P - 6T_5^P\right\} + \frac{4}{3}\frac{h\Delta t}{\rho c_P\Delta x}\left(T_\infty - T_5^P\right) + \frac{\dot{q}\Delta t}{\rho c_p}$$

$$\boxed{T_5^{P+1} = T_5^P + \frac{2}{3}Fo\left\{T_8^P + T_6^P + 2T_4^P + 2T_2^P - 6T_5^P\right\} + \frac{4}{3}\frac{h\Delta t}{\rho c_P\Delta x}\left(T_\infty - T_5^P\right) + \frac{\dot{q}\Delta t}{\rho c_p}}$$

Node 6

$$q_{36} = -k\frac{T_6^p - T_3^p}{\Delta y}\cdot\frac{\Delta x}{2}\cdot d$$

$$q_{56} = -k\frac{T_6^p - T_5^p}{\Delta x}\cdot\frac{\Delta y}{2}\cdot d$$

$$q_{conv} = h\left(T_\infty - T_6^p\right)\cdot\frac{\Delta x}{2}\cdot d + h\left(T_\infty - T_6^p\right)\cdot\frac{\Delta y}{2}\cdot d$$

$$\dot{E}_{gen} = \dot{q}\cdot V = \dot{q}\cdot\frac{\Delta x\Delta y}{4}\cdot d$$

$$\dot{E}_{store} = mC_p\frac{\partial T}{\partial t} = \rho\left(\frac{\Delta x\Delta y}{4}\cdot d\right)C_p\frac{T_6^{p+1} - T_6^p}{\Delta t}$$

$$\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} \quad\rightarrow\quad \dot{E}_{out} = 0 \quad and \quad \dot{E}_{in} = q_{36} + q_{56} + q_{conv}$$

$$\rho\left(\frac{\Delta x\cdot\Delta y}{4}\right)c_p\left(\frac{T_6^{P+1} - T_6^P}{\Delta t}\right) = k\frac{\Delta y}{2}\left(\frac{T_5^P - T_6^P}{\Delta x}\right) + \frac{k\Delta x}{2}\left(\frac{T_3^P - T_6^P}{\Delta y}\right) + h\left(\frac{\Delta x}{2} + \frac{\Delta y}{2}\right)\left(T_\infty - T_6^P\right) + \dot{q}\left(\frac{\Delta x\cdot\Delta y}{4}\right)$$

$$Fo = \frac{\alpha\Delta t}{\Delta x^2} = \frac{k\Delta t}{\rho c_p\Delta x^2} \quad\rightarrow\quad T_6^{P+1} = T_6^P + Fo\left\{2T_5^P + 2T_3^P - 4T_6^P\right\} + \frac{4h\Delta t}{\rho c_P\Delta x}\left(T_\infty - T_6^P\right) + \frac{\dot{q}\Delta t}{\rho c_p}$$

$$\boxed{T_6^{P+1} = T_6^P + Fo\left\{2T_5^P + 2T_3^P - 4T_6^P\right\} + \frac{4h\Delta t}{\rho c_P\Delta x}\left(T_\infty - T_6^P\right) + \frac{\dot{q}\Delta t}{\rho c_p}}$$

(b)

Stability limit for 2-D explicit method: $\Delta t \le \dfrac{(\Delta x)^2}{4\alpha}$

$$\frac{(\Delta x)^2}{4\alpha} = \frac{(\Delta x)^2}{4\dfrac{k}{\rho C_p}} = \frac{(0.01\,m)^2}{4\cdot\dfrac{(100\,W/m-K)}{(2000\,kg/m^3)(300\,J/kg-K)}} = 0.15\,\text{sec}$$

$\therefore \Delta t \le 0.15\,\text{sec}$

Δt = 0.1 sec is suitable because the stability limit for the given situation is Δt ≤ 0.15 sec.

(c)

$T_1^P = T_2^P = T_3^P = T_4^P = T_5^P = T_6^P = 300\ K \quad and \quad T_7^P = T_8^P = 400\ K$

$T_\infty = 500\ K$

$\Delta x = \Delta y = 0.01\,m \quad and \quad \Delta t = 0.1\,s$

$$Fo = \frac{k\Delta t}{\rho c_P \Delta x^2} = \frac{(100\,W/m-K)(0.1\,s)}{(2000\,kg/m^3)(300\,J/kg-K)(0.01\,m)^2} = 0.167$$

$\dot{q} = 10^6\ W/m^3 \quad and \quad h = 100\ W/m^2K$

$$T_4^{P+1} = 316.8\ K$$

$$T_5^{P+1} = 311.7\ K$$

$$T_6^{P+1} = 301.5\ K$$

Problem 2

(a)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k\,r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho C_p\frac{\partial T}{\partial t}$$

Assuming 1-D conduction and steady state condition,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k\,r\frac{\partial T}{\partial r}\right)+\dot{q}=0 \quad\rightarrow\quad \frac{d}{dr}\left(k\,r\frac{dT}{dr}\right)=-\dot{q}\,r$$

$$k\,r\frac{dT}{dr}=-\frac{\dot{q}\,r^2}{2}+C_1 \quad\rightarrow\quad \frac{dT}{dr}=-\frac{\dot{q}\,r}{2k}+\frac{C_1}{k\,r}$$

$$BC\,1:\quad \left.\frac{dT}{dr}\right|_{r=0}=0 \quad\rightarrow\quad \therefore C_1=0$$

$$\frac{dT}{dr}=-\frac{\dot{q}\,r}{2k} \quad\rightarrow\quad \int dT=-\int\frac{\dot{q}\,r}{2k}dr$$

$$T=-\frac{\dot{q}}{2k}\frac{r^2}{2}+C_2 \quad\rightarrow\quad T(r)=-\frac{\dot{q}\,r^2}{4k}+C_2$$

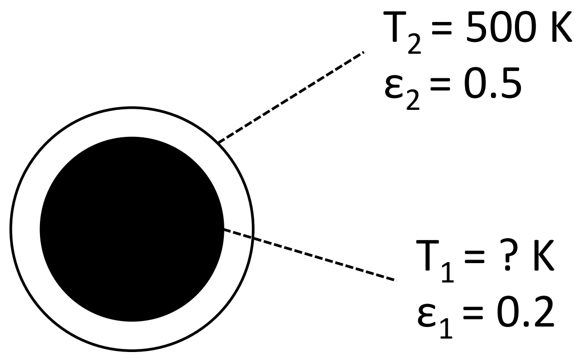
$$BC2:\quad T_{r=0}=T_c=C_2 \quad\rightarrow\quad \therefore C_2=T_c$$

$$\therefore T(r)=T_c-\frac{\dot{q}\,r^2}{4k}$$

$$T_1=T_{r=r_0}=T_c-\frac{\dot{q}\,r_0^2}{4k}$$

$$\boxed{T_1=T_c-\frac{\dot{q}\,r_0^2}{4k}}$$

(b)



$$\frac{1-\varepsilon_1}{A_1\varepsilon_1} \qquad \frac{1}{A_1F_{12}} \qquad \frac{1-\varepsilon_2}{A_2\varepsilon_2}$$

Assuming the length of the rod, L = 1m,

$$q = \dot{q}\cdot V = \dot{q}\cdot\frac{\pi D_1^2}{4}\cdot L = \left(20000\ W/m^3\right)\cdot\frac{\pi(0.05\ m)^2}{4}\cdot\left(1\ m\right) = 39.27\ W$$

$$\frac{E_{b,1}-E_{b,2}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}+\frac{1}{A_1F_{12}}+\frac{1-\varepsilon_2}{A_2\varepsilon_2}} = \frac{\sigma\left(T_1^4-T_2^4\right)}{\frac{1-0.2}{\pi\left(0.05\ m\right)\left(1\ m\right)\left(0.2\right)}+\frac{1}{\pi\left(0.05\ m\right)\left(1\ m\right)\left(1\right)}+\frac{1-0.5}{\pi\left(0.06\ m\right)\left(1\ m\right)\left(0.5\right)}}$$

$$= \frac{\left(5.67\times10^{-8}\ W/m^2-K^4\right)\left(T_1^4-500^4\right)K^4}{\frac{1-0.2}{\pi\left(0.05\ m\right)\left(1\ m\right)\left(0.2\right)}+\frac{1}{\pi\left(0.05\ m\right)\left(1\ m\right)\left(1\right)}+\frac{1-0.5}{\pi\left(0.06\ m\right)\left(1\ m\right)\left(0.5\right)}} = 39.27\ W \quad\rightarrow\quad T_1 = 544.99\ K$$

$$T_1 = T_c - \frac{\dot{q}\,r_0^2}{4k} = T_c - \frac{\left(20000\ W/m^3\right)(0.025\ m)^2}{4\cdot\left(15\ W/m-K\right)} = 544.99\ K \quad\rightarrow\quad T_c = 545.20\ K$$

$$\boxed{T_c = 545.20\ K \quad and \quad T_1 = 544.99\ K}$$

(c)

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{store}$$

$$\dot{E}_{in} = \dot{E}_{store} = 0$$

$$\therefore \dot{E}_{out} = \dot{E}_{gen}$$

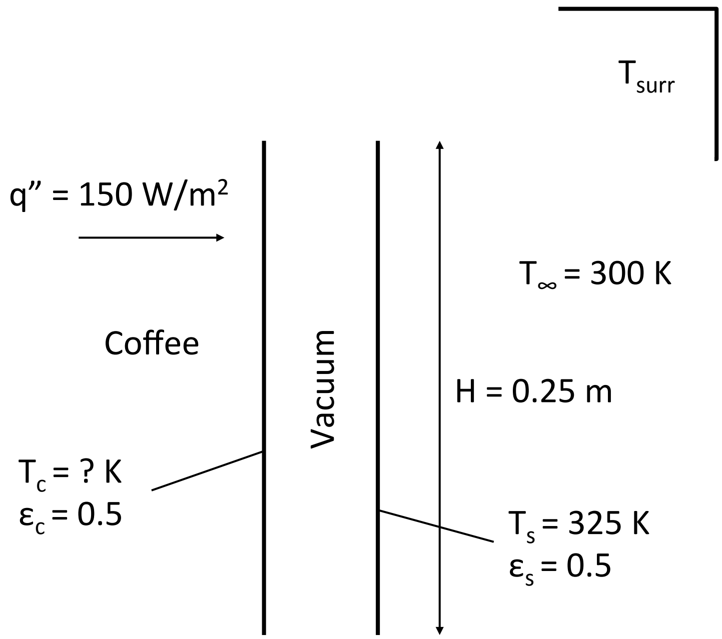
$$\dot{E}^{'}_{gen} = \dot{q} \cdot \frac{\pi D_1^2}{4} = \left(20000\ W/m^3\right) \cdot \frac{\pi (0.05\ m)^2}{4} = 39.27\ W/m$$

$$\therefore \dot{E}^{'}_{out} = q_1^{'} = \dot{E}^{'}_{gen} = 39.27\ W/m$$

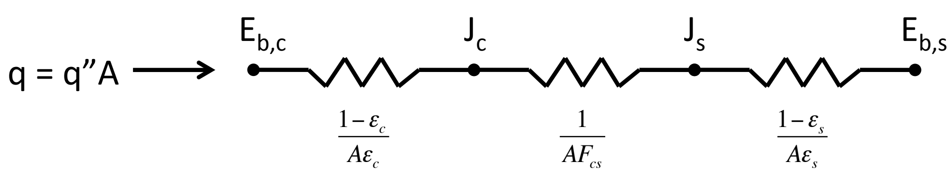
$$\boxed{\therefore q_1^{'} = 39.27\ W/m}$$

Problem 3

(a)



By neglecting the thickness of the container wall,



$$q''A = q_{rad} = \frac{\sigma\left(T_c^4 - T_s^4\right)}{\frac{1-\varepsilon_c}{\varepsilon_c A} + \frac{1}{AF_{cs}} + \frac{1-\varepsilon_s}{\varepsilon_s A}}\ ;\ F_{cs} = 1 \quad \rightarrow \quad q'' = \frac{\sigma\left(T_c^4 - T_s^4\right)}{\frac{1-\varepsilon_c}{\varepsilon_c} + 1 + \frac{1-\varepsilon_s}{\varepsilon_s}}$$

$$T_c = \left[\frac{q''}{\sigma}\left(\frac{1-\varepsilon_c}{\varepsilon_c} + 1 + \frac{1-\varepsilon_s}{\varepsilon_s}\right) + T_s^4\right]^{\frac{1}{4}} = 372\ K$$

$$\boxed{\therefore T_c = 372\ K}$$

(b)

$$q''_{conv} = h\left(T_s - T_\infty\right)$$

$$\mathrm{Ra} = \frac{g\beta\left(T_s - T_\infty\right)H^3}{\nu\alpha} = 3.01\times10^7$$

$$h = \mathrm{Nu}\frac{k}{H} = \frac{k}{H}\left(0.825 + \frac{0.387\,\mathrm{Ra}^{1/6}}{\left(1+\left(\frac{0.492}{\mathrm{Pr}}\right)^{9/16}\right)^{8/27}}\right)^2 \quad \rightarrow \quad h = 4.54\ W/m^2 - K$$

$$q''_{conv} = \left(4.54\,W/m^2 - K\right)\left(325\ K - 300\ K\right) = 114\ W/m^2$$

$$\therefore q''_{conv} = 114\ W/m^2$$

(c)

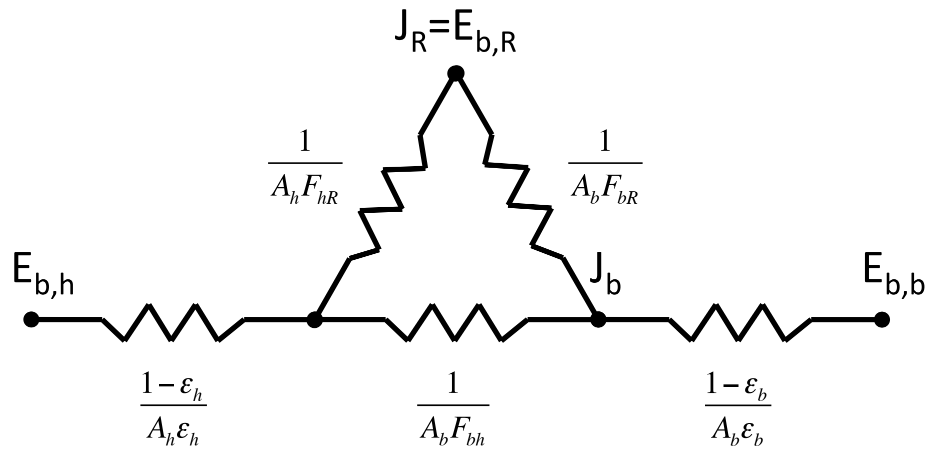
$$q'' = 150\ W/m^2 = q''_{conv} + q''_{rad} \quad \rightarrow \quad q''_{rad} = q'' - q''_{conv} = \varepsilon_s\sigma\left(T_s^4 - T_{surr}^4\right)$$

$$T_{surr} = \left(T_s^4 - \frac{q'' - q''_{conv}}{\varepsilon_s\alpha}\right)^{1/4} = 315\ K$$

$$\therefore T_{surr} = 315\ K$$

Problem 4

(a)



(b)

$$\overline{\mathrm{Nu}} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(0.4/\mathrm{Pr}\right)^{2/3}\right]^{1/4}} \cdot \left[1+\left(\frac{\mathrm{Re}}{282000}\right)^{5/8}\right]^{4/5}$$

$$\mathrm{Re} = \frac{\rho u D}{\mu} = \frac{\left(0.94\ kg/m^3\right)\left(5\ m/s\right)\left(0.2\ m\right)}{218\times10^{-7}\ N\cdot s/m^2} = 4311.93$$

$$\overline{\mathrm{Nu}} = 0.3 + \frac{0.62\left(4311.93\right)^{1/2}\left(0.7\right)^{3}}{\left[1+\left(0.4/0.7\right)^{2/3}\right]^{1/4}} \cdot \left[1+\left(\frac{4311.93}{282000}\right)^{5/8}\right]^{4/5} = 33.858 = \frac{\bar{h}\cdot D}{k} = \frac{\bar{h}\cdot\left(0.02\ m\right)}{\left(0.031\ W/mK\right)}$$

$$\therefore \bar{h} = 52.48\ W/m^2K$$

$$q_{conv} = \bar{h}\left(T_\infty - T_h\right)A_h; \quad A_h = \pi D_h L$$

$$q_{conv} = \bar{h}\left(T_\infty - T_h\right)A_h = \left(52.48\ W/m^2K\right)\left(375\ K - 350\ K\right)\pi\left(0.02\ m\right)\left(0.1\ m\right) = 8.24353\ W$$

$$\boxed{\therefore q_{conv} = 8.24\ W\ \ \text{(into the hotdog)}}$$

(c)

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{store}$$

$$\dot{E}_{gen} = \dot{E}_{out} = 0$$

$$\dot{E}_{store} = mC_p \frac{dT}{dt} = (0.03\, kg)(2500\, J/kg-K)(0.25\, K/s) = 18.75\, W$$

$$\dot{E}_{in} = q_{conv} + q_{rad}$$

$$\therefore \dot{E}_{store} = \dot{E}_{in} = q_{conv} + q_{rad} \quad \rightarrow \quad q_{rad} = \dot{E}_{store} - q_{conv} = \frac{E_{b,b} - E_{b,h}}{R_{tot}} = \frac{\sigma\left(T_b^4 - T_h^4\right)}{R_{tot}}$$

$$R_{tot} = \frac{1-\varepsilon_b}{\varepsilon_b A_b} + \frac{1-\varepsilon_h}{\varepsilon_h A_h} + \left[A_b F_{bh} + \left(\frac{1}{A_b F_{bR}} + \frac{1}{A_h F_{hR}}\right)^{-1}\right]^{-1}$$

$$F_{bR} = 1 - F_{bh} = 0.93$$

$$F_{hb} = \frac{A_b}{A_h} F_{bh} = 0.23$$

$$F_{hR} = 1 - F_{hb} = 0.77$$

$$\therefore R_{tot} = 558.2\, \frac{1}{m^2}$$

$$T_b = \left[\frac{\left(\dot{E}_{store} - q_{conv}\right) \cdot R_{tot}}{\sigma} + T_h^4\right]^{\frac{1}{4}} = 586.6\, K$$

$$\boxed{\therefore T_b = 586.6\, K}$$

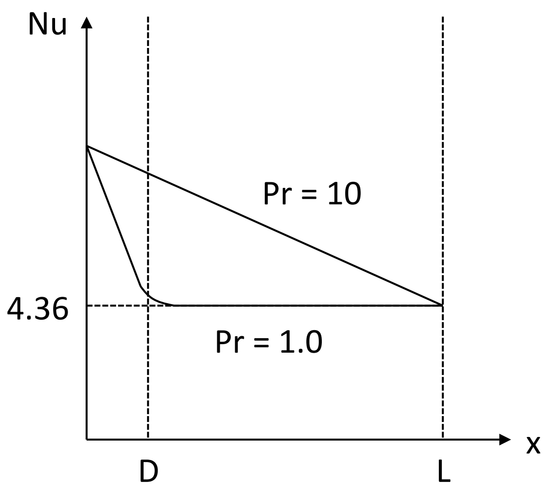
Problem 5

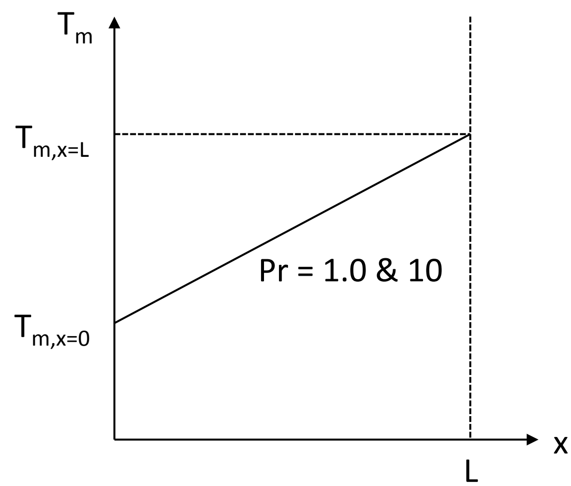
(a)

$$\left(\frac{x_{fd}}{D}\right)_{thermal} = 0.05\,\mathrm{Re}\,\mathrm{Pr} = \begin{cases} 0.05(20)(1.0) = 1 & (\mathrm{Pr} = 1.0) \\ 0.05(20)(10) = 10 & (\mathrm{Pr} = 10) \end{cases}$$

$$x_{fd,thermal} = \begin{cases} D & (\mathrm{Pr} = 1.0) \\ 10D & (\mathrm{Pr} = 10) \end{cases}$$

$$T_m = T_{m,x=0} + \frac{q''P}{\dot{m}c_p}x = T_{m,x=0} + (\mathrm{constant})\cdot x \quad \rightarrow \quad \textit{Linear Line}$$





(b)

$$T_m = \frac{\int_{A_c} \rho u c_p T \, dA_c}{\dot{m} c_p} = \frac{\rho u c_p \int_{A_c} T \, dA_c}{\rho u A_c c_p}$$

$$A_c = y \cdot d \quad and \quad \frac{dA_c}{dy} = d \quad \rightarrow \quad dA_c = d \cdot dy$$

$$T_m = \frac{\int_{-H}^{H} T \cdot d \, dy}{2H \cdot d} = \frac{\int_{-H}^{H} \frac{q''}{2k_f H}\left(y^2 - H^2\right) + T_w \, dy}{2H} = \frac{1}{2H}\left[\frac{q''}{2k_f H}\left(\frac{1}{3}y^3 - H^2 y\right) + T_w y\right]\Bigg|_{-H}^{H}$$

$$= \frac{1}{2H}\left\{\left[\frac{q''}{2k_f H}\left(\frac{1}{3}H^3 - H^3\right) + T_w H\right] - \left[\frac{q''}{2k_f H}\left(-\frac{1}{3}H^3 + H^3\right) - T_w H\right]\right\}$$

$$= \frac{1}{2H}\left[\frac{q''}{2k_f H}\left(-\frac{2}{3}H^3\right) + T_w H - \frac{q''}{2k_f H}\left(\frac{2}{3}H^3\right) + T_w H\right]$$

$$= \frac{1}{2H}\left[2T_w H - \frac{q''}{2k_f H}\left(\frac{4}{3}H^3\right)\right] = \frac{1}{2H}\left[2T_w H - \frac{q''}{k_f}\left(\frac{2}{3}H^2\right)\right] = \frac{1}{2H}\left[2T_w H - \frac{2q''}{3k_f}H^2\right]$$

$$= T_w - \frac{q''}{3k_f}H \quad \rightarrow \quad T_m = T_w - \frac{q''}{3k_f}H$$

$$q'' = h\left(T_w - T_m\right) \quad \rightarrow \quad h = \frac{q''}{T_w - T_m} = \frac{q''}{T_w - \left(T_w - \frac{q''}{3k_f}H\right)} = \frac{q''}{\frac{q''}{3k_f}H} = \frac{3k_f}{H}$$

$$\boxed{\therefore h = \frac{3k_f}{H}}$$