ME 315
Final Exam
Friday, May 3, 2013

- This is a closed-book, closed-notes examination. *There is a formula sheet provided at the end of this exam.*
- You must turn off all communications devices before starting this exam, and leave them off for the entire exam.
- Please write legibly and show all work for receiving partial credit. Please show your final answers in the boxes provided.
- State all assumptions.
- Please arrange all your sheets in the correct order. Make sure they are all included.

Name: ______________________________________
Last **First**

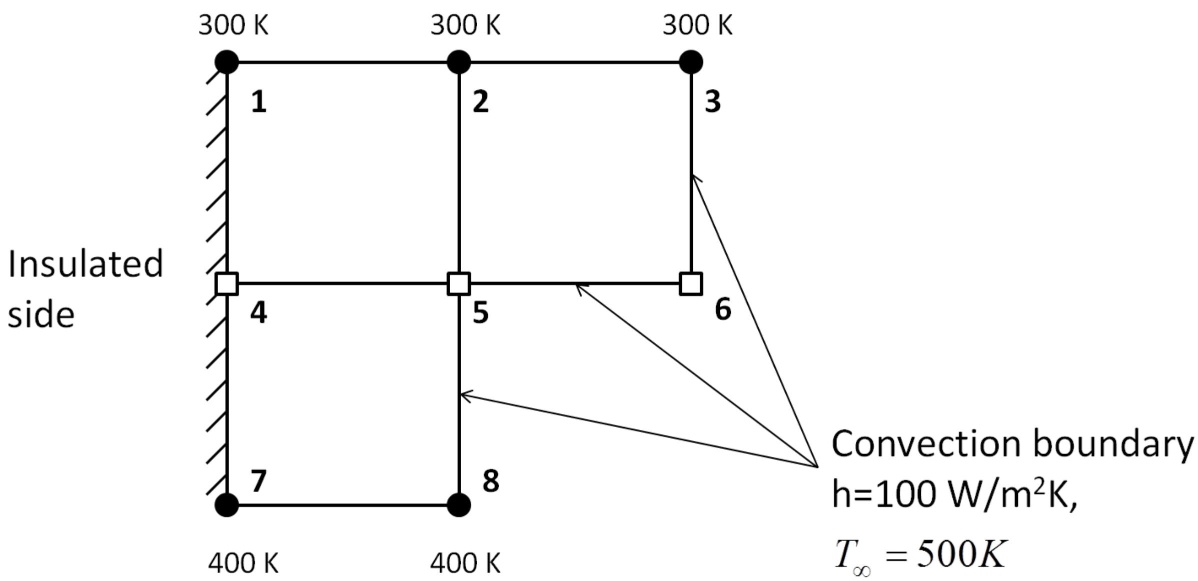
CIRCLE YOUR DIVISION

Div. 1 (9:30 am)
Prof. Choi

Div. 2 (12:30 pm)
Prof. Ritchey

Problem	Score
1 (20 Points)	
2 (20 Points)	
3 (20 Points)	
4 (20 Points)	
5 (20 Points)	
Total (100 Points)	

1. Consider unsteady heat conduction in a two-dimensional domain given below. The domain is meshed with a square mesh with $\Delta x = \Delta y = 1$ cm. At time t = 0, the boundary conditions given below are applied such that $T_1 = T_2 = T_3 = 300$ K, and $T_7 = T_8 = 400$ K. The initial temperatures at nodes 4, 5 and 6 can be assumed to be 300 K. Also assume that there is a uniform heat generation within the domain given by $\dot{q} = 10^6 W/m^3$. You are asked to determine how the temperatures of grid points 4, 5 and 6 change with time using an explicit time stepping scheme. You are given the following material properties: ρ = 2000 kg/m^3, k = 100 W/mK, c_p = 300 J/kgK.



(a) Using the energy balance method and an explicit time-stepping scheme, develop analytical expressions for T_4, T_5 and T_6 at time step t = Δt in terms of the neighbor temperatures, material properties and mesh parameters. Show <u>analytical expressions</u> for the discrete equations here.

T_4 (Δt) = T_5 (Δt) = T_6 (Δt) =

(b) You are given the choice of two time steps, $\Delta t = 0.1$ s and $\Delta t = 100$ s. Which one would you choose, and why?

$\Delta t =$ seconds
Reason:

(c) Using the value of Δt you chose in part (b), determine the numerical values of the temperatures T_4, T_5 and T_6 at $t=\Delta t$ using the explicit scheme.

$T_4 (\Delta t) =$	K
$T_5 (\Delta t) =$	K
$T_6 (\Delta t) =$	K

2. A long uniform rod of 50-mm diameter with a thermal conductivity of k = 15 W/mK is heated internally by volumetric energy generation of $\dot{q}$ = 20 kW/m^3. The rod is positioned coaxially within a larger circular tube of 60-mm diameter whose surface is maintained at T_2 = 500 K. The annular region between the rod and the tube is evacuated, and their surfaces are diffuse and gray. The emissivity of the rod is ε_1 = 0.2 and that of the tube is ε_2 = 0.5.

(a) Derive an analytical expression relating the center temperature of the rod, T_c, the surface temperature of the rod, T_1, and the volumetric heat generation rate, $\dot{q}$.

(b) Determine the center and surface temperatures of the rod.

T_c = K

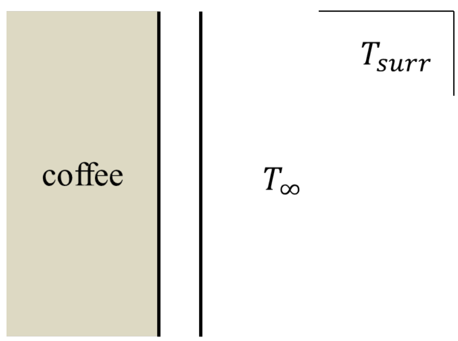
T_1 = K

(c) Determine the net radiation energy per unit length leaving the surface of the rod, q_1' (W/m).

q_1' = W/m

3. A container of coffee is placed in a room of quiescent air at T_∞ = 300 K. A very thin vertical flat plate radiation shield of T_s = 325 K with ε_s = 0.5 is placed around the container such that a vacuum exists between the thin wall of the container (ε_c = 0.5) and the shield. 150 W/m^2 of power is supplied to the coffee so that it maintains a constant temperature.

Find (a) the temperature of the coffee, (b) the heat flux due to free convection on the outer surface of the radiation shield, and (c) the temperature of the surroundings. The height of the walls is 25 cm.



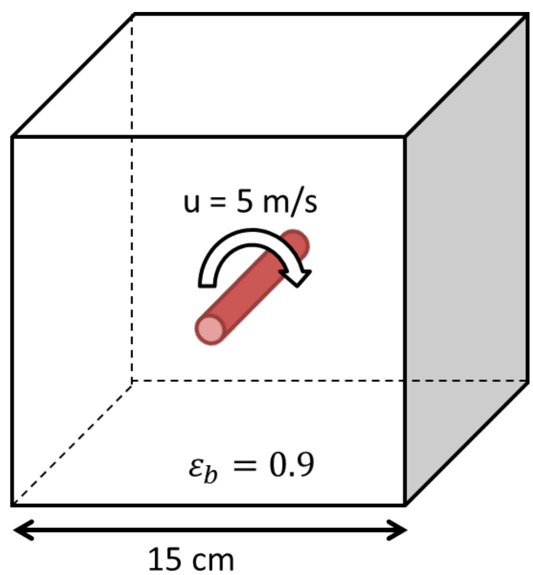
Air properties:
Pr = 0.7
ν = 17x10^{-6} m^2/s
α = 24x10^{-6} m^2/s
k = 0.027 W/mK

(a) T_c = K

(b) q''_{conv} = W/m^2

(c) T_{surr} = K

4. A single hotdog of mass m = 30 g is cooked inside a convection oven. The hotdog is represented as a cylinder with a diameter of 2 cm and length of 10 cm with an emissivity of ε_h = 0.3 and a specific heat of c_p = 2500 J/kgK. The convection oven is a cube with a side length of 15 cm. The bottom wall of the oven has an emissivity of ε_b = 0.9 while the other five walls are reradiating surfaces. At a given time, the hotdog has a temperature of T_h = 350 K and increases at a rate of dT/dt = 0.25 K/s. Air flows around the hotdog at a temperature of T_∞ = 375 K and velocity of u = 5 m/s. There is no convection heat transfer with the bottom wall or the end caps of the hotdog.



Air properties:
Pr = 0.7
ρ = 0.94 kg/m^3
μ = 218x10^{-7} Pa s
k = 0.031 W/mK

(a) Sketch the radiation network of the system showing the resistances using symbol.

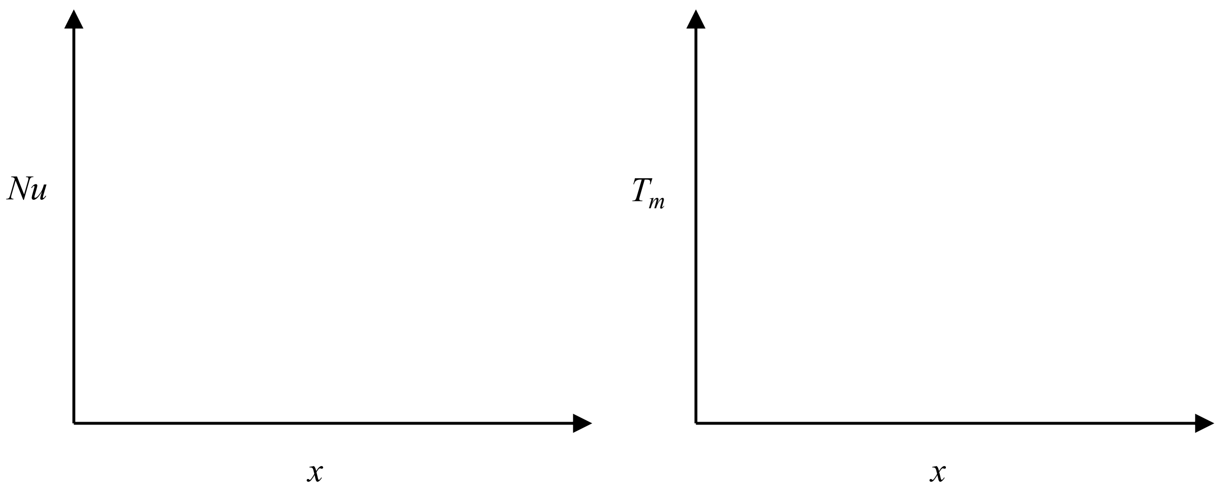
(b) Find the heat transfer rate due to convection around the hotdog.

q_{conv} = W

(c) Determine the temperature of the bottom surface. The view factor from the bottom surface to the hotdog is $F_{bh} = 0.07$.

T_b = K

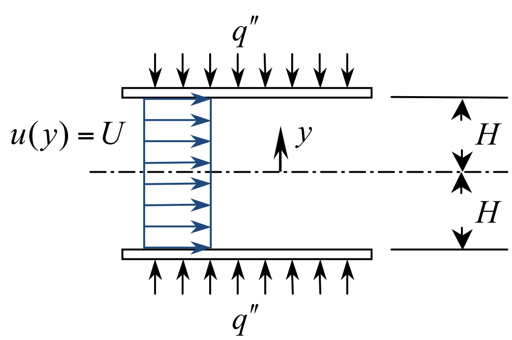
5. (a) Answer the following questions with respect to internal flow in a circular tube under constant surface heat flux and $Re_D = 20$ conditions. The length of the tube, $L = 10D$, where D is the diameter. On the first set of axes, sketch the variation of Nusselt number, $Nu(x)$, along the length of the tube for two values of $Pr = 1$ and 10. On the second set of axes, sketch the variation of the mean temperature, $T_m(x)$, along the length of the tube for $Pr = 1$ and 10. Clearly label all the plots.
Hint: The thermal entry length can be calculated as $(x_{fd}/D) = 0.05 Re_D Pr$.



(b) Find the convection coefficient h for a parallel-plates duct having the following characteristics:

- plate spacing $2H$
- a “uniform” velocity profile, *i.e.*, $u(y) = U$
- uniformly applied heat flux q'' at the walls
- a fully developed temperature profile, *i.e.*,

$$T(y) = \frac{q''}{2k_f H}\left(y^2 - H^2\right) + T_w$$



where T_w is the wall temperature, k_f is the fluid thermal conductivity, and the vertical coordinate y is zero at the centerline. Hint: calculate the mean fluid temperature T_m by utilizing its definition. The convection coefficient h can be related to heat flux q'' and the temperature difference $(T_w - T_m)$.

$h =$

ME 315 Final Exam
BASIC EQUATION SHEET

Conservation Laws

Control Volume Energy Balance: $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$; $\dot{E}_{st} = mC_p \, {dT}/{dt}$; $\dot{E}_{gen} = \dot{q}V$

Surface Energy Balance: $\dot{E}_{in} - \dot{E}_{out} = 0$

Conduction

Fourier's Law: $q''_{cond,x} = -k\frac{\partial T}{\partial x}$; $q''_{cond,n} = -k\frac{\partial T}{\partial n}$; $q_{cond} = q''_{cond}A$

Heat Flux Vector: $\vec{q''} = q''_x\vec{i} + q''_y\vec{j} + q''_z\vec{k} = -k\left[\frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k}\right]$

Heat Diffusion Equation:

Rectangular Coordinates: $\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho C_p\frac{\partial T}{\partial t}$

Cylindrical Coordinates: $\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho C_p\frac{\partial T}{\partial t}$

Spherical Coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho C_p\frac{\partial T}{\partial t}$$

Thermal Resistance Concepts:

Conduction Resistance: $R_{t,cond} \overset{plane\,wall}{=} \frac{L}{kA}$; $R_{t,cond} \overset{cylinder}{=} \frac{\ln\left(r_o/r_i\right)}{2\pi lk}$; $R_{t,cond} \overset{sphere}{=} \frac{\left(1/r_i\right) - \left(1/r_o\right)}{4\pi k}$

Convection Resistance: $R_{t,conv} \overset{plane\,wall}{=} \frac{1}{h_{conv}A}$; $R_{t,conv} \overset{cylinder}{=} \frac{1}{2\pi rlh_{conv}}$; $R_{t,conv} \overset{sphere}{=} \frac{1}{4\pi r^2 h_{conv}}$

Radiation Resistance: $R_{t,rad} \overset{plane\,wall}{=} \frac{1}{h_{rad}A}$; $R_{t,rad} \overset{cylinder}{=} \frac{1}{2\pi rlh_{rad}}$; $R_{t,rad} \overset{sphere}{=} \frac{1}{4\pi r^2 h_{rad}}$

Combined Convection and Radiation Surface: $\frac{1}{R_{conv+rad}} = \frac{1}{R_{t,conv}} + \frac{1}{R_{t,rad}}$

Contact Resistance: $R_{t,contact} = \frac{1}{h_{contact}A_{contact}}$

Thermal Energy Generation:

Plane wall:

$$T(x) - T_s = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right)$$

Cylinder:

$$T(r) - T_s = \frac{\dot{q}r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right)$$

Extended Surfaces:

Convective Tip: $\dfrac{\theta(x)}{\theta_b} = \dfrac{\cosh\left[m(L-x)\right] + (h/mk)\sinh\left[m(L-x)\right]}{\cosh(mL) + (h/mk)\sinh(mL)}$

$$q_{fin} = (hPkA_c)^{1/2}\,\theta_b \frac{\sinh(mL) + (h/mk)\cosh(mL)}{\cosh(mL) + (h/mk)\sinh(mL)}$$

Adiabatic Tip: $\dfrac{\theta(x)}{\theta_b} = \dfrac{\cosh\left[m(L-x)\right]}{\cosh(mL)}$; $q_{fin} = (hPkA_c)^{1/2}\,\theta_b \tanh(mL)$

Prescribed Tip Temperature: $\dfrac{\theta(x)}{\theta_b} = \dfrac{(\theta_L/\theta_b)\sinh(mx) + \sinh\left[m(L-x)\right]}{\sinh(mL)}$

$$q_{fin} = (hPkA_c)^{1/2}\,\theta_b \frac{\cosh(mL) - (\theta_L/\theta_b)}{\sinh(mL)}$$

Infinitely Long Fin: $\dfrac{\theta(x)}{\theta_b} = e^{-mx}$; $q_{fin} = (hPkA_c)^{1/2}\,\theta_b$

$$m^2 = \frac{hP}{kA_c};\ \theta_b = T_b - T_\infty;\ q_{fin} = q_{conv,finsurface} + q_{conv,tip};\ q_{conv,tip} = hA_c\theta_L$$

Fin Effectiveness: $\varepsilon_{fin} = \dfrac{q_{fin}}{hA_{c,b}\theta_b}$; $\varepsilon_{fin} = \dfrac{R_{t,conv-base}}{R_{t,cond-fin}}$

Fin Efficiency: $\eta_{fin} = \dfrac{q_{fin}}{hA_{fin}\theta_b}$; $\eta_{fin} \overset{adiabatic}{=} \dfrac{\tanh(mL)}{mL}$; $L_c = L + \dfrac{A_c}{P}$; $\eta_{fin} = \dfrac{\tanh(mL_c)}{mL_c}$

$$\eta_o = \frac{q_{total}}{hA_{total}\theta_b} = 1 - \frac{NA_{fin}}{A_{total}}\left(1 - \eta_{fin}\right);\ R_{t,cond-fin} = \frac{1}{\eta_{fin} hA_{fin}};\ R_{t,cond-fin\,array} = \frac{1}{\eta_o hA_{total}}$$

Two Dimensional Steady Conduction:

Conduction Shape Factor: $R_{t,cond} \overset{2D}{=} \dfrac{1}{Sk}$

Finite Difference Method
(uniform mesh, interior point, no gen., steady): $T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} = 4T_{i,j}$

Transient Conduction:

Lumped System Analysis: $Bi = \dfrac{R_{t-cond}}{R_{t-conv}} = \dfrac{h_{conv}L_c}{k_{solid}}$; $\dfrac{\theta}{\theta_i} = \dfrac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\dfrac{t}{\tau_t}\right)$; $Fo = \dfrac{\alpha t}{L_c^2}$

$$\frac{\theta}{\theta_i} = \exp\left[-\left(\frac{h_{conv}L_c}{k_{solid}}\right)\left(\frac{\alpha t}{L_c^2}\right)\right] = \exp\left[-(Bi)(Fo)\right];\ \tau_t = \frac{\rho V C_p}{h_{conv}A_s} = C_{t,solid}R_{t,conv};$$

Analytical Solutions: $\theta^* = \dfrac{\theta}{\theta_i} = \dfrac{T - T_\infty}{T_i - T_\infty}$; $x^* = \dfrac{x}{L}$; $r^* = \dfrac{r}{r_o}$; $t^* = \dfrac{\alpha t}{L^2}$

Plane Wall: $\theta^* \underset{plane\,wall}{\cong} C_1 \exp\left(-\zeta_1^2 Fo\right)\cos\left(\zeta_1 x^*\right)$; $\theta_o^* \overset{plane\,wall}{=} C_1 \exp\left(-\zeta_1^2 Fo\right)$;

$\frac{Q}{Q_o} \overset{plane\,wall}{=} 1-\theta_o^* \frac{\sin\left(\zeta_1\right)}{\zeta_1}$

Long Cylinder: $\theta^* \underset{cylinder}{\cong} C_1 \exp\left(-\zeta_1^2 Fo\right) J_0\left(\zeta_1 r^*\right)$; $\theta_o^* \overset{cylinder}{=} C_1 \exp\left(-\zeta_1^2 Fo\right)$;

$\frac{Q}{Q_o} \overset{cylinder}{=} 1-2\theta_o^* \frac{J_1\left(\zeta_1\right)}{\zeta_1}$

Sphere: $\theta^* \underset{sphere}{\cong} C_1 \exp\left(-\zeta_1^2 Fo\right)\frac{\sin\left(\zeta_1 r^*\right)}{\zeta_1 r^*}$; $\theta_o^* \overset{sphere}{=} C_1 \exp\left(-\zeta_1^2 Fo\right)$;

$\frac{Q}{Q_o} \overset{sphere}{=} 1-3\theta_o^* \frac{\left[\sin\left(\zeta_1\right)-\zeta_1 \cos\left(\zeta_1\right)\right]}{\zeta_1^3}$

Semi-infinite Solid:

Constant Surface Temperature: $\frac{T\left(x,t\right)-T_s}{T_i - T_s} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right)$; $q_s^{"} = -k \left.\frac{\partial T}{\partial x}\right|_{x=0} = \frac{k\left(T_s - T_i\right)}{\sqrt{\pi \alpha t}}$

Constant Surface Heat Flux: $T\left(x,t\right)-T_i = \frac{2q_0^{"}\left(\alpha t/\pi\right)^{1/2}}{k}\exp\left(-\frac{x^2}{4\alpha t}\right)-\frac{q_0^{"}x}{k} erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$

Convection: $\frac{T\left(x,t\right)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)-\left[\exp\left(\frac{hx}{k}+\frac{h^2 \alpha t}{k^2}\right)\right]\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$

Finite Difference Method:

Explicit Method: $T_{i,j}^{P+1} \overset{explicit}{=} \left(1-4Fo\right)T_{i,j}^{P} + Fo\left(T_{i+1,j}^{P}+T_{i-1,j}^{P}+T_{i,j+1}^{P}+T_{i,j-1}^{P}\right)$

Implicit Method: $T_{i,j}^{P} \overset{implicit}{=} \left(1+4Fo\right)T_{i,j}^{P+1} - Fo\left(T_{i+1,j}^{P+1}+T_{i-1,j}^{P+1}+T_{i,j+1}^{P+1}+T_{i,j-1}^{P+1}\right)$

Stability Limits: $\Delta t \underset{explicit}{\overset{1D}{\le}} \frac{\left(\Delta x\right)^2}{2\alpha}$; $\Delta t \underset{explicit}{\overset{2D}{\le}} \frac{\left(\Delta x\right)^2}{4\alpha}$; $\Delta t \underset{explicit}{\overset{3D}{\le}} \frac{\left(\Delta x\right)^2}{6\alpha}$

<u>Convection</u>

Newton's Law of Cooling: $q_{conv}^{"} = h_{conv}\left(T_s - T_\infty\right)$; $q_{conv} = q_{conv}^{"} A$

Mass Transfer: $n_A^{"} = h_m\left(\rho_{A,s}-\rho_{A,\infty}\right)$; $q_{evap} = n_A^{"} A h_{fg}$

Average Heat Transfer Coefficient: $\overline{h_{conv}} = \frac{1}{A_s}\int_{A_S} h_{conv} dA_s$

Average Mass Transfer Coefficient: $\overline{h_m} = \frac{1}{A_s}\int_{A_S} h_m dA_s$

Dimensionless Parameters:

Reynolds Number: $Re_{L_c} = \frac{\rho V L_c}{\mu} = \frac{V L_c}{\nu}$;

Prandtl Number: $Pr = \frac{\nu}{\alpha}$; Schmidt Number: $Sc = \frac{\nu}{D_{AB}}$; Lewis Number: $Le = \frac{\alpha}{D_{AB}}$

Nusselt Number: $Nu = \frac{h_{conv} L_c}{k_{fluid}}$; Sherwood Number: $Sh = \frac{h_m L_c}{D_{AB}}$

Boundary Layer Thickness: $\frac{\delta}{\delta_t} \approx Pr^n$; $\frac{\delta}{\delta_c} \approx Sc^n$; $\frac{\delta_t}{\delta_c} \approx Le^n$

Heat-Mass Analogy: $\frac{Nu}{Sh} = \frac{Pr^n}{Sc^n}$; $\frac{h}{h_m} = \frac{k}{D_{AB} Le^n} = \rho C_p Le^{1-n}$

External Flow:

Flat Plate

Flat Plate (Laminar Local): $\delta \overset{flat\ plate}{\underset{laminar}{=}} \frac{5x}{Re_x^{1/2}}$; $C_{f,x} \overset{flat\ plate}{\underset{laminar}{=}} 0.664 Re_x^{-1/2}$;

$Nu_x \overset{isothermal\ flat\ plate}{\underset{laminar}{=}} 0.332 Re_x^{1/2} Pr^{1/3}$

Flat Plate (Laminar Average): $\overline{C_{f,L}} \overset{flat\ plate}{\underset{laminar}{=}} 1.328 Re_L^{-1/2}$; $\overline{Nu_L} \overset{isothermal\ flat\ plate}{\underset{laminar}{=}} 0.664 Re_L^{1/2} Pr^{1/3}$;

$\overline{Sh_L} \overset{flat\ plate}{\underset{laminar}{=}} 0.664 Re_L^{1/2} Sc^{1/3}$

Flat Plate (Turbulent Local): $\delta \overset{flat\ plate}{\underset{turbulent}{=}} \frac{0.37x}{Re_x^{1/5}}$; $C_{f,x} \overset{flat\ plate}{\underset{turbulent}{=}} 0.0592 Re_x^{-1/5}$;

$Nu_x \overset{isothermal\ flat\ plate}{\underset{turbulent}{=}} 0.0296 Re_x^{4/5} Pr^{1/3}$;

Flat Plate (Turbulent Average): $\overline{C_{f,L}} \overset{flat\ plate}{\underset{turbulent}{=}} 0.074 Re_L^{-1/5}$;

$\overline{Nu_L} \overset{isothermal\ flat\ plate}{\underset{turbulent}{=}} 0.037 Re_L^{4/5} Pr^{1/3}$

Flat Plate (Mixed Average): $\overline{C_{f,L}} \overset{flat\ plate}{\underset{mixed}{=}} 0.074 Re_L^{-1/5} - 1742 Re_L^{-1}$;

$\overline{Nu_L} \overset{isothermal\ flat\ plate}{\underset{mixed}{=}} \left(0.037 Re_L^{4/5} - 871\right) Pr^{1/3}$

Cylinder:

Cylinder: $\overline{Nu_D} \overset{cylinder}{=} 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{\left[1 + \left(0.4/Pr\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282000}\right)^{5/8}\right]^{4/5}$ for $ReDPr$>0.2;

Sphere:

$$\bar{N}u_D = 2 + \left(0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3}\right)\mathrm{Pr}^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4}$$

Internal Flow:

Mean Velocity: $u_m = \dfrac{\int_{A_c} \rho u(r,x)\,dA_c}{\rho A_c}$; $u_m \overset{circular}{=} \dfrac{2}{r_o^2}\int_0^{r_o} u(r,x)\,r\,dr$

Reynolds Number: $Re_{D_h} = \dfrac{u_m D_h}{\nu}$; $D_h = \dfrac{4A_c}{P}$; $Re_D \overset{circular}{=} \dfrac{u_m D}{\nu}$

Turbulent: $Re_D \geq 2{,}300$

Hydrodynamic Entrance Lengths: $\left(\dfrac{x_{fd,hydrodynamic}}{D}\right)^{laminar} = 0.05Re_D$; $60 > \left(\dfrac{x_{fd,hydrodynamic}}{D}\right)^{turbulent} > 10$

Thermal Entrance Lengths: $\left(\dfrac{x_{fd,thermal}}{D}\right)^{laminar} = 0.05Re_D Pr$; $60 > \left(\dfrac{x_{fd,thermal}}{D}\right)^{turbulent} > 10$

Mean (Bulk) Temperature: $T_m = \dfrac{\int_{A_c} \rho u C_p T\,dA_c}{\dot{m} C_p}$; $T_m \overset{circular}{=} \dfrac{2}{u_m r_o^2}\int_0^{r_o} uT(r,x)\,r\,dr$

Constant Heat Flux: $T_m(x) = T_{m,i} + \dfrac{q''_{conv}P}{\dot{m}C_p}x = T_{m,i} + \dfrac{q''_s P}{\dot{m}C_p}x$; $q_{conv} = q''_s A_s = q''_s(PL)$

Constant Surface Temperature: $\dfrac{T_s - T_{m,x}}{T_s - T_{m,i}} = \exp\left(-\dfrac{Px}{\dot{m}C_p}\overline{h_{conv}}\right)$; $\Delta T_{LMTD} = \dfrac{\Delta T_o - \Delta T_i}{\ln\left(\dfrac{\Delta T_o}{\Delta T_i}\right)}$

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}C_p}\overline{h_{conv}}\right);\ q_{conv} = \overline{h_{conv}}A_s\Delta T_{LMTD} = \dot{m}C_p\left(T_{m,o} - T_{m,i}\right)$$

Circular Pipe

Fanning Friction Factor :

$f^{laminar} = \dfrac{64}{Re_D}$; $f \overset{turbulent}{=} \dfrac{0.316}{Re_D^{1/4}}$ for $Re_D < 2\mathrm{x}10^4$; $f \overset{turbulent}{=} \dfrac{0.184}{Re_D^{1/5}}$ for $Re_D > 2\mathrm{x}10^4$

Laminar Fully-developed Region: $Nu_D \underset{q''=constant}{\overset{laminar}{=}} 4.36$; $Nu_D \underset{T_s=constant}{\overset{laminar}{=}} 3.66$

Laminar Entrance Region: $\overline{Nu_D} \overset{T_s=constant}{=} 3.66 + \dfrac{0.0668(D/L)Re_D Pr}{1 + 0.04\left[(D/L)Re_D Pr\right]^{2/3}}$;

$\overline{Nu_D} \overset{T_s=constant}{=} 1.86\left(\dfrac{Re_D Pr}{L/D}\right)^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$ (Both applicable to q_s'' = constant)

Turbulent Fully-Developed Region: $Nu_D \overset{turbulent}{=} 0.023Re_D^{4/5}Pr^n$; $n \overset{fluid\ cooling}{=} 0.3$

$n \overset{fluid\ heating}{=} 0.4$ (Applicable to q_s'' = constant and T_s = constant)

Free Convection:

Boundary Layer Parameters: $\eta = \frac{y}{x}\left(\frac{Gr_x}{4}\right)^{1/4}$; $u = \frac{2\nu}{x}\left(Gr_x\right)^{1/2} f'\left(\eta\right)$

$Gr_x = \frac{g\beta\left(T_s - T_\infty\right)x^3}{\nu^2}$; $Ra_x = Gr_x Pr = \frac{g\beta\left(T_s - T_\infty\right)x^3}{\nu\alpha}$

Vertical Flat Plate: $\overline{Nu_L} \overset{isothermal\ vertical\ plate}{=} \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + \left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$

Horizontal Flat Plate: $L_c = A_s/P$; $\overline{Nu_{L_c}} \underset{upper\,hot\,/\,lower\,cold}{\overset{isothermal\ horizontal\ plate}{=}} 0.54 Ra_{L_c}^{1/4}$;

$\overline{Nu_{L_c}} \underset{upper\,hot\,/\,lower\,cold}{\overset{isothermal\ horizontal\ plate}{=}} 0.15 Ra_{L_c}^{1/3}$; $\overline{Nu_{L_c}} \underset{lower\,hot\,/\,upper\,cold}{\overset{isothermal\ horizontal\ plate}{=}} 0.27 Ra_{L_c}^{1/4}$

Horizontal Cylinder: $\overline{Nu_D} \overset{isothermal\ horizontal\ cylinder}{=} C Ra_D^n$

Sphere: $\overline{Nu_D} \overset{isothermal\ sphere}{=} 2 + \frac{0.589 Ra_D^{1/4}}{\left[1 + \left(0.469/Pr\right)^{9/16}\right]^{4/9}}$

Boiling:

Basic Equation: $q_{boiling} = h_{boiling} A \Delta T_e = h_{boiling} A\left(T_s - T_{sat}\right)$

Nucleate Boiling: $q_s'' \underset{boiling}{\overset{nucleate}{=}} \mu_l h_{fg}\left[\frac{g\left(\rho_l - \rho_v\right)}{\sigma}\right]^{1/2}\left[\frac{c_{p,l}\Delta T_e}{C_{sf} h_{fg} Pr_l^n}\right]^3$;

$q_{max}'' \underset{boiling}{\overset{nucleate}{=}} \frac{\pi}{24} h_{fg}\rho_v\left[\frac{\sigma g\left(\rho_l - \rho_v\right)}{\rho_v^2}\right]^{1/4}$

Minimum Heat Flux: $q_{min}'' = 0.09 h_{fg}\rho_v\left[\frac{\sigma g\left(\rho_l - \rho_v\right)}{\left(\rho_l + \rho_v\right)^2}\right]^{1/4}$

Film Boiling: $\overline{Nu_D} = \frac{\overline{h_D} D}{k_v} \underset{boiling}{\overset{film}{=}} C\left[\frac{g\left(\rho_l - \rho_v\right) h_{fg}' D^3}{\nu_v k_v\left(T_s - T_{sat}\right)}\right]^{1/4}$;

$h_{fg}' = h_{fg} + 0.8 C_{p,v}\left(T_s - T_{sat}\right)$; $C \overset{cylinder}{=} 0.62$; $C \overset{sphere}{=} 0.67$

Condensation:

Basic Equation: $q_{condensation} = h_{condensation} A \Delta T_d = h_{condensation} A\left(T_{sat} - T_s\right)$; $\dot{m}_{condensation} = \frac{q_{condensation}}{h_{fg}'}$

Film Condensation:

Vertical Flat Plate: $\overline{Nu_L} = \frac{\overline{h_L} L}{k_l} \overset{laminar}{\underset{film\ condensation}{=}} 0.943 \left[\frac{g\left(\rho_l - \rho_v\right) h_{fg}^{'} L^3}{\nu_l k_l \left(T_{sat} - T_s\right)} \right]^{1/4};$

$$h_{fg}^{'} = h_{fg} + 0.68 C_{p,l} \left(T_{sat} - T_s\right)$$

Heat Exchangers:

$$\Delta T_{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}} \quad q = UA \Delta T_{LMTD}$$

Heat exchanger effectiveness: $\varepsilon = \frac{q}{q_{\max}} = \frac{q}{C_{\min}\left(T_{hi} - T_{ci}\right)}$

Number of transfer units: NTU = $\frac{UA}{C_{\min}}$

Radiation

Two Surface Interaction:

$$d\omega_{2-1} = \frac{dA_{2,normal}}{r^2} = \frac{A_2 cos\theta_2}{r^2}; \ q_{1-2} = I_e \left(A_1 cos\theta_1\right) d\omega_{2-1}$$

Emissive Power:

$$E_\lambda(\lambda) = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi) cos\theta sin\theta d\theta; \ E = \int_0^\infty E_\lambda(\lambda) d\lambda$$

$$E_\lambda(\lambda) \overset{diffuse\ \ emitter}{=} \pi I_{\lambda,e}(\lambda); \ E \overset{diffuse\ \ emitter}{=} \pi I_e$$

Irradiation:

$$G_\lambda(\lambda) = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi) cos\theta sin\theta d\theta; \ G = \int_0^\infty G_\lambda(\lambda) d\lambda$$

$$G_\lambda(\lambda) \overset{diffuse\ irradiation}{=} \pi I_{\lambda,i}(\lambda); \ G \overset{diffuse\ irradiation}{=} \pi I_i$$

Radiosity:

$$J_\lambda(\lambda) = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,e+r}(\lambda,\theta,\phi) cos\theta sin\theta d\theta; \ J = \int_0^\infty J_\lambda(\lambda) d\lambda$$

$$J_\lambda(\lambda) \overset{diffuse\ emitter}{\underset{diffuse\ reflector}{=}} \pi I_{\lambda,e+r}(\lambda); \ J \overset{diffuse\ emitter}{\underset{diffuse\ reflector}{=}} \pi I_{e+r}$$

Black Body Emission:

$$E_{\lambda,b}(\lambda,T) \overset{Black\ Body}{=} \frac{C_1}{\lambda^5 \left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right]}; \ E_b(T) \overset{Black\ Body}{=} \int_0^\infty E_{\lambda,b}(\lambda,T) d\lambda \overset{Black\ Body}{=} \sigma T^4;$$

$$E_b \overset{Black\ Body}{=} \pi I_b;$$

Wein's displacement law: $\lambda_{max} T \overset{Black\ Body}{=} 2898\mu mK$

Radiative Properties:

Emissivity: $\varepsilon_{\lambda,\theta} = \dfrac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{\lambda,eb}(\lambda,T)};$

$$\varepsilon_\lambda = \frac{E_\lambda}{E_{\lambda,b}} = \frac{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi,T) cos\theta sin\theta d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,eb}(\lambda,T) cos\theta sin\theta d\theta}; \quad \varepsilon = \frac{\int_0^\infty E_\lambda(\lambda,T)d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda,T)d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda,T)d\lambda}{\sigma T^4}$$

Absorptivity: $\alpha_{\lambda,\theta} = \dfrac{I_{\lambda,i\ absorbed}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)};$ $\quad \alpha_\lambda = \dfrac{G_{\lambda,absorbed}(\lambda)}{G_\lambda(\lambda)} = \dfrac{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i\ absorbed}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi) cos\theta sin\theta d\theta};$

$$\alpha = G_{absorbed}/G; \quad \alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda(\lambda)d\lambda}{\int_0^\infty G_\lambda(\lambda)d\lambda}$$

Reflectivity: $\rho_{\lambda,\theta} = \dfrac{I_{\lambda,i\ reflected}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)};$ $\quad \rho_\lambda = \dfrac{G_{\lambda,reflected}(\lambda)}{G_\lambda(\lambda)} = \dfrac{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i\ reflected}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi) cos\theta sin\theta d\theta};$

$$\rho = G_{reflected}/G; \quad \rho = \frac{\int_0^\infty \rho_\lambda G_\lambda(\lambda)d\lambda}{\int_0^\infty G_\lambda(\lambda)d\lambda}$$

Transmissivity: $\tau_{\lambda,\theta} = \dfrac{I_{\lambda,i\ transmitted}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)};$ $\quad \tau_\lambda = \dfrac{G_{\lambda,transmitted}(\lambda)}{G_\lambda(\lambda)} = \dfrac{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i\ transmitted}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi) cos\theta sin\theta d\theta}$

$$;\tau = G_{transmitted}/G; \tau = \frac{\int_0^\infty \tau_\lambda G_\lambda(\lambda)d\lambda}{\int_0^\infty G_\lambda(\lambda)d\lambda}$$

Semi-transparent Surface: $\alpha_\lambda + \rho_\lambda + \tau_\lambda \overset{semi-transparent}{=} 1;\ \alpha + \rho + \tau \overset{semi-transparent}{=} 1$

Opaque Surface: $\alpha_\lambda + \rho_\lambda \overset{opaque}{=} 1;\ \alpha + \rho \overset{opaque}{=} 1$

Kirchhoff's Law: $\varepsilon_{\lambda,\theta} = \alpha_{\lambda,\theta}$

Gray Surface: $\varepsilon_\lambda \neq f(\lambda);\ \alpha_\lambda \neq f(\lambda);$

Diffuse-Gray Surface: $\varepsilon = \alpha$

View Factor: $F_{ij} = \frac{q_{i-j}}{A_i J_i} = \int_{A_i}\int_{A_j} \frac{cos\theta_i cos\theta_j}{\pi r^2} dA_i dA_j$; $F_{ji} = \frac{q_{j-i}}{A_j J_j} = \int_{A_i}\int_{A_j} \frac{cos\theta_i cos\theta_j}{\pi r^2} dA_i dA_j$

Reciprocity: $A_i F_{ij} = A_j F_{ji}$

Summation: $\sum_{j=1}^{N} F_{ij} = 1$; $F_{ii} \overset{\text{convex surface}}{\underset{\text{plane surface}}{=}} 0$; $F_{ii} \overset{\text{concave surface}}{\neq} 0$

Surface with multiple sub-surfaces: $F_{(j)i} = \frac{\sum_{k=1}^{n} A_k F_{ki}}{\sum_{k=1}^{n} A_k}$

Radiative Exchange: $q_{ij} \overset{\text{Black Body}}{=} A_i F_{ij} \sigma \left(T_i^4 - T_j^4 \right)$;

Diffuse-gray enclosure: $q_i = \frac{E_{bi} - J_i}{(1-\varepsilon_i)/A_i \varepsilon_i}$; $q_i = \sum_{j=1}^{N} \frac{J_i - J_j}{1/A_i F_{ij}}$

Radiation Shields: $q_{12} \overset{\text{Single Shield}}{=} \frac{E_{b,1} - E_{b,2}}{\frac{(1-\varepsilon_1)}{A_1\varepsilon_1} + \frac{1}{A_1 F_{13}} + \frac{(1-\varepsilon_{31})}{A_3 \varepsilon_{31}} + \frac{(1-\varepsilon_{32})}{A_3 \varepsilon_{32}} + \frac{1}{A_3 F_{32}} + \frac{(1-\varepsilon_2)}{A_2 \varepsilon_2}}$

Net Radiation Balance: $q''_{rad} = J - G$

Large isothermal surroundings: $G \overset{\text{large isothermal}}{=} J^{\text{large isothermal}} = \sigma T_{surr}^4$

$$q''_{rad} \overset{\text{large isothermal}}{=} \varepsilon\sigma\left(T_s^4 - T_{surr}^4\right) = h_{rad}\left(T_s - T_{surr}\right)$$

$$h_{rad} = \varepsilon\sigma\left(T_s + T_{surr}\right)\left(T_s^2 + T_{surr}^2\right)$$

Useful Constants

σ = Stefan-Boltzmann's Constant = $5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2\text{K}^4}$

R_u = Universal gas constant = 8,314 J/ kmol·K

Geometry

Cylinder: $A = 2\pi r l$; $V = \pi r^2 l$

Sphere: $A = 4\pi r^2$; $V = \frac{4}{3}\pi r^3$

Triangle: *A= bh/2* *b*: base *h*: height