ME 315
Final Examination Solution
8:00 -10:00 AM
Friday, May 8, 2009

- This is a closed-book, closed-notes examination. *There is a formula sheet at the back.*
- You must turn off all communications devices before starting this exam, and leave them off for the entire exam.
- Please write legibly and show all work for your own benefit. Please show your final answers in the boxes provided.
- State all assumptions.
- Please arrange all your sheets in the correct order. Make sure they are all included.

Name: ___________________________________
Last First

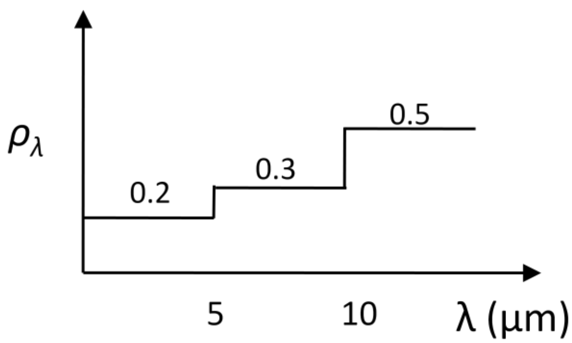
CIRCLE YOUR DIVISION

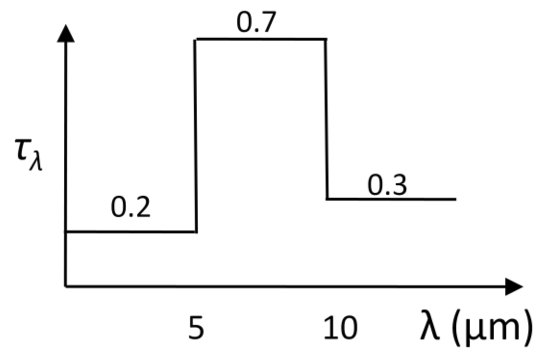
Div. 1 (9:30 am)
Prof. Murthy

Div. 2 (12:30 pm)
Prof. Choi

Problem	Score
1 (30 Points)	
2 (35 Points)	
3 (40 Points)	
4 (45 Points)	
Total (150 Points)	

1. (30 points) A small, *diffuse* plate is initially at 300 K and has radiative properties shown below. The plate is suddenly inserted in a large furnace at T_f = 2000 K.





(a) Sketch the spectral absorptivity of the plate, α_λ.

(b) Calculate total hemispherical absorptivity and emissivity.

α =

ε =

(c) Is this plate approximately gray? Circle one answer and explain your choice.

Gray vs. non-gray

Reason:

(d) After a very long time in the furnace, the radiative properties of this plate could be changed. What do you expect in relative magnitudes of absorptivity and emissivity? Circle one answer and explain your choice.

(i) $\alpha > \varepsilon$
(ii) $\alpha = \varepsilon$
(iii) $\alpha < \varepsilon$
(iv) There will be no change in α or ε.

Reason:

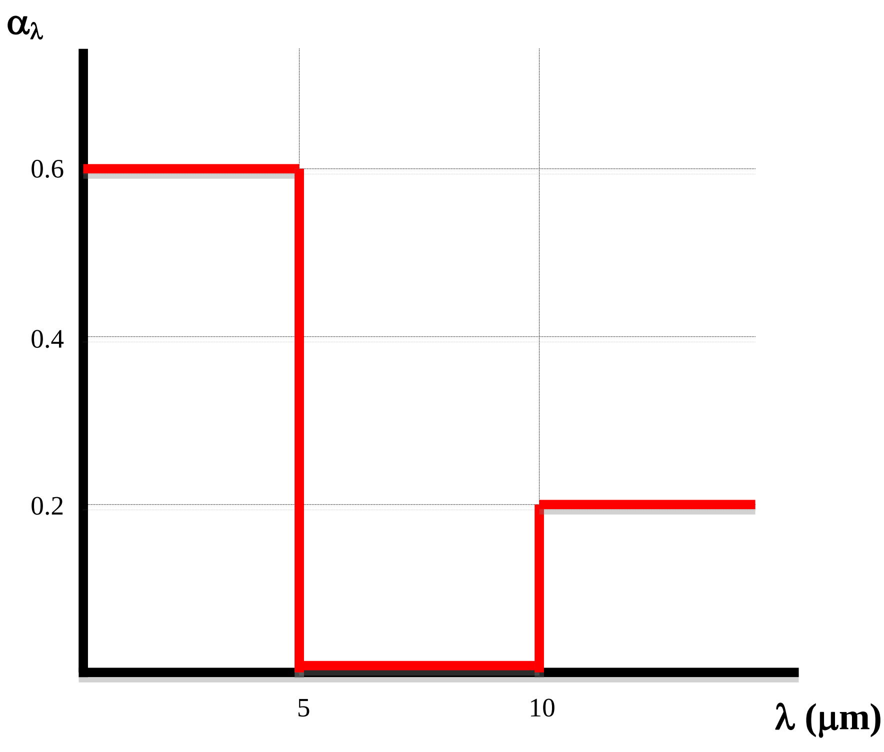
Solution

(a)

$$\alpha_\lambda + \varepsilon_\lambda + \tau_\lambda = 1$$

λ (μm)	ρ_λ	τ_λ	α_λ
0 – 5	0.2	0.2	0.6
5 – 10	0.3	0.7	0
10 - ∞	0.5	0.3	0.2

Therefore,



(b)

Diffuse surface $\Rightarrow \alpha_\lambda = \varepsilon_\lambda$

$T_{surface}$ = 300 K T_{surr} = 2000 K

$\lambda_1 \cdot T_{surr}$ = 5 μm · 2000 K = 10000 μm ·K : $F_{(0 \to \lambda 1)}$ = 0.914199

$\lambda_2 \cdot T_{surr}$ = 10 μm · 2000 K = 20000 μm ·K : $F_{(0 \to \lambda 2)}$ = 0.985602

$\alpha = \alpha_{\lambda 1} \cdot F_{(0 \to \lambda 1)} + \alpha_{\lambda 2} \cdot [F_{(0 \to \lambda 2)} - F_{(0 \to \lambda 1)}] + \alpha_{\lambda 3} \cdot [1 - F_{(0 \to \lambda 2)}]$

$= (0.6) \cdot (0.914199) + 0 + (0.2) \cdot (1 - 0.0985602) = 0.551399$

$$\boxed{\therefore \alpha = 0.551399}$$

$\lambda_1 \cdot T_{surface}$ = 5 μm · 300 K = 1500 μm ·K : $F_{(0 \to \lambda 1)}$ = 0.013759

$\lambda_2 \cdot T_{surface}$ = 10 μm · 300 K = 3000 μm ·K : $F_{(0 \to \lambda 2)}$ = 0.273232

$\varepsilon = \varepsilon_{\lambda 1} \cdot F_{(0 \to \lambda 1)} + \varepsilon_{\lambda 2} \cdot [F_{(0 \to \lambda 2)} - F_{(0 \to \lambda 1)}] + \varepsilon_{\lambda 3} \cdot [1 - F_{(0 \to \lambda 2)}]$

$= (0.6) \cdot (0.013759) + 0 + (0.2) \cdot (1 - 0.273232) = 0.\ 153609$

$$\boxed{\therefore \varepsilon = 0.153609}$$

(c)

$\varepsilon \neq \alpha$ $\boxed{\therefore \text{ Non-gray surface}}$

(d)

As $t \to \infty$, $T_{surface} \to T_{surr}$. Therefore, $\varepsilon \to \alpha$ since both the source and emitter temperature become the same, and therefore the same part of the spectrum determines both.

$$\boxed{\therefore \varepsilon = \alpha}$$

2. (35 points) An engineer seeks to dry a contoured surface using *only* a hot dry air stream, as shown below. A liquid film of water exists on the contoured surface, and dry air at 350 K and a velocity of 10 m/s is passed over it in order to dry it. The temperature of the contoured surface is measured to be 300 K. Furthermore, the engineer also makes measurements of the average Nusselt number and finds that it is of the form:

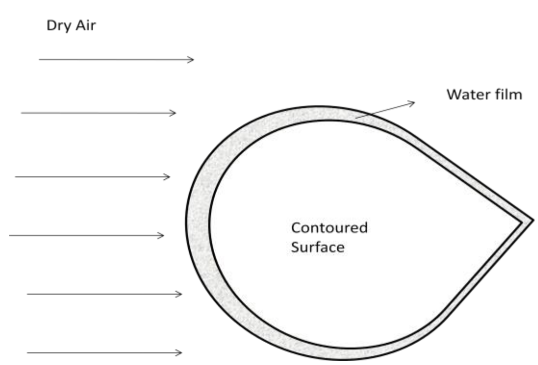
$$\overline{Nu_L} = 0.5 Re_L^{0.6} Pr^{0.4}$$

All properties in the correlation are to be found at the film temperature. Furthermore, the surface area is given to be 2 m^2 and the characteristic length L is given to be 1 m.

You are given the following properties:

Air properties at 350K	**Value**
ρ	0.9950 kg/m^3
μ	208.2×10^{-7} Ns/m^2
C_p	1.009 kJ/kgK
k	0.03 W/mK
Air properties at 300 K	
ρ	1.1614 kg/m^3
μ	184.6×10^{-7} Ns/m^2
C_p	1.007 kJ/kgK
k	0.0263 W/mK
Diffusivity of water vapor in mixture, D_{AB}	
D_{AB}	0.26×10^{-4} m^2/s

The properties of saturated water are given in the formula sheet.



(i) Find the rate of water vapor evaporated from the surface in kg/s.

$\dot{m}_{evap}$ =	kg/s

(ii) Find the required rate of convective heat transfer to the surface in Watts in order for this rate of water vapor evaporation to occur.

q_{conv} = W

Solution

(i)

Assume linear interpolations are valid for air properties.

$$T_{film} = \frac{T_\infty + T_s}{2} = \frac{350 + 300}{2} = 325\ K$$

$$\rho_{air,325K} = \frac{\rho_{300k} + \rho_{350k}}{2} = \frac{1.1614 + 0.9950}{2} = 1.0782\ kg/m^3$$

$$\mu_{air,325K} = \frac{\mu_{300k} + \mu_{350k}}{2} = \frac{184.6 \times 10^{-7} + 208.2 \times 10^{-7}}{2} = 196.4 \times 10^{-7}\ N-s/m^2$$

$$c_{p,air,325K} = \frac{c_{p,air,300k} + c_{p,air,350k}}{2} = \frac{1.007 + 1.009}{2} = 1.008\ kJ/kg-K$$

$$k_{air,325K} = \frac{k_{300k} + k_{350k}}{2} = \frac{0.0263 + 0.03}{2} = 0.02815\ W/m-K$$

$$Re_L = \frac{\rho U L_c}{\mu} \quad ; \quad L_c = 1\ m \quad U = 10\ m/s$$

$$\therefore Re_L = \frac{(1.0782\ kg/m^3)\cdot(10\ m/s)\cdot(1\ m)}{196.4 \times 10^{-7}\ N-s/m^2} = 548981.67$$

$$Pr = \frac{\nu}{\alpha} \qquad \alpha = \frac{k}{\rho \cdot c_p} \qquad \nu = \frac{\mu}{\rho}$$

$$\therefore Pr = \frac{\mu}{\rho}\cdot\frac{\rho \cdot c_p}{k} = \frac{\mu \cdot c_p}{k} = \frac{(196.4 \times 10^{-7}\ N-s/m^2)\cdot(1.008 \times 10^3 J/kg-K)}{0.02815\ W/m-K} = 0.703272$$

$$\overline{Nu_L} = 0.5\ Re_L^{0.6}\ Pr^{0.4} = 0.5\ (548981.67)^{0.6}\ (0.703272)^{0.4} = 1206.57 = \frac{\bar{h}\cdot L_c}{k_{air}}$$

$$\therefore \bar{h} = \frac{\overline{Nu_L}\cdot k_{air}}{L_c} = \frac{(1206.57)\cdot(0.02815\ W/m-K)}{(1\ m)} = 33.9651\ W/m^2-K$$

$$\frac{h}{h_m} = \frac{k_{air}}{D_{AB}\ Le^n} \quad ; \quad n = 0.4$$

$$Le = \frac{\alpha}{D_{AB}} = \frac{k}{\rho \cdot c_p} \cdot \frac{1}{D_{AB}} = \frac{0.02815\,W/m-K}{(1.0782\ kg/m^3)\cdot(1.008\times10^3\,J/kg-K)} \cdot \frac{1}{(0.26\times10^{-4}\ m^2/s)} = 0.996197$$

$$\frac{h}{h_m} = \frac{33.9651\,W/m^2-K}{h_m} = \frac{0.02815\,W/m-K}{(0.26\times10^{-4}\ m^2/s)\cdot(0.996197)^{0.4}}$$

$$\therefore h_m = 0.031323\,m/s$$

$$\dot{m}_{evap} = h_m \cdot A \cdot [\rho_{A,s} - \rho_{A,\infty}] \quad ; \quad A = 2\ m^2 \quad \rho_{A,\infty} = 0$$

$$\rho_{A,s} = \rho_{water\ vapor,300K} = \frac{1}{v_{water\ vapor,300K}} = \frac{1}{39.13\,m^3/kg} = 0.025556\ kg/m^3$$

$$\therefore \dot{m}_{evap} = (0.031323\,m/s)\cdot(2\ m^2)\cdot(0.025556\ kg/m^3) = 0.001601\,kg/s$$

$$\boxed{\therefore \dot{m}_{evap} = 0.001601\,kg/s}$$

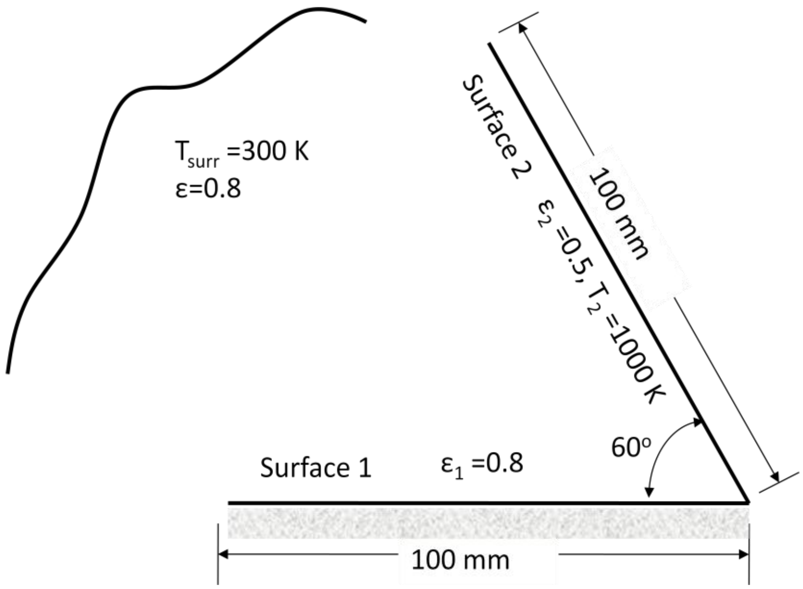
(ii)

$$q_{evap} = \dot{m}_{evap} \cdot h_{fg} \quad ; \quad h_{fg,water,300K} = 2438\ kJ/kg$$

$$\therefore q_{evap} = (0.001601)\cdot(2438\times10^3\,J/kg) = 3903.24\,W$$

$$\boxed{\therefore 3903.24 \text{ W is required}}$$

3. (40 points) Consider two inclined diffuse-gray plates of area A_1 and A_2 respectively, as shown below. The plates are infinitely long into the page and are at an angle of 60^o, as shown. The two plates are exposed to a large surroundings at a uniform temperature of 300 K and an emissivity ε =0.8. Surface 1 is insulated, and has an emissivity of ε_1 =0.8. Surface 2 is at a temperature T_2 =1000 K, and has an emissivity of ε_2=0.5.



(i) Find the view factor F_{12}

F_{12} =

(ii) Draw a radiation circuit describing the radiation exchange between the surfaces, showing the <u>symbolic form</u> of all relevant resistances and potentials.

Surroundings

Surface 1

Surface 2

(iii) Find the heat transfer rate q_2 leaving surface 2 in W/m.

q_2 = W/m

(iv) Find the irradiation G_2 on surface 2 in W/m^2.

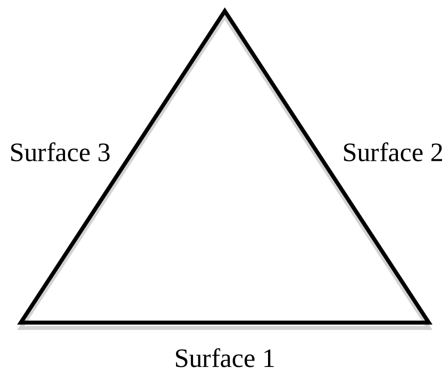
G_2 = W/m^2

(v) Find the net radiation heat transfer rate from surfaces 1 and 2 to the surroundings in W/m.

Heat transfer rate = W/m

Solution

(i)



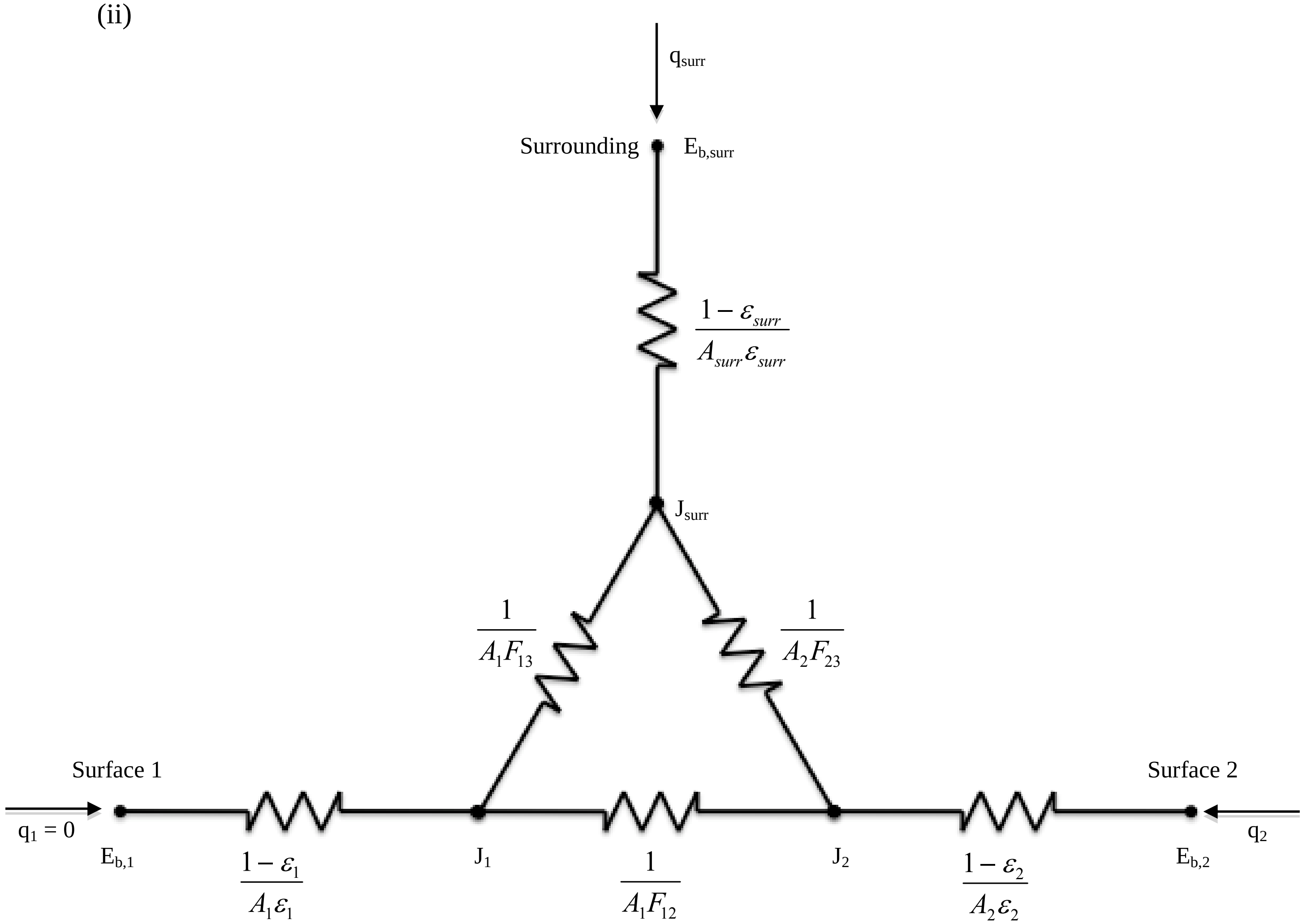
$$F_{11} + F_{12} + F_{13} = 1$$

$$F_{11} = 0$$

By symmetry, $F_{12} = F_{13}$

$$\boxed{\therefore F_{12} = F_{13} = 0.5}$$

(ii)



(iii)

Surrounding is large →$J_{surr} = E_{b,surr}$

$$q_2 = \frac{E_{b,2} - E_{b,surr}}{\frac{1-\varepsilon_2}{A_2\varepsilon_2} + \frac{1}{A_2F_{23} + \left[\frac{1}{A_1F_{12}} + \frac{1}{A_1F_{13}}\right]^{-1}} + \cancel{\frac{1-\varepsilon_{surr}}{A_{surr}\varepsilon_{surr}}}}$$

$$q_2^{'} = \frac{E_{b,2} - E_{b,surr}}{\frac{1-\varepsilon_2}{L\cdot\varepsilon_2} + \frac{1}{L\cdot F_{23} + \left[\frac{1}{L\cdot F_{12}} + \frac{1}{L\cdot F_{13}}\right]^{-1}}}$$

$$= \frac{(5.67\times10^{-8}W/m^2-K^4)\cdot(1000^4-300^4)K^4}{\frac{1-0.5}{(0.1\,m)\cdot(0.5)} + \frac{1}{(0.1\,m)\cdot(0.5) + \left[\frac{1}{(0.1\,m)\cdot(0.5)} + \frac{1}{(0.1\,m)\cdot(0.5)}\right]^{-1}}}$$

$$= 2410.317\ W/m$$

$$\boxed{\therefore q_2^{'} = 2410.317\ W/m}$$

(iv)

$$q_2 = \frac{E_{b,2} - J_2}{\frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$q_2^{'} = \frac{E_{b,2} - J_2}{\frac{1-\varepsilon_2}{L\cdot\varepsilon_2}} = \frac{(5.67\times10^{-8}W/m^2-K^4)\cdot(1000\,K)^4 - J_2}{\frac{1-0.5}{(0.1\,m)\cdot(0.5)}} = 2410.317\ W/m$$

$$\therefore J_2 = 32596.83\ W/m^2$$

$q_2 = A_2 \cdot (J_2 - G_2)$

$q_2^{'} = L \cdot (J_2 - G_2)$

$2410.317\ W/m = (0.1\ m) \cdot (32596.83\ W/m^2 - G_2)$

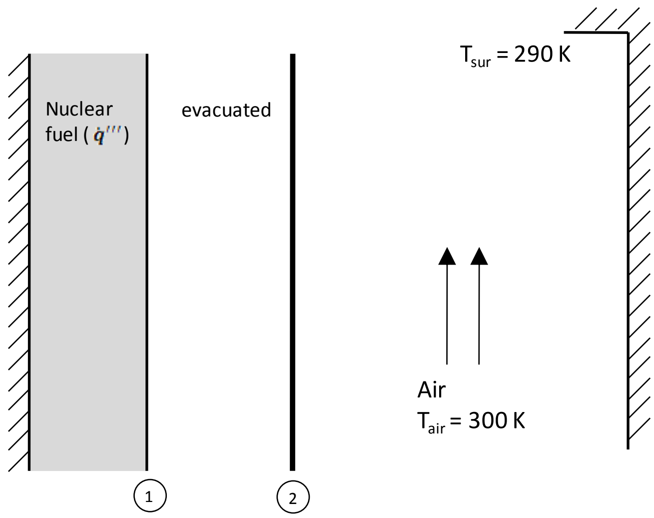
$$\boxed{\therefore G_2 = 8493.66\ W/m^2}$$

(v)

$q_{1\&2 \to surr}^{'} = q_2^{'} \quad as \quad q_1^{'} = 0$

$$\boxed{\therefore q_{1\&2 \to surr}^{'} = 2410.317\ W/m}$$

4. (45 points) A vertically-oriented large nuclear fuel solid with a thickness of t = 0.1 m generates uniform volumetric heat at $\dot{q}'''$ (W/m^3). One of its surfaces is perfectly insulated, while the other is facing a thin plate with the medium between them evacuated. Both materials are diffuse-gray with ε_1=0.8 and ε_2=0.01, and are placed in a large room at 290 K. The other side of the thin plate is exposed to free convection by air at T_∞ = 300 K as shown below. The plate temperature is measured to be T_2 = 400 K. You are given air properties evaluated at 350 K: ν =20.92×10^{-6} m^2/s, α = 29.9 ×10^{-6} m^2/s, β = 2.725×10^{-3} K^{-1}, and k_{air} = 30.0×10^{-3} W/mK. You may assume that both sides of plate 2 have the same emissivity.



(a) Determine net heat flux between surfaces 1 and 2.
You may assume a characteristic length, L=1 m, for natural convection.

q''_{12} = W/m^2

(b) Find the temperature of surface 1.

T_1 = K

(c) Determine the volumetric heat generation rate.

$\dot{q}$= W/m^3

<u>Solution</u>

(a)

$$\overline{Nu_L} = \left\{ 0.825 + \frac{0.387\ Ra_L^{1/6}}{\left[1 + (0.492 / Pr)^{9/16}\right]^{8/27}} \right\}^2$$

$$Ra = \frac{g\,\beta\,(T_s - T_\infty)\,L_c^3}{\nu\,\alpha} = \frac{(9.81\ m/s)\cdot(2.725\times10^{-3}K^{-1})\cdot(400-300)K\cdot(1\ m)^3}{(20.92\times10^{-6}m^2/s)\cdot(29.9\times10^{-6}m^2/s)} = 4.27369\times10^9$$

$$Pr = \frac{\nu}{\alpha} = \frac{20.92\times10^{-6}m^2/s}{29.9\times10^{-6}m^2/s} = 0.699666$$

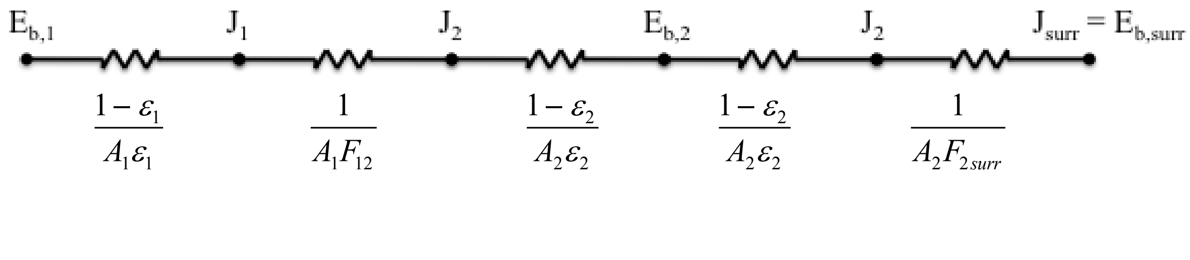
$$\therefore \overline{Nu_L} = \left\{ 0.825 + \frac{0.387\cdot(4.27369\times10^9)^{1/6}}{\left[1 + (0.492/0.699666)^{9/16}\right]^{8/27}} \right\}^2 = 192.644 = \frac{\bar{h}\cdot L_c}{k_{air}} \quad ; \quad k_{air,350K} = 30.0\times10^{-3}\ W/m-K$$

$$\therefore \bar{h} = \frac{\overline{Nu_L}\cdot k_{air}}{L_c} = \frac{(192.644)\cdot(30.0\times10^{-3}\ W/m-K)}{1\ m} = 5.77932\ W/m^2-K$$

$$q''_{12} = q''_{conv} + q''_{rad}$$

$$q''_{conv} = \bar{h}_{conv}\cdot(T_2 - T_\infty) = (5.77932\ W/m^2-K)\cdot(400-300)K = 577.932\ W/m^2$$

Surrounding is large $\rightarrow J_{surr} = E_{b,surr}$



$$q''_{rad} = \frac{E_{b,2} - E_{b,surr}}{\frac{1-\varepsilon_2}{\varepsilon_2} + \frac{1}{F_{2surr}}} \quad ; \quad F_{2surr} = 1$$

$$q''_{rad} = \frac{\sigma \cdot (T_2^4 - T_{surr}^4)}{\frac{1-\varepsilon_2}{\varepsilon_2} + \frac{1}{F_{2surr}}} = \frac{(5.67 \times 10^{-8} W / m^2 - K^4) \cdot (400^4 - 290^4) K^4}{\frac{1-0.01}{0.01} + 1} = 10.5049 \ W / m^2$$

or

For a small surface in a large surrounding (based upon Kirchhoff's law),

$$q''_{rad} = \varepsilon_2 \cdot \sigma \cdot (T_2^4 - T_{surr}^4) = (0.01) \cdot (5.67 \times 10^{-8} W / m^2 - K^4) \cdot (400^4 - 290^4) K^4 = 10.5049 \ W / m^2$$

$$q''_{12} = q''_{conv} + q''_{rad} = 577.932 \ W / m^2 + 10.5049 \ W / m^2 = 588.437 \ W / m^2$$

$$\boxed{\therefore q''_{12} = 588.437 \ W / m^2}$$

(b)

$$q''_{12} = \frac{E_{b,1} - E_{b,2}}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2}} \quad ; \quad F_{12} = 1$$

$$q''_{12} = \frac{\sigma \cdot (T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2}} = \frac{(5.67 \times 10^{-8} W / m^2 - K^4) \cdot (T_1^4 - 400^4) K^4}{\frac{1-0.8}{0.8} + 1 + \frac{1-0.01}{0.01}} = 588.437 \ W / m^2$$

$$\boxed{\therefore T_1 = 1016.11 \ K}$$

(c)

$$q''_{12} = \dot{q} \cdot t \quad ; \quad t = 0.1\, m$$

$$588.437\, W / m^2 = \dot{q} \cdot (0.1\, m)$$

$$\boxed{\therefore \dot{q} = 5884.37\, W / m^3}$$